International Journal of Humanoid Robotics
Vol. 1, No. 1 (2004) 157–173

ZERO-MOMENT POINT — THIRTY FIVE YEARS OF ITS LIFE

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Received 24 October 2003
Accepted 8 January 2004

This paper is devoted to the permanence of the concept of Zero-Moment Point, widely-known by the acronym ZMP. Thirty-five years have elapsed since its implicit presentation (actually before being named ZMP) to the scientific community and thirty-three years since it was explicitly introduced and clearly elaborated, initially in the leading journals published in English. Its first practical demonstration took place in Japan in 1984, at Waseda University, Laboratory of Ichiro Kato, in the first dynamically balanced robot WL-10RD of the robotic family WABOT. The paper gives an in-depth discussion of source results concerning ZMP, paying particular attention to some delicate issues that may lead to confusion if this method is applied in a mechanistic manner onto irregular cases of artificial gait, i.e. in the case of loss of dynamic balance of a humanoid robot.

After a short survey of the history of the origin of ZMP a very detailed elaboration of ZMP notion is given, with a special review concerning “boundary cases” when the ZMP is close to the edge of the support polygon and “fictious cases” when the ZMP should be outside the support polygon. In addition, the difference between ZMP and the center of pressure is pointed out. Finally, some unresolved or insufficiently treated phenomena that may yield a significant improvement in robot performance are considered.

Keywords: Biped locomotion; zero-moment point; dynamically balanced gait; support polygon.

1. Introduction

Biped locomotion has been a focus of researchers for decades. Theoretical studies from various aspects have been accompanied by a lot of simulation work and various practically realized systems, from the simplest cases of planar mechanisms to the Honda and Sony humanoid robots, the most advanced biped locomotion robots designed up to now. Irrespective of their structure and number of Degrees Of Freedom (DOFs) involved, the basic characteristics of all biped locomotion systems are: (i) the possibility of rotation of the overall system about one of the foot edges caused by strong disturbances, which is equivalent to the appearance of an unpowered (passive) DOF, (ii) gait repeatability (symmetry), which is related to regular gait only, and (iii) regular interchangeability of single- and double-support phases. During walking, two different situations arise in sequence: the statically stable double-support phase in which the mechanism is supported on both feet simultaneously, and statically unstable single-support phase, when only one foot of the mechanism is in contact with the ground while the other is being transferred from the back to front positions. Thus, the locomotion mechanism changes its structure during a single walking cycle from an open to a closed kinematic chain. All these circumstances have to be taken into account in artificial gait synthesis.

All of the biped mechanism joints are powered and directly controllable except for the contact between the foot and the ground (which can be considered as an additional passive DOF), where the interaction of the mechanism and environment only takes place. This contact is essential for the walk realization because the mechanism's position with respect to the environment depends on the relative position of the foot/feet with respect to the ground.

The foot cannot be controlled directly but in an indirect way, by ensuring the appropriate dynamics of the mechanism above the foot. Thus, the overall indicator of the mechanism behavior is the point where the influence of all forces acting on the mechanism can be replaced by one single force. This point was termed the *Zero-Moment Point* (*ZMP*).[1–6] Recognition of the significance and role of ZMP[a] in the biped artificial walk was a turning point in gait planning and control. The seminal method for gait synthesis (semi-inverse method) was proposed by Vukobratović and Juričić.[2,3] It should be noted that despite of the limitation that the motion can be synthesized only for as many joints as the zero-moment conditions can be preset,[b] this method has remained for a long time the only procedure for biped gait synthesis. The ZMP is also indispensable in biped control, for establishing the practically unavoidable feedback[7] with respect to dynamic ground reaction forces.

In this paper we review some basic issues related to biped locomotion with particular attention paid to the ZMP because of its crucial importance for gait analysis, synthesis and control. Despite the fact that the notion of ZMP has never been introduced in the form of a formal definition, in the course of almost three and a half

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[a] It should be noted that in first two papers[1,2] neither the term ZMP (the mechanism had a pin-point foot and no support area was employed) nor semi-inverse method were explicitely mentioned. However, the compensational dynamics was obtained on the basis of the semi-inverse method and the ZMP concept, although the possible positions of ZMP in this case were reduced to the tip of the pin-point foot. A couple of years later, when a spatial link was used instead of the pin-point foot, the notion of ZMP was formally introduced.

[b] Actually, we can set up zero-moment conditions for any passive (unpowered) DOF of the mechanism. For example, apart from the foot-ground contact we can set up zero-moment conditions for the shoulder joint for freely swinging arms (passive DOFs), while the motion at all other joints has to be prescribed.

decades this concept has been involved in very diverse applications related to numerous anthropomorphic locomotion mechanisms of different degrees of complexity.

