Swing leg retraction helps biped walking stability

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Abstract— In human walking, the swing leg moves backward just prior to ground contact, i.e. the relative angle between the thighs is decreasing. We hypothesized that this swing leg retraction may have a positive effect on gait stability, because similar effects have been reported in passive dynamic walking models, in running models, and in robot juggling. For this study, we use a simple inverted pendulum model for the stance leg. The swing leg is assumed to accurately follow a time-based trajectory. The model walks down a shallow slope for energy input which is balanced by the impact losses at heel strike. With this model we show that a mild retraction speed indeed improves stability, while gaits without a retraction phase (the swing leg keeps moving forward) are consistently unstable. By walking with shorter steps or on a steeper slope, the range of stable retraction speeds increases, suggesting a better robustness. The conclusions of this paper are therefore twofold; (1) use a mild swing leg retraction speed for better stability, and (2) walking faster is easier.

Index Terms— Swing leg trajectory, dynamic walking, biped, swing leg retraction

I. INTRODUCTION

In human walking, the swing leg moves forward to maximal extension and then it moves backward just prior to ground contact. This backward motion is called 'swing leg retraction'; the swing foot stops moving forward relative to the floor and it may even slightly move backward. In biomechanics it is generally believed that humans apply this effect (also called 'ground speed matching') in order to reduce heel strike impacts. However, we believe that there is a different way in which swing leg retraction can have a positive effect on stability; a fast step (too much energy) would automatically lead to a longer step length, resulting in a larger energy loss at heel strike. And conversely, a slow step (too little energy) would automatically lead to a shorter step length, resulting in less heel strike loss. This could be a useful stabilizing effect for walking robots.

The primary motivation to study swing leg retraction comes from our previous work on passive dynamic walking [15], [4], [18]. Passive dynamic walking [11] robots can demonstrate stable walking without any actuation or control. Their energy comes from walking downhill and their stability results from the natural dynamic pendulum motions of the legs. Interestingly, such walkers possess two equilibrium gaits, a 'long period gait' and a 'short period gait' [12], [5]. The long period gait has a retraction phase, and this gait is the only one that can be stable. The short period gait has no swing leg retraction. This solution is usually dismissed, as it never provides passively stable gaits.

More motivation stems from work on juggling and running, two other underactuated dynamic tasks with intermittent con-

tact. The work on juggling [14] featured a robot that had to hit a ball which would then ballistically follow a vertical trajectory up and back down until it was hit again. The research showed that stable juggling occurs if the robot hand is following a well chosen trajectory, such that its upward motion is decelerating when hitting the ball. The stable juggling motion required no knowledge of the actual position of the ball. We feel that the motion of the hand and ball is analogous to that of the swing leg and stance leg, respectively. Also analogous is the work on a simple point-mass running model [16]. It was shown that the stability of the model was significantly improved by the implementation of a retraction phase in the swing leg motion. It has been suggested [13] that this effect also appears in walking.

In this paper we investigate the stabilizing influence of the swing leg retraction speed just prior to heel strike impact. We use a Poincaré map analysis of a simple point-mass model (Section II). The results are shown in Section III, including a peculiar asymmetric gait that is more stable than any of the symmetric solutions. Section IV reports that the results are also valid for a model with a more realistic mass distribution. The discussion and conclusion are presented in Sections V and VI.

II. SIMULATION MODEL AND PROCEDURE

The research in this paper is performed with an inverted pendulum model consisting of two straight and massless legs (no body) and a single point mass at the hip joint, see Fig. 1. Straight legged ('compass gait') models are widely used as an approximation for dynamic walking [7], [6], [9], [8], [5].
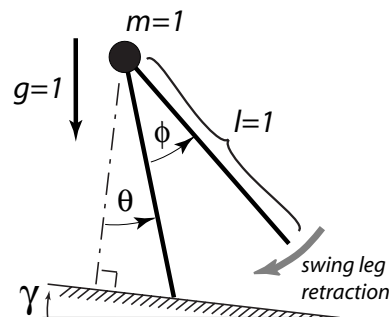

Fig. 1. Our inverse pendulum model, closely related to the 'Simplest Walking Model' of [5].

