Nonlinear feedback control of a biped walking robot

K.Mitobe, N.Mori, K.Aida and Y.Nasu
Faculty of Engineering, Yamagata University
Jo-nan 4-3-16, Yonezawa, Japan

Abstract

An implementation of a biped robot which is capable of dynamic walking by a simple nonlinear control algorithm is presented. Four D.C. servo motors actuate the knee and ankle joints of the legs of the robot. The biped is constrained to the sagital plane, and the motion generation is reduced to a problem of controlling the position and velocity of the robot's center of gravity. They are controlled by a nonlinear feedback controller, based on a simple feedback linearization method. Several design issues including mechanical structure, leg actuation, and control system of the robot are discussed. Experimental results demonstrate the effectiveness of the algorithm.

1 Introduction

This paper presents an implementation of a biped robot which is capable of dynamic walking by a simple nonlinear control algorithm. Control of biped walking robots requires efficient motion generation of walking gaits which ensure stable movement of the robot's center of gravity. Since the motion of a biped robot with many degrees of freedom is governed by a high order nonlinear differential equation, significant simplification of the dynamical equation is required to generate the control inputs in real time. For example, Miyazaki and Arimoto [2] reduced the order of the dynamical equation by the singular perturbation method, and Mita, et.al.[3] linearized the equation around the commanded position of the robot. On the other hand, in many approaches to ensure the stable movement of the robot's center of gravity, the dynamic model of the robot's single leg support phase is reduced to the dynamic model of an inverted pendulum, so as to control the robot's center of gravity based upon a simple robot model. However, the inverted pendulum is inherently an unstable system, and hence the trajectory of the robot's center of gravity depends significantly on the initial conditions of each single support phase. To maintain the stable walking gait, the timing of switching the support leg must be determined strictly by the dynamic model. It is difficult to vary the walking speed during walking, because the walking cycle of the robot is determined by the natural frequency of the inverted pendulum. To overcome this difficulty, Sano and Furusho[1] controlled the angular momentum of the robot around the ankle of the supporting leg, Kajita and Tani[4] fixed the vertical height of the robot's center of gravity to obtain the linear inverted pendulum mode.

In this paper, the motion generation is reduced to a problem of controlling the position and velocity of the robot's center of gravity. By this reduction, walking gait of the robot is generated efficiently without linear approximation or order reduction of the dynamical equation. The position and velocity are controlled by a nonlinear feedback controller, in which the measurements of the position and velocity are nonlinearly fed back to obtain the motor torque inputs. This control law provides a stable trajectory of the robot's center of gravity in the single support phase of the walking.

The biped is constrained to the sagittal plane. Four D.C. servo motors actuate the knee and ankle joints of the legs of the robot. Several design issues including mechanical structure, leg actuation, and control system of the robot are discussed.

2 Controller structure

This paper considers a biped robot consists of a torso and two legs. Each leg has two degrees of freedom. The robot is modelled as Fig.1(a), where the following simplifying assumptions are made.

A1. The movement of the robot is restricted to the sagittal plane.

A2. The pitch angle of the torso θ_5 is fixed while walking.

A3. The contact between the foot of the support leg and the ground is realized by the full foot.

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In the experiments, the mechanical structure of the robot is designed so as to satisfy these assumptions. For example, to satisfy the assumption A2, each leg is constructed by using a pair of parallel links.

By the assumption A2, the movement of the torso is pure translational movement, and hence the angular momentum of the torso can be ignored. Then, the motion of the robot obeys the dynamical equation of a rigid robot:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{1}$$

where $q = (\theta_1, \theta_2, \theta_3, \theta_4)^T$. Vectors, $C(q,\dot{q})\dot{q}, G(q)$ and $\tau \in R^{4\times1}$ represent the centrifugal and Coriolis terms, gravitational terms, and joint inputs, respectively. $M(q) \in R^{4\times4}$ represents the inertia matrix.

Precisely, to satisfy the assumption A3, longitudinal distribution of the pressure on the foot of the support leg must be considered[5]. However, for simplicity, we assume the condition without detailed discussion.