The aim of this work is primarily to remind the reader of the seminal results related to ZMP, whose first practical application was in the realization of the dynamically balanced biped gait in 1984 (performed by the WL-10RD robot, developed in Ichiro Kato's laboratory at Waseda University), and which was reported 16 years after the appearance of the ZMP concept.[8] Besides this, after inspecting numerous papers, published especially in the proceedings of international conferences devoted to humanoid robots, we have found some insufficiently precise, and sometimes incomplete, definitions of ZMP that might potentially lead to an inappropriate understanding of this concept, especially by younger researchers, though this concept has gained unequivocal confirmation through a great number of sophisticated realizations of humanoid robots. Hence, this paper aims at refreshing the ZMP notion, re-stressing its basic meaning, and mentioning some new, but very essential, phenomena that have still remained from the focus of the studies on gait dynamics and control. Finally, we touch upon some other forms of locomotion-manipulation activities considered as extremely complex contact tasks.

2. The ZMP Notion

Apart from the realization of the relative motion of the mechanism's links, the most important task of a locomotion mechanism during the gait is to preserve its dynamic balance (some "new" authors use the term "stability"!), which is achieved by ensuring the foot's whole area, and not only the edge, is in contact with the ground. The foot relies freely on the support and the only contact with the environment is realized via the friction force and vertical force of the ground reaction.

Let us consider the locomotion mechanism in the single-support phase [Fig. 1(a)], with the whole foot being on the ground. To facilitate the analysis we can neglect the part of the mechanism above the ankle of the support foot (point A) and replace its influence by the force $\mathbf{F}_\mathrm{A}$ and moment $\mathbf{M}_\mathrm{A}$ [Fig. 1(b)], whereby the weight of the foot itself acts at its gravity center (point G). The foot also experiences the ground reaction at point P, whose action keeps the whole mechanism in equilibrium.

In general, the total ground reaction consists of three components of the force $\mathbf{R}$ $(R_\mathrm{x}, R_\mathrm{y}, R_\mathrm{z})$ and moment $\mathbf{M}$ $(M_\mathrm{x}, M_\mathrm{y}$ and $M_\mathrm{z})$. Since the friction force acts at the point of contact of the foot with the ground, and the foot on the ground is at rest, those components of the force $\mathbf{R}$ and moment $\mathbf{M}$ that act in the horizontal plane will be balanced by friction. Therefore, the horizontal reaction force $(R_\mathrm{x}, R_\mathrm{y})$ represents the friction force that is balancing the horizontal component of the force $\mathbf{F}_\mathrm{A}$, whereas the vertical reaction moment M_z represents the moment of friction reaction forces [Fig. 1(c)] that balances the vertical component of the moment $\mathbf{M}_\mathrm{A}$ and the moment induced by the force $\mathbf{F}_\mathrm{A}$. Thus, if we assume the foot-floor contact is without sliding, the static friction will compensate for the horizontal force components $(R_\mathrm{x}, R_\mathrm{y})$ and vertical reaction torque (M_z). The vertical

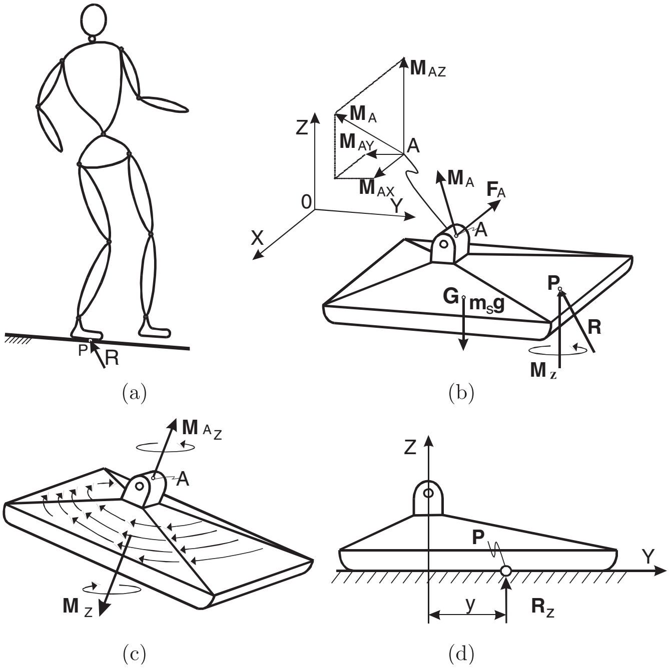

Fig. 1. Biped mechanism and forces acting on its sole.

reaction force R_z represents the ground reaction that balances vertical forces. It remains to consider the balancing of the horizontal component of the foot load moment. However, due to a unidirectional nature of the connection between the foot and the ground (it is obvious that the ground reaction force induced by foot action is always oriented upwards) horizontal components of all active moments can be compensated for only by changing position of the reaction force **R** within the support polygon. Therefore, the horizontal component of the moment $\mathbf{M}_A$ will shift the reaction force to the corresponding position, to balance the additional load. This is illustrated in Fig. 1(d), where, for the sake of simplicity, we present a simple planar case in the y–z plane. The moment M_{Ax} is balanced by shifting the acting point of the force R_z, whose intensity is determined from the equation of balance of all the forces acting on the foot, by the corresponding distance y. It is necessary to emphasize that all the time the reaction force is within the area covered by the foot, the increase in the ankle moment will be compensated for by changing the position of this force, and no horizontal components of the moments M_x and M_y will exist. This is the reason why in Fig. 1(b) at point P only the M_z component exists.