A. Stance leg

The stance leg is modeled as a simple inverted pendulum of length 1 (m) and mass 1 (kg) (Fig. 1). It undergoes

gravitational acceleration of 1 (m/s^2) at an angle of γ following the common approach to model a downhill slope in passive dynamic walking. It has one degree of freedom denoted by θ, see Fig. 1. The foot is a point and there is no torque between the foot and the floor. The equation of motion for the stance leg is:

$$\ddot{\theta} = \sin(\theta - \gamma) \qquad (1)$$

which is integrated forward using a 4th order Runge-Kutta integration routine with a time step of 0.001 (s).

B. Swing leg

The swing leg is modeled as having negligible mass. Its motion does not affect the hip motion, except at the end of the step where it determines the initial conditions for the next step. A possible swing leg motion is depicted in Fig. 2 with a dashed line. As is standard with compass gait walkers, we ignore the brief but inevitable foot scuffing at midstance.
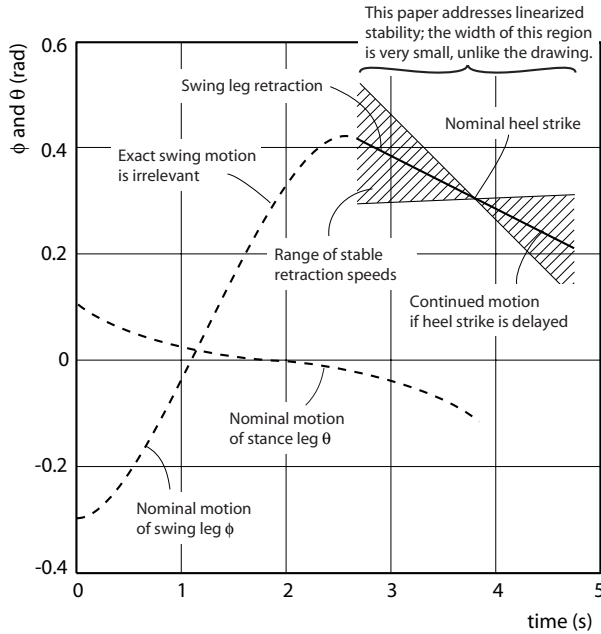

Fig. 2. The figure shows an example trajectory and it shows the stable region (hatched area) for retraction speeds. Only the swing leg trajectory around heel strike is important; the swing leg by itself has no dynamic effect on the walking motion other than through foot placement.

The swing leg motion at the end of the step is a function of time which we construct in two stages. First we choose at which relative swing leg angle ϕ (See Fig. 1) heel strike should take place, ϕ_{lc}. This is used to find a limit cycle (an equilibrium gait), which provides the appropriate step time, T_{lc}. Second, we choose a retraction speed $\dot{\phi}$. The swing leg angle ϕ is then created as a linear function of time going through the point $\{T_{lc}, \phi_{lc}\}$ with slope $\dot{\phi}$.

C. Transition

The simulation transitions from one step to the next when heel strike is detected, which is the case when $\phi = -2\theta$. An additional requirement is that the foot must make a downward

motion, resulting in an upper limit for the forward swing leg velocity $\dot{\phi} < -2\dot{\theta}$ (note that $\dot{\theta}$ is always negative in normal walking, and note that swing leg *retraction* happens when $\dot{\phi} < 0$). In our simulation, we use a third order polynomial to interpolate between two simulation data points in order to accurately find the exact time and location of heel strike.

The transition results in an instantaneous change in the velocity of the point mass at the hip, see Fig. 3. All of the velocity in the direction along the new stance leg is lost in collision, the orthogonal velocity component is retained. This results in the following transition equation:

$$\dot{\theta}^+ = \dot{\theta}^- \cos\phi \qquad (2)$$

in which $\dot{\theta}^-$ indicates the rotational velocity of the old stance leg, and $\dot{\theta}^+$ that of the new stance leg. At this instant, θ and ϕ flip sign (due to relabeling of the stance and swing leg). Note that in Eq. 2 ϕ could equally well be replaced with 2θ.
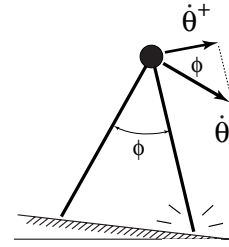

Fig. 3. At heel strike the velocity of the point mass is redirected. All velocity along the length of the new stance leg is lost, so that $\dot{\theta}^+ = \dot{\theta}^- \cos\phi$.

