16-299 Lecture 4: Numerical Integration

Suppose we have a continuous time model of a system:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \tag{1}$$

How can we simulate (predict the behavior of) the system into the future? This is done with numerical integration.

We have already seen a simple form of numerical integration, Euler integration, when we generated a discrete time state space model from a continuous time state space model:

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta\dot{\mathbf{x}}_i \tag{2}$$

where Δ is a small time step. To predict the future, we just keep applying this equation:

$$\mathbf{x}_{i+n+1} = \mathbf{x}_{i+n} + \Delta\dot{\mathbf{x}}_{i+n} \tag{3}$$

A slight modification of this formula, averaging the old and new velocities, is slightly more accurate. It requires splitting up the equations for the new accelerations from the equations for the new velocities. The first step is to use:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \tag{4}$$

The second step is to pull the accelerations out of $\dot{\mathbf{x}}$. Then:

$$velocities_{new} = velocities_{old} + \Delta accelerations \tag{5}$$

and

$$positions_{new} = positions_{old} + \Delta(velocities_{new} + velocities_{old})/2 \tag{6}$$

There are more complex integration methods (see the Matlab documentation) that permit a larger time step Δ. However, these methods depend on smooth dynamics and smooth controls (the policy is differentiable). In robotics, there are contacts being made and broken which make the dynamics discontinuous, and the policies often have discontinuities due to condition checking (if-then statements). The simulation time step is often limited to being the same or smaller than the control time step, and the benefit of more complex integration methods is minimal. If the controller has internal continuous states, those must be added to the "plant" dynamics for methods that evaluate the dynamics multiple times per step. If the controller has internal discrete states, the additional complexity of managing those discrete states when the dynamics are evaluated multiple times at the same time point, or time even goes backwards, discourages the use of anything other than the methods described above. In any event, the default numerical integration method of a simulation package is often used.