However, if the real support polygon is not large enough to encompass the appropriate position of the force **R** to balance the action of external moments, the force **R** will act at the foot edge and the uncompensated part of the horizontal component of the reaction moment will cause the mechanism's rotation about the foot edge, which can result in the mechanism's overturning. Therefore, we can say that the necessary and sufficient condition for the locomotion mechanism to be in dynamic equilibrium is that for the point P on the sole where the ground reaction force is acting,

$$\begin{aligned} M_{\mathrm{x}} &= 0, \\ M_{\mathrm{y}} &= 0. \end{aligned} \tag{1}$$

Since both components relevant to the realization of dynamic balance are equal to zero, a natural choice to name this point was Zero-Moment Point. Or, in other words, all the time the reaction of the ground due to the foot resting on it can be reduced to the force $\mathbf{R}$ and vertical component of the moment M_{z}; the point P at which the reaction force is acting represents ZMP.

Now, a logical question can be posed: given the mechanism dynamics, what should the ZMP position be that would ensure dynamic equilibrium? It should be noted that in view of the fact that the entire mechanism is supported on the foot, a prerequisite for the mechanism's dynamic equilibrium is that the foot rests fully on the floor. Thus, to answer the previous question let us state the static equilibrium equations for the supporting foot [Fig. 1(b)]:

$$\mathbf{R} + \mathbf{F}_{\mathrm{A}} + \mathbf{m}_{\mathrm{s}}\mathbf{g} = 0, \tag{2}$$

$$\overrightarrow{\mathbf{OP}} \times \vec{\mathbf{R}} + \overrightarrow{\mathbf{OG}} \times \mathbf{m}_{\mathrm{s}}\mathbf{g} + \mathbf{M}_{\mathrm{A}} + M_{\mathrm{z}} + \overrightarrow{\mathbf{OA}} \times \mathbf{F}_{\mathrm{A}} = 0, \tag{3}$$

where $\overrightarrow{\mathbf{OP}}$, $\overrightarrow{\mathbf{OG}}$ and $\overrightarrow{\mathbf{OA}}$ are radius vectors from the origin of the coordinate system O_{xyz} to the ground reaction force acting point (P), foot mass center (G), and ankle joint (A), respectively, while the foot mass is m_{s}. If we place the origin of the coordinate system at the point P and project Eq. (3) onto the z-axis, then the vertical component of the ground reaction moment[c] (actually, it is the ground friction moment) will be

$$M_{\mathrm{z}} = M_{\mathrm{fr}} = -(M_{\mathrm{A}}^{\mathrm{z}} + (\overrightarrow{\mathbf{OA}} \times \mathbf{F}_{\mathrm{A}})^{\mathrm{z}}).$$

In a general case, this moment is different from zero and can be reduced to zero only by the appropriate dynamics of the overall mechanism. However, the projection of

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[c] Although no rotational slippage of the foot over the ground surface will occur in a normal walk, to cover this (very hypothetical) possibility it can be requested that during the mechanism motion even the third moment component is equal to zero ($M_{\mathrm{z}} = 0$). To achieve this,[9] the mechanism should perform some additional movements, for example, by the trunk about the vertical axis to ensure that $M_{\mathrm{A}}^{\mathrm{V}} = 0$, where the superscript V stands for the vertical component of the moment at the ankle jont. However, for a regular motion and a normal friction coefficient between foot and ground the requirement $M_{\mathrm{z}} = 0$ is not necessary because this moment is intrinsically compensated by the friction force.

Eq. (3) onto the horizontal plane gives

$$(\overrightarrow{\mathbf{OP}} \times \vec{\mathbf{R}})^{\mathrm{H}} + \overrightarrow{\mathbf{OG}} \times \mathbf{m}_{\mathrm{s}}\mathbf{g} + \mathbf{M}_{\mathrm{A}}^{\mathrm{H}} + (\overrightarrow{\mathbf{OA}} \times \mathbf{F}_{\mathrm{A}})^{\mathrm{H}} = 0. \tag{4}$$

This equation is a basis for computing the position of the ground reaction force acting point (P). Equation (4), representing the equation of the foot equilibrium, answers the above question concerning the ZMP position that will ensure dynamic equilibrium for the overall mechanism dynamics, but it does not answer the inverse question: whether for the given motion the mechanism is in dynamic equilibrium?