The instant of transition is used as the start of a new step for the swing leg controller; in the case that a disturbance would make step n last longer than usual, then the start of the swing leg trajectory for step $n + 1$ is postponed accordingly. Thus, although the swing leg motion is a time-based trajectory independent of the state of the stance leg, it does depend on foot contact information for the start of the trajectory.

D. Finding limit cycles

The model exhibits a limit cycle if the initial conditions of step $n+1$ are exactly equal to those of step n. For this model, the only independent initial conditions are the stance leg angle (θ) and its velocity ($\dot{\theta}$). The motion of the swing leg is fully trajectory controlled; we assume that it accurately follows the desired trajectory.

The first step of finding a limit cycle is to guess initial conditions that are near a hypothesized limit cycle, either through experience or by starting from a known limit cycle for similar parameter values. This provides initial guess $\{\theta_0, \dot{\theta}_0\}$. Then a Newton-Raphson gradient-based search algorithm is applied on the difference between $\{\theta_0, \dot{\theta}_0\}$ and the initial conditions of the next step, which we obtain through forward simulation. The search algorithm terminates when the norm of the difference is smaller than $1e^{-9}$. The search algorithm uses a numerically obtained gradient $\mathbf{J}$ which is also used for the stability analysis as described in the next paragraph. Note that this procedure can find unstable as well as stable limit cycles.

E. Poincaré stability analysis

The stability of the gait is analyzed with the Poincaré mapping method, which is a linearized stability analysis of the equilibrium gait. The Poincaré mapping method perturbs the two independent initial conditions and monitors the effect on the initial conditions for the subsequent step. Assuming linear behavior, the relation between the original perturbations at step n and the resulting perturbations at step $n+1$ is captured in the Jacobian matrix $\mathbf{J}$, as in:

$$\left[\begin{array}{c} \Delta\theta_{n+1} \\ \Delta\dot\theta_{n+1} \end{array} \right] = \mathbf{J} \left[\begin{array}{c} \Delta\theta_n \\ \Delta\dot\theta_n \end{array} \right] \tag{3}$$

If the magnitudes of both of the eigenvalues of $\mathbf{J}$ are smaller than 1, then errors decay step after step and the gait is stable. The eigenvalues could have imaginary parts, as was the case for the passive model [5], but in the model with trajectory control they have no imaginary parts.

F. Nominal gait

We have chosen the steady passive gait with a slope of $\gamma = 0.004$ rad as a basis of reference for walking motions. For the passive model, γ is the only parameter, and for $\gamma = 0.004$ there exists only one unstable equilibrium gait (the 'short period solution') and one stable equilibrium gait (the 'long period solution'). We use the latter as our reference gait. The initial conditions and the step time of that gait are listed in Table I.

TABLE I

INITIAL CONDITIONS AND STEP TIME FOR STEADY WALKING OF THE PASSIVE WALKING MODEL AT A SLOPE OF $\gamma = 0.004$ RAD. NOTE THAT THE INITIAL VELOCITY FOR THE SWING LEG $\dot\phi$ IS IRRELEVANT FOR OUR STUDY, BECAUSE THE SWING LEG MOTION IS FULLY TRAJECTORY CONTROLLED.

θ	0.1534 rad
$\phi = -2\theta$	0.3068 rad
$\dot\theta$	-0.1561 rad/s
$\dot\phi$	0.0073 rad/s
step time	3.853 s

III. RESULTS

A. Nominal limit cycle

For the given gait of Table I on a given slope of $\gamma = 0.004$ rad, the only parameter that we can vary is the retraction speed $\dot\phi$; how fast is the swing leg moving rearward (or forward, depending on the sign) just prior to heel strike. This parameter has no influence on the nominal gait, but it does change the behavior under small disturbances as captured by the Poincaré stability analysis. Note that the swing leg will follow a fixed time-based trajectory independent of the disturbances on the initial conditions.