The origin of the world frame O is defined in each single support phase at the point where support leg contacts to the ground. In the world frame, position of the joint at the bottom of the torso, (See Fig.1(a)) is expressed in terms of the joint angles as

$$x_1 = l_1 sin(\theta_1) + l_2 sin(\theta_2), \tag{2}$$

$$y_1 = l_1 cos(\theta_1) + l_2 cos(\theta_2), \tag{3}$$

where l_1 and l_2 represent the link parameters defined as Fig.1(b). Since the robot's center of gravity is expressed as $(x_1, y_1 + a_5)$, the point (x_1, y_1) can be regarded as the robot's center of gravity. In the following sentences, (x_1, y_1) is referred to as the position of the robot's center of gravity.

For the control of the lifted leg, a frame O', with the origin fixed at the robot's center of gravity, is defined as Fig.1(a). In this frame, the position of the lifted foot is expressed as

$$x_2 = l_2 sin(\theta_3) + l_1 sin(\theta_4), \tag{4}$$

$$y_2 = l_2 cos(\theta_3) + l_1 cos(\theta_4). \tag{5}$$

The dynamical equation (1) can be rewritten in terms of a vector x $(= (x_1, y_1, x_2, y_2)^T)$ as

$$\begin{aligned} &M(q)J^{-1}(q)\ddot{x} \\ &\quad + \{C(q,\dot{q}) - M(q)J^{-1}(q)\dot{J}(q,\dot{q})\}\dot{q} \\ &\quad + G(q) = \tau, \end{aligned} \tag{6}$$

where J is the Jacobian matrix defined by

$$\dot{x} = J\dot{q}. \tag{7}$$

If the matrix MJ^{-1} is nonsingular, by choosing the control input τ as

$$\tau = MJ^{-1}u + \{C - MJ^{-1}\dot{J}\}\dot{q} + G, \tag{8}$$

eq.(6) is reduced to

$$\ddot{x} = u, \tag{9}$$

where u is a new input to the system. For the trajectory tracking, u is given as

$$u = \ddot{x}_d + K_v(\dot{x}_d - \dot{x}) + K_p(x_d - x), \tag{10}$$

where x_d represents a desired trajectory for x: the position of robot's center of gravity and the foot of the lifted leg. The tracking error $e = x_d - x$ satisfies the following error equation.

$$\ddot{e} + K_v\dot{e} + K_p e = 0. \tag{11}$$

K_v and K_p are positive matrices, chosen such that eq.(11) is an exponentially stable differential equation. The block diagram of the control system is shown in Fig.2.