To answer this question we have to consider the relationship between the computed position of P and the support polygon. If the position of point P, computed from Eq. (4), is within the support polygon, the system is in dynamic equilibrium. However, in reality, the point P cannot exist outside the support polygon, as in that case the reaction force **R** cannot act on the system at all. From this follows a straightforward but very important conclusion: in reality, in order to ensure dynamic equilibrium, a point P that satisfies Eq. (4) must be within the support polygon.

If we suppose for a moment that the point P is outside the support polygon, let us consider what would then be the meaning of this point. In view of the fact that this position of P was obtained from the condition $M_{\mathrm{x}} = M_{\mathrm{y}} = 0$, we can consider it as a fictitious ZMP (FZMP).[d] Therefore, in reality, ZMP can exist only within the support polygon, and this point we can term regular ZMP, or ZMP for short, and all the calculated positions of the point P outside the support polygon represent fictitious locations.

Let us explain this in more detail. It is clear from Eqs. (2) and (3) that the ZMP position depends on the mechanism dynamics (i.e. on $\mathbf{F}_{\mathrm{A}}$ and $\mathbf{M}_{\mathrm{A}}$). In the situation when the mechanism dynamics changes so that the ZMP approaches the support polygon edge (in either single-support or double-support phases) let us focus our attention on the moment when the ZMP is just reaching the support polygon edge. The corresponding point will remain the ZMP only if no additional moments are acting at this point.[e] However, if an additional moment appeared, the locomotion mechanism would start to rotate about the foot edge[f] and the mechanism would collapse. In such a situation, the acting point of ground reaction force would be on

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[d] The term Foot-Rotation Indicator (FRI) Point has been suggested.[10]

[e] Obviously, in a regular gait, it is wholly undesirable to have the ZMP on the support polygon edge (or close to it), as an additional moment that would cause the mechanism to overturn easily. In that case an urgent action of the biped control system would be required to bring the ZMP back to the "safety zone." This can be achieved by appropriate intervention movements.[11,12]

[f] In reality, the foot is not ideally rigid but deformable, and in the case of its initial rotation inclination, the edge will transform into a new surface, e.g. into a narrow strip. With increasing inclination angle, the size and position of the contact surface will change, and consequently, a new contact area (strip) will be established. If the ZMP is within the new contact area (strip), the mechanism's dynamic equilibrium might be preserved even if the contact area is outside the previous support polygon. In other words, the condition $M_{\mathrm{x}} = M_{\mathrm{y}} = 0$ will be fulfilled until the ZMP is within the instantenous contact area, irrespective of whether it is within or outside the previous support polygon that existed before the inclination. In this way, i.e. by foot inclination, it is possible to compensate for a wider span of disturbance. Hence, the elucidation, modeling and realization of

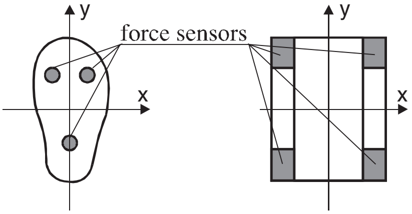


Fig. 2. Examples of the disposition of force sensors on the mechanism's sole.

the foot edge (the reaction force must oppose the action force at the same point!), but this point would not be ZMP[g] any more, since both conditions $M_{\mathrm{x}} = 0$ and $M_{\mathrm{y}} = 0$ would not be fulfilled simultaneously.

To further clarify the meaning of the ZMP outside the support polygon (FZMP) let us be reminded that there are two different cases in which the ZMP plays a key role:

(i) in determining the proper dynamics of the mechanism above the foot to ensure a desired ZMP position,
(ii) in determining the ZMP position for the given mechanism motion.

Case (i) belongs to the task of gait synthesis[2,3,7] and will not be further elaborated here, whereas Case (ii) refers to the gait control, where the ZMP position is a key indicator of the mechanism dynamic equilibrium.

Thus, a crucial question is how to determine the ZMP position. In the case of a real walking mechanism, information about ZMP position can be obtained by measuring forces acting at the contact of the ground and the mechanism,[7] with the aid of force sensors on the mechanism's sole. It should be noticed that measurement could be performed only if all force sensors (see Fig. 2) are in contact with the ground. If some of the sensors deployed from the ground surface, the mechanism as a whole would rotate about the foot edge and overturn. To overcome such a situation it is necessary to change the control strategy.