The stability results are shown in Fig. 4; the eigenvalues of $\mathbf{J}$ on the vertical axis versus the retraction speed $\dot\phi$ at the horizontal axis. A positive value for $\dot\phi$ indicates that the swing leg keeps moving forward, a value of zero means that the swing leg is being held at the heel strike value $\phi = 0.3068$
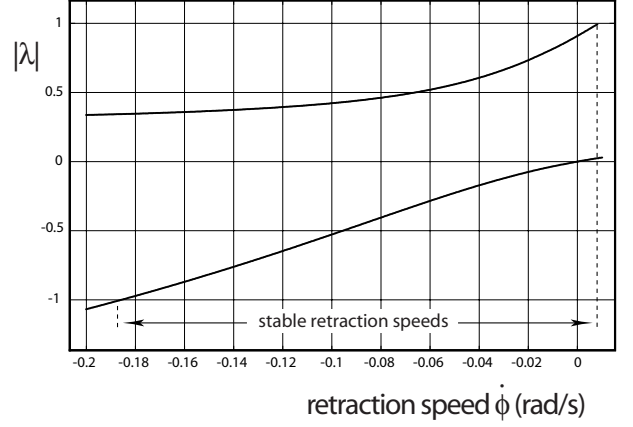

Fig. 4. The graph shows the range of stable retraction speeds. The stability is indicated with eigenvalues of $\mathbf{J}$ on the vertical axis. The walking motion is stable if the eigenvalues are between -1 and 1. The horizontal axis contains the retraction speed $\dot\phi$. A positive value for $\dot\phi$ indicates that the swing leg keeps moving forward, a value of zero means that the swing leg is being held at the heel strike value $\phi = 0.3068$ and so the foot comes down vertically. A negative value for $\dot\phi$ indicates the presence of a retraction phase. Stable gaits exist for retraction speeds between -0.18 and +0.009 rad/s. In words, this graph shows that relative hip angle should be decreasing around the instant that heel strike is expected.

and so the foot comes down vertically. A negative value for $\dot\phi$ indicates the presence of a retraction phase.

Fig. 4 shows that stable gaits emerge for retraction speeds of $-0.18 < \dot\phi < 0.009$, and that the fastest convergence will be obtained with $\dot\phi = -0.09$ since the maximum absolute eigenvalue is minimal at that point. In other words, swing leg retraction is not necessary for stable walking, but errors will definitely decay faster if the swing leg motion does include a retraction phase. Also, even though some forward swing leg motion ($\dot\phi > 0$) is allowable, this would make the walker operate very close to instability characterized by a rapidly growing eigenvalue.

An interesting data point is $\dot\phi = 0$. One of the eigenvalues there is zero ($\lambda_1 = 0$); any errors in the initial condition θ will be completely eliminated within one step, because it is certain that the step will end with $\phi = 0.3068$ as the swing leg will be held at that value until heel strike occurs. The other eigenvalue can also be calculated manually. Although the derivation is a little more involved, the result simply reads $\lambda_2 = \cos^2\phi$. A system with $\dot\phi = 0$ is dynamically equivalent to the 'Rimless Wheel' [10], [2].

B. The influence of step length

We repeat the stability analysis of the previous subsection still using the same slope $\gamma = 0.004$ but varying the step length of the gait. For example, we chose a much faster and shorter step starting with $\theta_0 = -0.1317$. The limit cycle belonging to that value starts with $\dot\theta_0 = -0.17$ while the step time is $2s$ (this is what we tuned for). The resultant eigenvalues are shown in Fig. 5. Clearly there is a much larger range of stable retraction speeds, at the cost of slightly slower convergence. The optimal retraction speed is $\dot\phi = -0.71$ which produces eigenvalues of 0.8, i.e. errors decrease 20% per step.
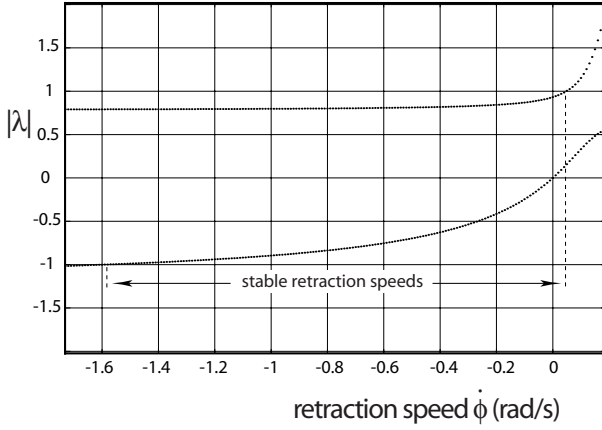
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Fig. 5. A faster walking motion (T = 2 s) leads to a much larger range of stable retraction speeds.

Fig. 7. Effect of floor slope on the range of stable retraction speeds for a step length of $\phi = 0.3068$. The gray area shows that steeper slopes (and thus faster steps) are better. The graph also shows the retraction speed with the smallest eigenvalues.