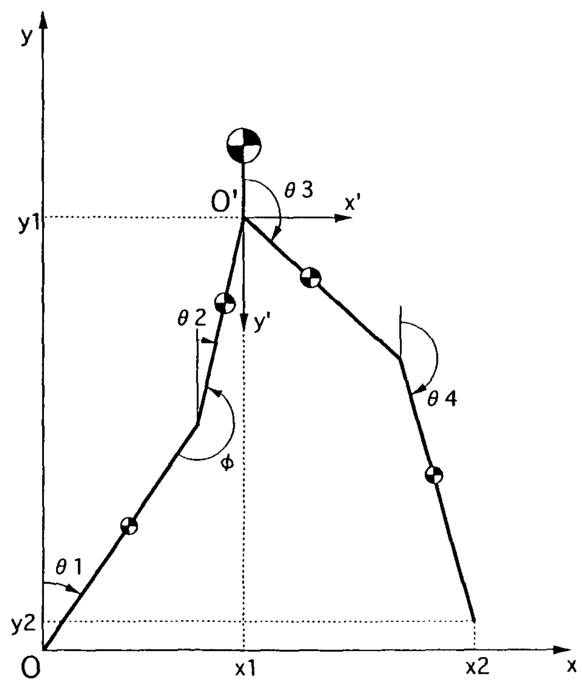


Fig.1(a) Model of the biped robot

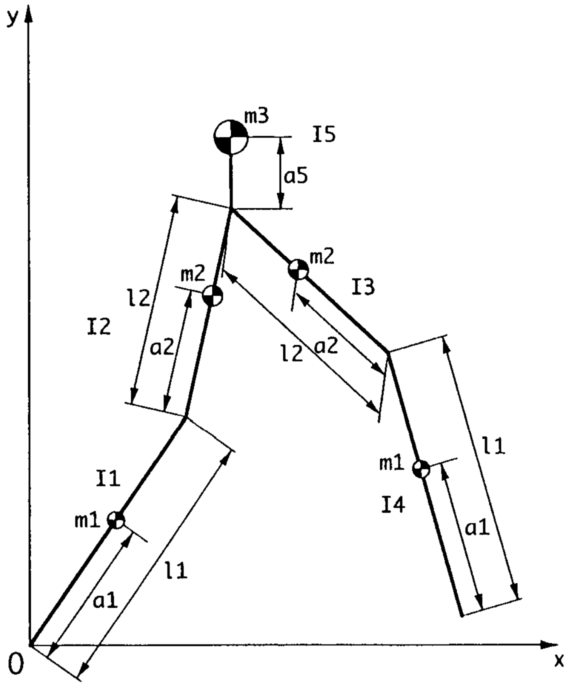
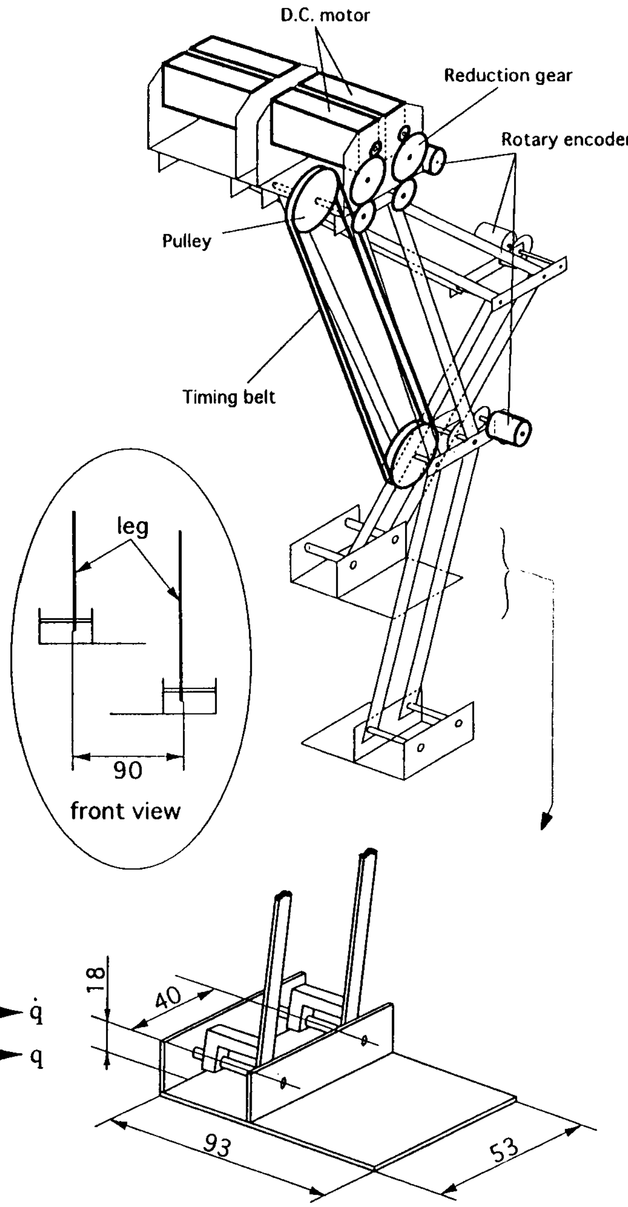


Fig.1(b) Link parameters



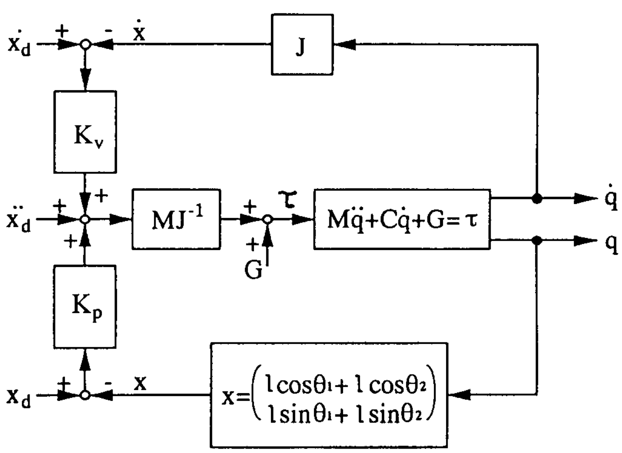


Fig.2 Controller for the support leg

Fig.3 Mechanical structure of the robot

3 Mechanical design and control law

Practical implementation of the control law (8) requires precise measurement of the parameters in the dynamical equation (1). This requirement, however, is difficult to satisfy because of the unavoidable measurement errors. Especially, inertia moments of each link and the actuator gains are difficult to obtain precisely. In this paper, this difficulty is avoided by choosing other link parameters properly, as well as reducing the mass of the legs of the robot, so that the control law does not contain the value of inertia moment nor the actuator gains.