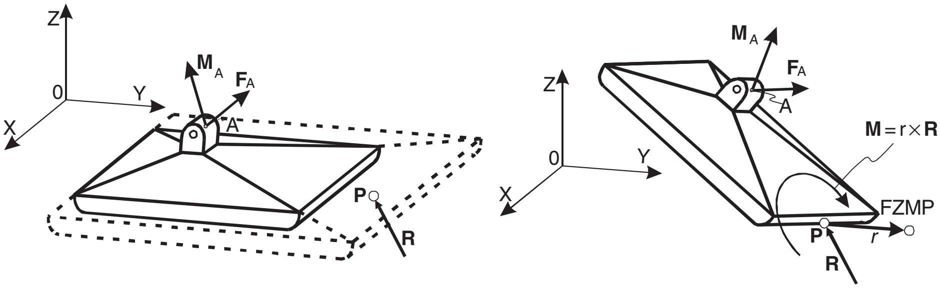
However, if the biped gait is investigated using a dynamic model, the ZMP position must be computed. For a given mechanism motion, the force and moment at the ankle joint ($\mathbf{F}_{\mathrm{A}}$ and $\mathbf{M}_{\mathrm{A}}$) can be obtained from the model of the mechanism dynamics,[7] and all elements in Eq. (4) except for $\overrightarrow{\mathbf{OP}}$ will be known.

The procedure for determining ZMP position consists of two steps.

Step 1. Compute $\overrightarrow{\mathbf{OP}}$ from Eq. (4) (see Fig. 1). Let us call the obtained position of the point P *computed ZMP position.* Note again that at this moment we actually

the foot as a flexible structure, having a soft contact surface, is an important and complex task that remains to be properly resolved.

[g] The only situation when a dynamically balanced gait is performed while the ground reaction force is intentionally kept within a very narrow area (the tiptoe) occurs in a balletic motion, but this does not belong to a regular biped gait.



(a) (b)

Fig. 3. Illustration of the determination of ZMP position: (a) Step 1, and (b) Step 2.

do not know whether this position of point P [see Fig. 3(a)] will be within the real support polygon or outside it.

Step 2. The computed ZMP position is just a candidate to be a regular ZMP and its position should be compared with the real support polygon size. If the computed ZMP is outside the support polygon, this means that the ground reaction force acting point (P) is actually on the edge of the support polygon and the mechanism rotation about the support polygon edge will be initiated by the unbalanced moment, whose intensity depends on the distance from the support polygon edge to the computed position of ZMP, i.e. to the FZMP position.

The above procedure is illustrated in Fig. 3. In Step 1, we obtain an answer to the question concerning the ZMP location for the given dynamics not taking into account the real foot size [see Fig. 3(a)], whereas in Step 2, we obtain the answer whether, regarding the foot size (more precisely, the support polygon size), the mechanism is really balanced or not, and where the regular ZMP (provided it exists) is located. If the computed acting point of the ground reaction force is within the real support polygon, this point is ZMP and the mechanism is in equilibrium. If this is not the case, the ground reaction force acting point will be on the support polygon border (the ground reaction force cannot exit the support polygon!) and the distance from it to the computed ZMP position is proportional to the intensity of the perturbation moment that acts on the foot [Fig. 3(b)].

The ZMP concept has been properly comprehended by researchers, widely used, and very frequently cited. It can be noted that, although being essentially correct, all the ZMP definitions differ significantly in the extent of their detail.

To illustrate this we give just two interpretations. The first interpretation is basically the same in two papers:[13,14]

> **ZMP interpretation 1.** *ZMP is defined as that point on the ground at which the net moment of the inertial forces and the gravity forces has no component along the horizontal axes.*

The other interpretation[15] is:

> **ZMP interpretation 2.** *p is the point that $T_x = 0$ and $T_y = 0$, T_x, T_y represent the moments around x- and y-axis generated by reaction force F_r and reaction torque T_r, respectively. The point p is defined as the Zero Moment Point (ZMP). When ZMP exists within the domain of the support surface, the contact between the ground and the support leg is stable:*
>
> $$p_{ZMP} = (x_{ZMP}, y_{ZMP}, 0) \in S,$$
>
> *where p_{ZMP} denotes a position of ZMP. S denotes a domain of the support surface. This condition indicates that no rotation around the edges of the foot occurs.*

Primarily because of those younger researchers that are just beginning their work in this field and who often have had no insight into the original works in which the ZMP notion was introduced, let us notice that ZMP has often been insufficiently precisely related to the ground surface (a surface of practically unlimited size), even without mentioning the support polygon. Also, it has often been missed to stress that a ZMP outside the support polygon practically has no sense, as in ZMP *de facto* does not exist, and in reality the mechanism in such situations falls by rotating about the edge of the support polygon.

Here we have to point out another important issue, and this is the difference between the center of pressure (CoP) and ZMP, as it is very important to make a clear distinction between the two notions, which must not generally be regarded as identical. The pressure between the foot and the ground[4,6,7] can always be replaced by a force acting at the center of pressure — CoP. If this force balances all active forces acting on the mechanism during the motion (inertia, gravitation, Coriolis and centrifugal forces and moments) its acting point is ZMP. Thus, in the case of a dynamically balanced gait, CoP and ZMP coincide. When the gait is not dynamically balanced, ZMP does not exist and the mechanism colapses about the foot edge.

To make the ZMP notion and its relationship with CoP perfectly clear we will summarize our previous discussion in three characteristic cases for a non-rigid foot in contact with the ground, as sketched out in Fig. 4.