Fig. 6 provides an overview of the effect of step length on stable range of retraction speeds and the optimal retraction speed and accompanying eigenvalues. The stable range decreases for larger step lengths until it is zero for $\phi = 0.3155$. Above that value no limit cycles exist, because the energy supply from gravity cannot match the large impact losses. Near this value, the walker is operating dangerously close to a state in which it does not have sufficient forward energy to pass the apex at midstance, resulting in a fall backward. The main conclusion from this graph is that it is wise to operate well away from a fall backward, i.e. walk fast!
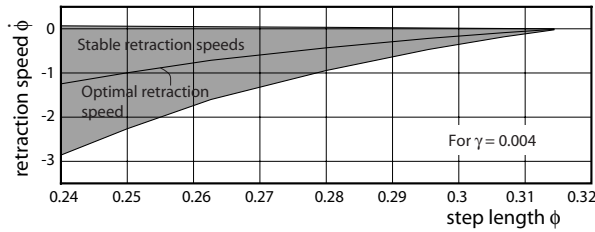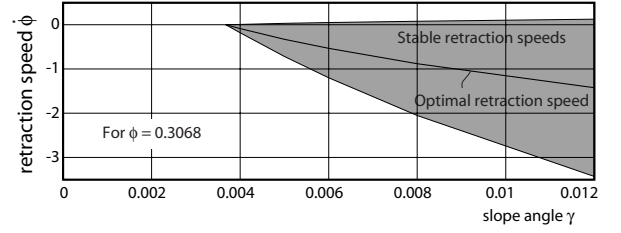

Fig. 6. Effect of step length on the range of stable retraction speeds for a floor slope of $\gamma = 0.004$. The gray area shows that shorter steps are better. The graph also shows the retraction speed with the smallest eigenvalues.

C. The influence of the slope angle

The influence of the slope angle is similar to the that of the step length. A steeper slope provides more energy input and thus the resultant gait is faster, an effect similar to decreasing the step length. Fig. 7 shows how the range of stable retraction speeds depends on the slope angle, for the nominal step length $\phi = 0.3068$.

D. Asymmetric gait is more stable

The results in the previous sections show that the retraction speed can change the eigenvalues, but it doesn't ever seem to obtain eigenvalues of all zeros. The explanation is simple; the system uses one control input (the swing leg angle ϕ at heel strike) with which it must stabilize two states (stance leg angle θ and its velocity $\dot{\theta}$). Although this mismatch cannot result

in 'deadbeat control' (all eigenvalues zero) within a single step, it should be possible to find a deadbeat controller for a succession of two steps. Here we present such a controller for our nominal situation of $\gamma = 0.004$ and $\phi = 0.3068$.

The previous solutions were all symmetric, i.e. the trajectory of the swing leg was the same each step. We found that a purposefully induced asymmetric gait can result in eigenvalues of all zeros. The swing leg trajectories (one for leg 1 and another for leg 2) are shown in Fig. 8. Leg 1 always goes to $\phi = 0.3068$ and does not have a retraction phase (i.e. the foot comes straight down). Leg 2 always follows a trajectory with a retraction speed of $\dot{\phi} = -0.125245$. If one calculates the eigenvalues over a series of steps of Leg 1 - Leg 2 - Leg 1 (or more), all eigenvalues are zero. This means that any disturbance will be completely eliminated after three steps of this asymmetric gait.
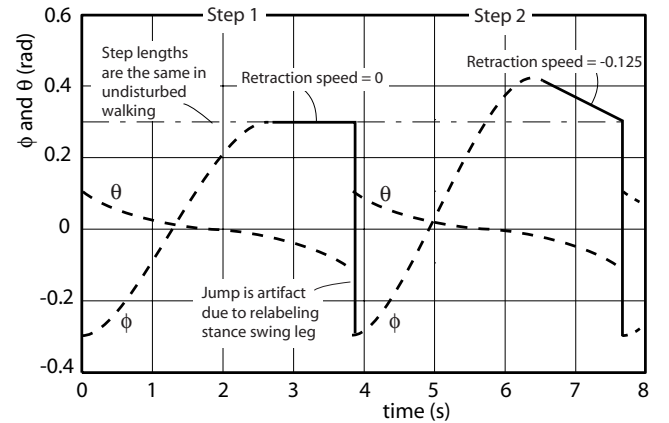

Fig. 8. An asymmetric gait can lead to two-step deadbeat control, i.e. to two eigenvalues of zero.