All D.C. motors for the actuation of each joint are located in the torso, so as to reduce the mass and inertia of other links. By assuming that the mass and inertia of the legs are sufficiently small compared to the mass of the torso, the lifted leg in the single supporting leg can be ignored.

$$m_1 = m_2 = I_1 = I_2 = 0. \tag{12}$$

Moreover, if we choose the length of the upper and lower leg equivalent, $l_1 = l_2 = l$, the second term in the right hand side of (6) is equal to 0. Then the control inputs to the support leg reduce to

$$\begin{aligned} \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} &= m_3 l \begin{pmatrix} cos(\theta_1) & -sin(\theta_1) \\ cos(\theta_2) & -sin(\theta_2) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ &+ m_3 l g \begin{pmatrix} sin(\theta_1) \\ sin(\theta_2) \end{pmatrix}, \end{aligned} \tag{13}$$

where τ_1 and τ_2 represent the input to the ankle and knee joint of the support leg. The merit of this simplification is that the matrix in the right hand side of (13) contains no link parameters. Although the control law contains m_3 and l, these parameters are not required to be known precisely, because the stability of the whole system can be guaranteed when m_3 and l are positive, and this condition is always true. Assuming eq.(12), by using (13), (x_1, y_1) satisfies

$$\ddot{x}_1 = u_1, \tag{14}$$
$$\ddot{y}_1 = u_2. \tag{15}$$

To track a desired trajectory (x_{1d}, y_{1d}), the new inputs (u_1, u_2) are determined as

$$\begin{aligned} u_1 = \ddot{x}_{1d} &- k_{11}(\dot{x}_{1d} - \dot{x}_1) \\ &- k_{12}(x_{1d} - x_1), \\ u_2 = \ddot{y}_{1d} &- k_{21}(\dot{y}_{1d} - \dot{y}_1) \\ &- k_{22}(y_{1d} - y_1), \end{aligned} \tag{16}$$

where $k_{11}, k_{12}, k_{21}, k_{22}$ are positive constants, so that x_1 and y_1 converge to x_{1d} and y_{1d} respectively.

To realize a constant walking speed with a fixed vertical height of the robot's center, the inputs (u_1, u_2) can be simplified as

$$\begin{aligned} u_1 &= k_{11}(v_{ref} - \dot{x}_1), \\ u_2 &= -k_{21}\dot{y}_1 - k_{22}(h_{ref} - y_1), \end{aligned} \tag{17}$$

where v_{ref} is the reference for the walking speed and h_{ref} is the reference for the height of the robot's center. In this case, the motion of the support leg is generated simpler than the case of eq.(16), because the reference input is given by only v_{ref} and h_{ref}, instead of a desired trajectory. Moreover, the walking speed can be easily varyed by changing the value of v_{ref}.

Although double support phase is known to exists in the human gait, in this paper, we assume that the support leg switches instantaneously so that every moment of the walking belong to the single support phase. Experimental results show that this assumption does not destroy the stable walking, because the control law eq.(16) or eq.(17) provide a stable trajectory of the robot's center of gravity.