In a balanced gait, the ZMP coincides with CoP [Fig. 4(a)]. In the case of a disturbance that brings the acting point of the ground reaction force to the foot edge, the perturbation moment will cause rotation of the biped system about the foot edge (as we already mentioned, the foot edge is in fact a very narrow strip as the shoe sole is not totally rigid) and its overturning. In that case we can speak only of the fictitious ZMP, whose distance from the foot edge represents the intensity of the perturbation moment [Fig. 4(b)]. However, it is possible to realize the biped motion, for example, on the toe tips [Fig. 4(c)] with special shoes having a pinpoint area (balletic motion), while keeping the ZMP position within the pinpoint area.

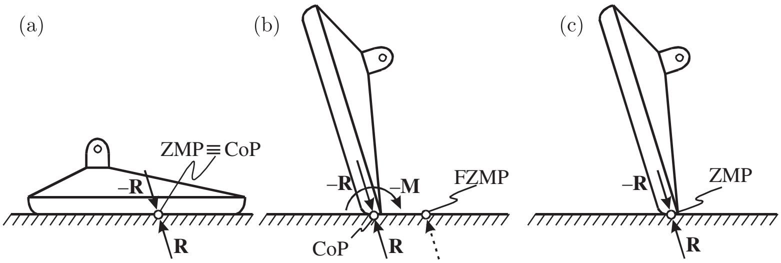


Fig. 4. Possible relations between ZMP and CoP for a non-rigid foot: (a) dynamically balanced gait, (b) unbalanced gait where ZMP does not exist and the ground reaction force acting point is CoP while the point where $M_{\mathrm{x}} = 0$ and $M_{\mathrm{y}} = 0$ is outside the support polygon (FZMP). The system as a whole rotates about the foot edge and overturns, and (c) tiptoe dynamic balance ("balletic motion").

Although the ZMP now coincides with CoP, it is not a regular gait, and the person should be specially trained to perform it.

Here, it is necessary to be reminded that the task of deriving a model of nominal dynamics of a humanoid robot is concerned with satisfying a certain number of dynamic connections. This is in fact the so-called mixed type of task, when the link's motion and the driving torques are both partly known and their complements are sought. In the case of investigating the dynamics of biped structure, the motion of the links performing a given type of gait is known, while the known moments are equal to zero. The latter follows from the equilibrium conditions holding for a selected point within the support polygon and for the joints of passive links. Therefore, there are two types of zero-moment points. Both of them serve to form the model of nominal dynamics of the humanoid robot, but those within the support polygon are practically unavoidable in gait synthesis as well as for the overall control of dynamically balanced gait.

To relate the ZMP notion only to CoP is not correct as the ZMP can exist at some other points in the system, e.g. at the shoulder jonts if we consider arms as freely-swinging pendulums with no actuators at the joints.

In summary, the ZMP always coincides with the CoP (dynamically balanced gait), but the CoP is not always ZMP (dynamically unbalanced gait). However, the FZMP never coincides with the CoP because CoP cannot, naturally, exist outside the support polygon.

3. Some Further Notes

3.1. *ZMP and FZMP*

It is of crucial importance to explain the significance and role of the real ZMP and its fictitious position outside the support polygon — FZMP.

In humanoid gait realization, the task of primary importance is undoubtedly to constantly maintain dynamic equilibrium, i.e. to perform dynamic stabilization. Hence the basic task of the control system is to keep the ZMP within the support polygon, to prevent it from coming too close to the support polygon edge, and thus avoid the loss of equilibrium of the overall system in the case of a sudden disturbance. However, the question remains what to do if such a situation still arose and whether potential information about the FZMP could be of any help.

In the gait performed by a walking mechanism, at the moment of the occurrence of an external disturbance, the contact of the humanoid mechanism with the ground will be reduced to a narrow strip on the foot edge, and that very moment will cease the possibility of regular maintaining of the mechanism's dynamic balance. Namely, by losing regular contact of its foot with the ground, the humanoid loses the force feedback of the ground dynamic reaction, i.e. the possibility of stabilizing itself as a whole. Such a situation can arise both in the single-support and double-support phases of the gait. In that case an emergency-coping strategy can be applied, which primarily assumes the movements of the arms in an attempt to diminish the perturbation moment, combined with an increase in stride and moving the leg aside to enlarge the trace. This eventually can bring about the enlargement of the support polygon within which is to be located a new, "emergency" ZMP. If the critical situation (i.e. the robot's overturning) is thus overcome, further robot motion may be interrupted and restarted in the form of a regular gait, or, if possible, the motion will not be interrupted but, after several transitional steps, continued in the same manner as prior to the occurrence of the disturbance.