Preliminary research suggests that this 'deadbeat' solution even pertains to large errors, although that requires a non-constant retraction speed. We intend to investigate such large-error solutions in the future. Note that in steady gait, the motion of the swing legs is asymmetric (one retracts and the other does not) but the step length and step time are still symmetric. Also note that due to the asymmetry, the eigenvalues cannot be divided up into 'one-step' eigenvalues.

IV. Automated optimization on a more realistic mass distribution also results in swing leg retraction

The theoretical results in the previous section are based on a point mass model for walking. One of the main assumptions there is that the mass of the swing leg is negligible. Obviously, in real walking systems this is not true. The reaction forces and torques from a non-massless swing leg will influence the walking motion. In our experience, the main effect is energy input. Driving the swing leg forward also pumps energy into the gait. A benefit of this effect is that a downhill slope is no longer required, but the question is whether it breaks the stabilizing effect of swing leg retraction. Or, even if it does still help stability, whether the stability gain outweighs the added energetic cost for accelerating the swing leg. We study these questions using a model with a more realistic mass distribution, based on a prototype we are currently experimenting with [1] (Fig 9). The swing leg trajectory is optimized both for stability and for efficiency.
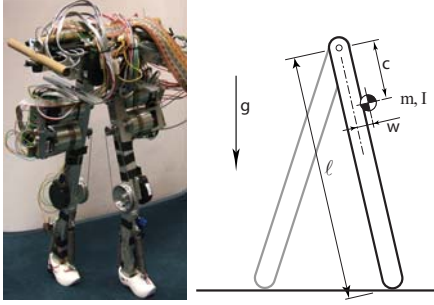

Fig. 9. Our current experimental biped and a straight-legged model with mass distribution based on the experimental robot, see Table II.

TABLE II
Parameter values for a model with a more realistic mass distribution in the legs.

gravity	g	9.81 m/s^2
floor slope	γ	0 rad
leg length	l	0.416 m
leg mass	m	3 kg
vertical position CoM	c	0.027 m
horizontal position CoM	w	0 m
moment of inertia	I	0.07 kgm^2

The model (Fig. 9) has the same topology as our initial model (Fig. 1). However, instead of a single point mass at the hip, the model now has a distributed mass over the legs, see Table II. The swing leg follows the desired trajectory with reasonable accuracy using a PD controller on the hip torque:

$$T = k(\phi - \phi_{des}(t)) + d\dot{\phi} \qquad (4)$$

with gain values $k = 1500$ and $d = 10$. The swing leg trajectory is parameterized with two knot points defining the start and the end of the retraction phase (Fig. 10). The trajectory before the first knot point is a third order spline which starts with the actual swing leg angle and velocity just after heel strike. The trajectory between the two knot points is a straight line. This parametrization provides the optimizer

with ample freedom to vary the retraction speed, the nominal step length, and the duration of the retraction phase.
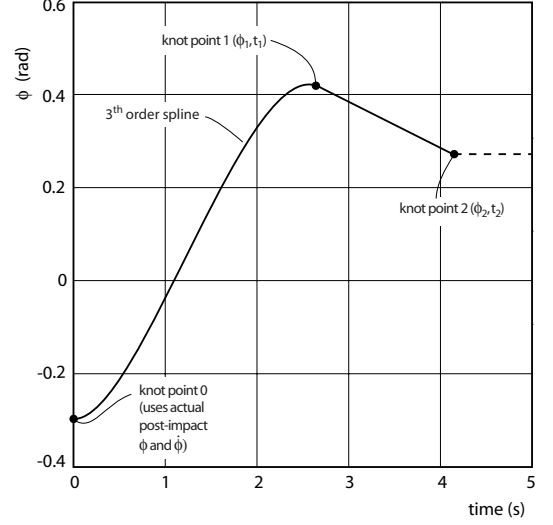

Fig. 10. The desired trajectory for the swing leg is parameterized with two knot points (four parameters).