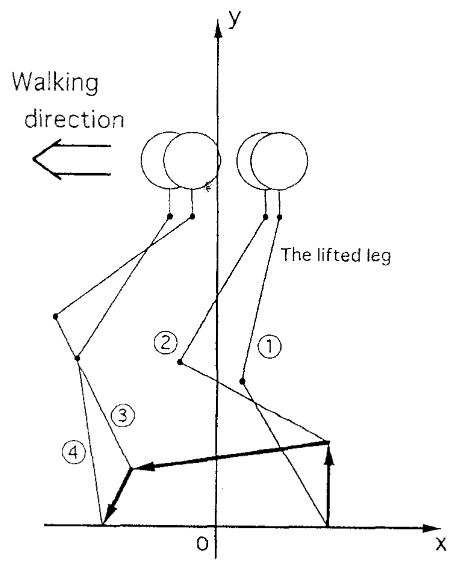


Fig.4 Trajectory of the lifted leg

Table 1 Basic specifications of the experimental biped

	length[mm]		mass[g]
l1	138	m1+m2	320
l2	138		
l3	48	m3	910

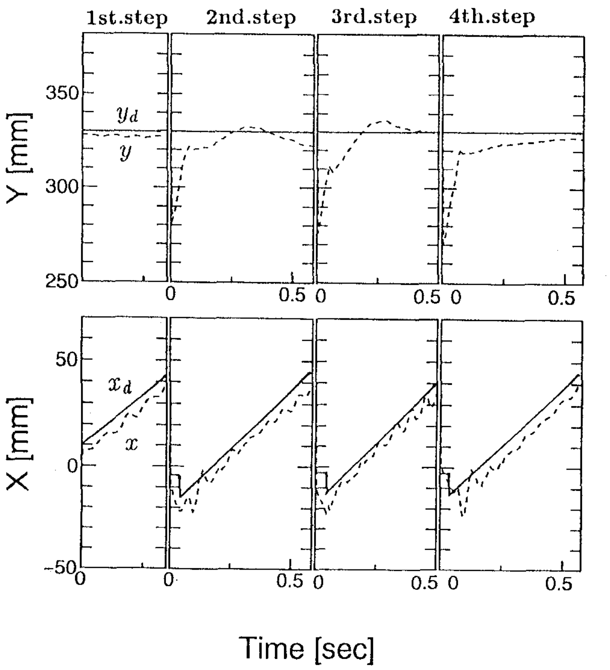


Fig.5(a) Response of the robot's center of gravity for the control law (16)

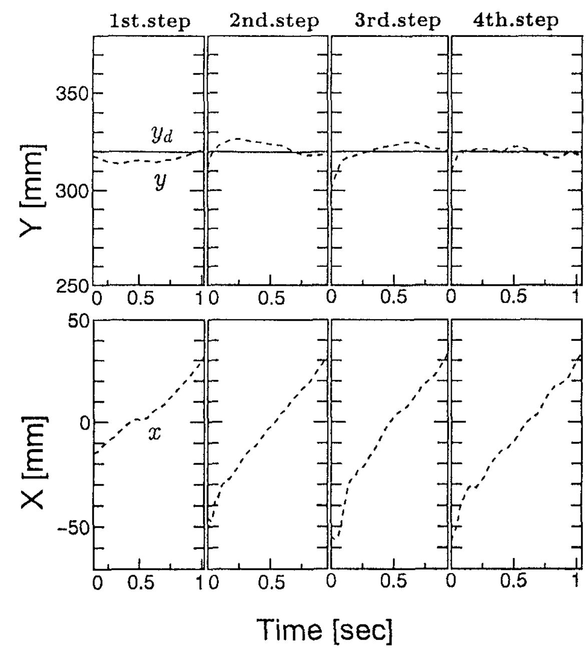


Fig.6(a) Response of the robot's center of gravity for the control law (17)

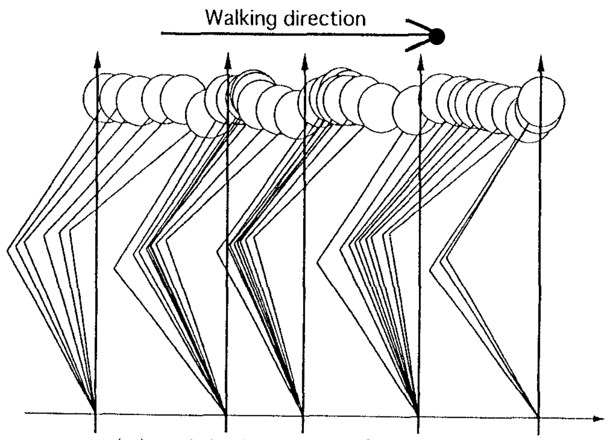


Fig.5(b) Stick diagram of the support leg for the control law(16)