It should be emphasized that this outlined "emergency-copying" strategy is an extremely delicate task, requiring special sensors like gyroscopes and other high-tech transducers, as well as very powerful control units capable of updating actuator data in microseconds. Instead of using special highly sophisticated sensors and fast microprocessor control units to stabilize the humanoid robot in real time in the case of emergency, the problem of dynamic intervention can be solved in another, less sophisticated way. The procedure would consist of arm motion by which some additional contacts would be made (the mechanism may lean using its hands against some object in its surroundings), resolving thus the problem of the momentary loss of dynamic balance of the previous anthropomorphic configuration. Preventing the robots's overturning can also be achieved by temporary reconfiguration into a quadruped using the upper extremities, followed by re-establishing the motion in the form of regular dynamically balanced biped gait. Namely, by ensuring additional support points static equilibrium may be re-established and the dynamically balanced gait continued. This procedure of re-establishing dynamic equilibrium might be considered as a kind of total compliance procedure.

3.2. *Some prospective tasks*

The expectations to be met by humanoid robots are constantly growing both in number and specificity. Already today we can envisage the ambitious use of service robotics in the widest sense, from helping (or replacing) humans in hazardous situations and hostile environments to entertainment and "socialization" of man-robot communication.

Hence it is necessary to make certain improvements and refinements to humanoid robots, both in the domain of complexity of their mechanisms (DOFs) and ensuring new performance, which on the other hand would demand the inclusion of some new, previously neglected phenomena in the modeling and control of humanoid robots. Let us mention only those phenomena (limited to the dynamics-control domain only) whose presence could yield new, significantly improved, performance and capabilities of humanoid robots:

- Elasto-dynamics and increased complexity of the robot foot for more realistic description of the contact task Robot-Dynamic Environment, to enable appropriate dynamic control with respect to position and contact force of the dynamic reaction.
- Softness of the two-link semi-rigid foot instead of the conventionally treated rigid foot, as this plays an important role in the appearance of unpowered DOFs between the foot and the ground. Namely, instead of the unnatural edge about which the mechanism would rotate in the case of a large disturbance, it is more realistic to consider the contact in the form of some narrower area of the artificial foot appearing as a result of the mentioned foot elasticity. This phenomenon is important because of the ZMP position, which, in contrast to the conservative case of sudden rotation about the foot edge (theoretical line), is found on a border strip of the foot, giving thus a higher chance of using a more effective control strategy in the critical regimes of the synthesized gait.
- Elasticity of humanoid robot joints, especially of the ankle joints, where appropriate active absorbers could be built in, whose damping would change depending on the impact of the robot's foot against the ground. This phenomenon has been initially considered in some papers from the domain of humanoid robotics. In addition to the variable damping coefficient, care should also be paid to the realization of variable active stiffness, which represents a somewhat more complex case for implementation in humanoid robots.
- Synthesis of dynamic position-force control of artificial gait in the case of elastic actively damped joints and the elastodynamic character of the foot.
- Resolving the problem of quasi-continuous transition of the ZMP from the single-support to the double-support gait phase. It should also be borne in mind the quasi-continuity of the ZMP trajectories that are approximately realized at their discrete locations. When analyzing the influence of the character of ZMP trajectories in respect of the degree of anthropomorphism of humanoid robots, the latter characteristic should also be considered in relation to the increase in the number of DOFs.
- Smooth transition from one walking pattern to another (e.g. transition from walking on a flat surface to walking upstairs and downstairs, avoiding obstacles, walk acceleration/deceleration, etc.). A special challenge represents the "independent" use of hands for another task during the walk (e.g. taking objects from the table in passing without stopping and their manipulation, carrying heavy objects, etc.).

At this point we will also briefly formulate another major topic that seems inevitable or prospective. As some of these problems have already been recognized and elaborated, we will mention only those tasks that have not been resolved yet.

- The term "ground" usually means something immobile (perhaps deformable, but immobile). However, a general approach requires the option of walking on a mobile support. Moreover, such support should not be considered as a pure nonstationary constraint but rather as a dynamic system that interacts with the walker. Thus, a mobile platform that has its own dynamics has to be introduced. The platform should have up to six DOFs. It is clear that the platform can be modeled in different ways. A constructive approach may refer to the use of special Stewart platform structures,[16] as shown in Fig. 5(a).

In describing the effects that should be taken into account when working on human or humanoid dynamics we come to the configuration of the system in the dynamics simulator GHDS (General Human/Humanoid Dynamics Simulator).[17]

It is important to make a distinction between the GHDS and test bed that involves the real devices: robot, cameras, etc. [sketched in Fig. 5(b)]. The fusion of these two systems may be considered as an ultimate goal.

The above topics concerning humanoid robots, being still in a modest initial stage or not yet formulated, are of crucial importance to achieve those capabilities of humanoid robots that could realistically meet high requirements of their envisaged applications.