The optimization procedure is set up as follows. The model is started with manually tuned initial conditions, after which a forward dynamic simulation is run for 20 simulated seconds. The resulting motion is then rated for average velocity and efficiency:

$$cost = \Sigma_{20s}(w_T T^2 + w_v(\dot{x} - \dot{x}_{des})^2 + w_v \dot{y}^2) \qquad (5)$$

with the weight for the torque penalty $w_T = 0.1$, the weight for the velocity penalty $w_v = 1$, and the desired forward velocity $\dot{x}_{des} = 0.3$, summed over a trial interval of 20 s. During the motion, random noise with uniform distribution is added to the hip torque. In this way, the model is indirectly rated for robustness; if the noise makes the walker fall, then the resultant average walking velocity is low and so the penalty for not achieving $\dot{x}_{des}$ is high.

A simulated annealing procedure optimized the cost function of Eq. (5) by adjusting the four knot point parameters for the swing leg trajectory. Fig. 11 shows that for a wide range of noise levels and initialization values, the optimization procedure consistently settles into gaits a retraction phase. These results fully concur with the theoretical results for the model with massless legs. Therefore, we conclude that the analysis is valid and the conclusions hold: walk fast and use a mild retraction speed.

V. Discussion

This work was limited to a small error analysis. Our future work consists of analyzing the effect of the swing leg motion when under large disturbances, i.e. an analysis of the basin of attraction must be added to the present linearized stability analysis [18]. We also intend to investigate the effects of an increased model complexity by adding knees, feet, and an upper body.

Observations of the gait of our previously developed passive-based walking robots [17] show that they all walk with
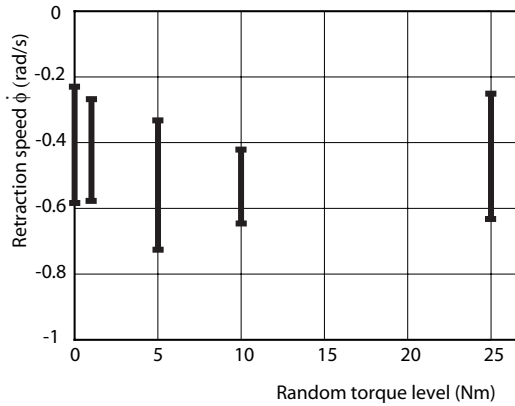
Fig. 11. Retraction speed $\dot{\phi}$ as a function of the level of random torque disturbances. A negative value for $\dot{\phi}$ indicates the presence of a retraction phase. The results are obtained with an optimization algorithm which was initialized with trajectories *with* a retraction phase for some runs and *without* one for others. Irrespective of the initialization and the level of disturbances, the optimization always settles into a trajectory *with* a retraction phase, i.e. $\dot{\phi}$ is always negative just prior to foot contact.

a retraction phase in the swing leg motion. For illustration, Fig 12 shows the motion of the swing foot with respect to the floor, as measured with a motion capture system on our most recent prototype Denise [3]. The measurements (an average of over 150 steps) show that there exists a clear retraction phase just prior to heel strike.
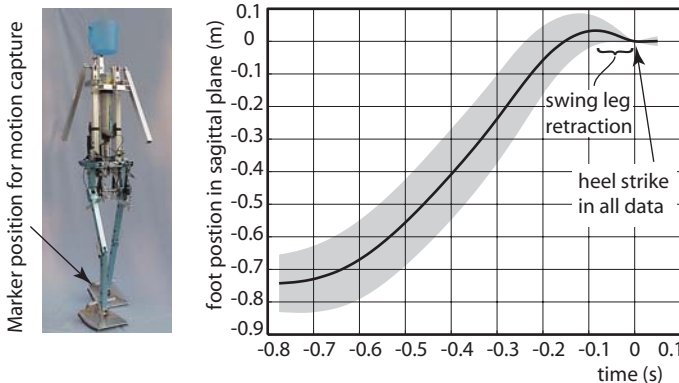

Fig. 12. Left: our prototype Denise. Right: the graph shows the clear existence of a retraction phase. The graphs shows a measurement of the motion of Denise's heel with respect to the floor. We measured over 150 steps from several trials. The data is time-synchronized using heel strike at the end as the reference, and the final position after heel strike is defined as 0 meters. Then we calculated the mean and standard deviation, which are shown in the graph.

VI. CONCLUSION

In this paper we research the effect of swing leg retraction on gait stability. The conclusions are straightforward:

1) **Walk fast**; this decreases the sensitivity of the gait to the swing leg motion just prior to heel contact.
2) **Use mild swing leg retraction**; by moving the swing leg rearward just prior to heel contact, one avoids the highly unstable effects that occur when the swing leg is still moving forward at heel contact.

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