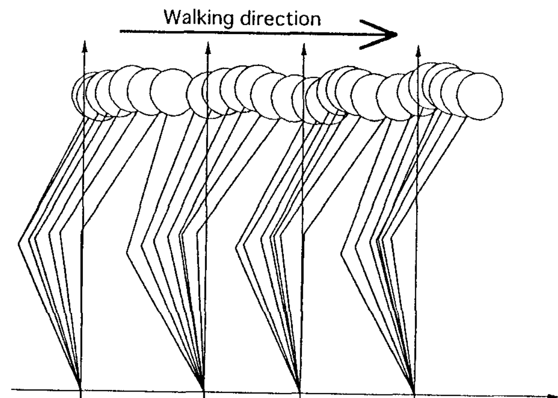


Fig.6(b) Stick diagram of the support leg for the control law(17)

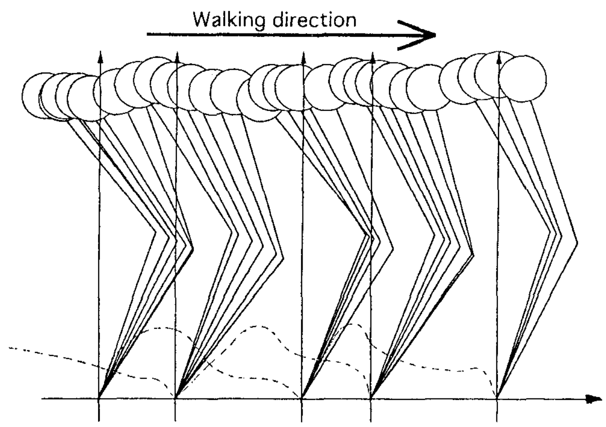


Fig.7 Stick diagram of the support leg for the control law(17)

4 Experimental results

Mechanical structure of the biped robot is illustrated in Fig.3. The robot includes four D.C. servo motors which actuate the joints of the legs. The knee and ankle joints are driven by the motors through reduction gears and timing belts. Each joint angle is measured with the rotary encoder attached at each joint. To reduce the weight of the legs, we selected a small size encorder with 100[P/R] resolution, which is not sufficient to derive the measurement of the angular velocity through a conventional F/V converter. The angular velocities of each joint is detected by measuring the pulse width of the encoder signal. Basic specifications of the experimental biped are shown in Table 1.

In the experiments, the biped robot is controlled to walk at a constant speed with a fixed height of the center of gravity, whereas the reference trajectory for the lifted foot is given as Fig.4. The step length is140[mm] and the length of the foot is 50[mm].

In the first experiment, the support leg is controlled using the control input eq.(16). Fig.5(a) shows the response of the robot's center of gravity (x_1, y_1) and the reference trajectory (x_{1d}, y_{1d}). Small tracking error remaining in each single support phase is considered to be the effect of the frictional forces existing in the motors and reduction gears. Fig.5(b) shows the stick diagram of the support leg traced from the video tape of the experiment. This result shows that a stable gait can be accomplished by controlling the trajectory of the center of gravity.

In the next experiment, the support leg of the robot is controlled using the control input eq.(17). Fig.6(a) shows the response of (x_1, y_1). Fig.6(b) shows the stick diagram of the experimental result.

While walking, the knee joint angle ϕ of the support leg (See Fig.1(a)) remains less than π[rad], and consequently, the walking gait is similar to that of birds. However, this is not a result from the control algorithm eq.(16) or eq.(17), because the same control law is also possible to generate a human-like gait by setting the initial condition of the knee joint angle greater than π[rad]. Fig.7 shows another experimental result using the control law eq.(17), where the knee joint angle remains greater than π[rad]. In this case, the walking gait looks like a human gait. In Fig.7, the trajectory of the lifted foot is indicated by the dotted line.

Small oscillation observed in the experimental results is considered to be an effect of ignored mass and inertia momentum of the lifted leg. It does not prevent the walking at this walking speed, however, for the smooth walking in various speeds, the mass of the legs should be included in the consideration of the control law.

5 Concluding remarks

An implementation of a biped robot which is capable of dynamic walking by a simple nonlinear control algorithm is presented, where the walking gait of the robot is generated by controlling the trajectory of the robot's center of gravity.

After the control law is developed using a simple feedback linearization method, the relationship between the control law and mechanical design is considered. The control law is simplified by the proper choice of the link parameters, as well as reducing the weight of the legs.

Experimental responses demonstrated the effectiveness of the algorithm. A merit of this method is simplicity of the command signals. For instance, it is possible for the walking velocity to be varied by changing the higher level command, with a fixed lower level controller.

Designing the optimal trajectory for the lifted leg is left for further investigation.

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