4. Conclusion

The concept of ZMP has and will have an essential role in both theoretical considerations and the practical development of humanoid robots and biped locomotion. After several decades of its application it can be noticed that in referring to it, probably because the notion has become truly accepted and commonly known, the ZMP has sometimes been defined in an insufficiently precise and over-simplified way. Having in mind that the original works in which the ZMP concept was introduced are not easily accessible to all researchers, especially to younger ones, we thought it useful to refresh this notion and remind readers of its original meaning and thus avoid its superficial understanding and possible confusion. Besides this, it is evident that the development of humanoid robotics is going in the direction of incorporating robots into "intimate" human environments, coexistence and co-operation with humans (even as a partner on the same task), so that it is rightly expected that the performance of robots will become closer to, and in some segments even better than, those of humans. Also, we have to mention the important area of service robotics and the role of robots in hostile environments. Hence, in its last sections this paper touches upon some important but still unresolved locomotion-manipulation issues.

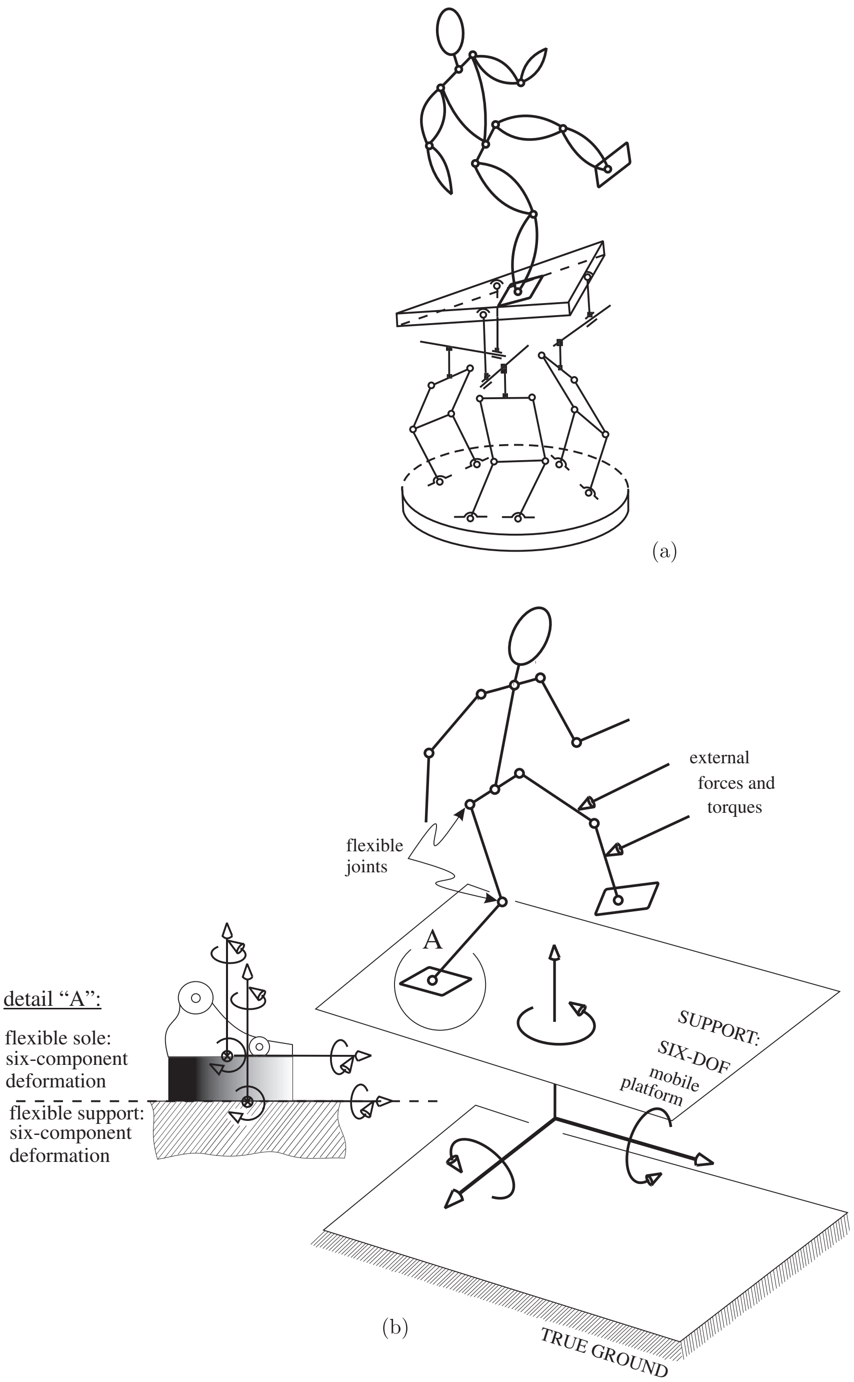


Fig. 5. Model of the general task for ensuring biped's dynamic equilibrium.

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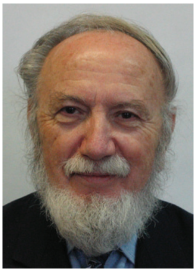

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