16-299 Lecture 9: The Frequency Domain Point of View

Why spend time on the frequency domain point of view?

For linear systems and regulators for smooth nonlinear systems, the frequency domain provides insight on the effect of modeling error and robust control.

Understanding the frequency domain point of view is also useful for understanding the history of control.

What is the frequency response of a typical plant?

[Diagram of typical frequency response]

What is the goal of feedback control?

Frequency domain controller design techniques are useful for linear systems, delay, and smooth nonlinearities. They are not as useful for hard nonlinearities such as impact, signal or control saturation, or other rapid changes in the plant dynamics.

The goal of feedback control is to pass through the reference or desired state input with a gain of 1.

However, modeling and fundamental uncertainty limits the use of feedback control at high frequencies.

[Frequency domain performance vs. uncertainty picture]

What makes feedback control go unstable?

[Feedback control diagram]

In the simple linear single gain feedback system above, state feedback is subtracted from a reference signal (or desired state) to compute an error. The gain of the feedback controller amplifies that error, generating the input to the plant. For a linear system, we can consider how feedback affects each frequency independently. A particularly sensitive frequency is one where the phase lag through the plant is 180°. This phase lag inverts the input signal to the plant. Since negative feedback also inverts the feedback signal, in this case the feedback signal **adds** to the reference signal, amplifying it. If the gain around the loop is more than 1, the loop becomes unstable.

The frequency where the phase lag goes through 180° is known as the "crossover" frequency. A measure of how robust feedback control is is the amount the plant gain could be changed at the crossover frequency and remain below 1. This is known as the "gain margin". Gain margins of 2 or more are considered good. A similar measure is the amount the phase lag can be changed at the frequency where the open loop gain is 1. This is known as the "phase margin".

[Gain and phase margin diagram]

Frequency domain controller design: Lead and lag compensators

Before the widespread use of computers as feedback controllers, controllers were designed using analog computing techniques. In addition to a frequency independent gain, circuits could be added to boost gain at low frequencies and reduce gain at high frequencies. Simple versions of this type of compensation have one zero and one pole:

$$\frac{s-z}{s-p} \tag{1}$$

where z determines the zero location and p determines the pole location.

From “Building blocks of linear systems”, Mark A. Haidekker, in Linear Feedback Controls (Second Edition), 2020: A lag compensator has the pole closer to the origin, that is, $|z| > |p|$, whereas the lead compensator has the positions reversed $(|z| < |p|)$. Lead- and lag-compensators have an asymptotically constant magnitude for very high and very low frequencies, but the lead compensator raises the high-frequency magnitude, whereas the lag compensator lowers it. Between the pole and the zero, a lead compensator raises the phase asymptote by 90°, and a lag compensator lowers it by the same amount. Phase-lead compensators can often be used to improve the stability of feedback control systems with a small gain margin or with time delays, because they raise the phase between their zero and their pole. Lag compensators are mainly used to raise the lower-frequency gain.

Loop shaping and H-infinity control design methods

Loop shaping and H-infinity control design methods optimize the frequency response of the controlled system.