16-299 Lecture 12: Nonlinear control

Examples of nonlinearities

Hardening spring
Softening spring
My shower control
Impact - chattering
Oscillators

Things that look like nonlinearities

Things that look like nonlinearities, but typically involve hidden states:
Hysteresis

Dynamic concepts

Attractors (equilibrium points)

Repellors (opposite of attractors)

Phase portraits (flow fields). Pendulum example, with multiple attactors, and repellers.

Saddle points

Limit cycles (draw phase portrait)

Webb Telescope orbit

What is stability?

Flows that stay in a region. (Region of attraction)

Flows that parallel a trajectory.

Asymptotic stability:

Flows that approach a point.

Flows that approach a trajectory (phase does not converge). Autonomous system.

Flows that approach a moving point. (phase converges). There is a clock somewhere.

Nonlinear stability tests

Local linearization at equilibrium points: pendulums and inverted pendulums.
Lyapunov's method: Given a system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{1}$$

we can prove a nonlinear system is stable if we can find what is known as a Lyapunov function, often called $L(\mathbf{x})$ or $V(\mathbf{x})$ which only decreases. We assume the origin is an equilibrium point $\mathbf{f}(0) = 0$, and that $V(0) = 0$ too. $V(\mathbf{x}) > 0$ everywhere else. If $\dot{V} \leq 0$ everywhere except the origin then the origin is a stable equilibrium point, but the system may not get there. If $\dot{V} \leq 0$ everywhere except the origin then the origin is an asymptotically stable equilibrium point, and the system is guaranteed to get to the equilibrium.

The energy in a system is usually a good Lyapunov function. So are value functions (which we will talk about more later).

Other concepts

An important concept in dynamical systems is “bifurcation”
Entrainment. Clocks on the wall.
Feedback linearization.
Linearization about a trajectory.
Divergence of KF, Extended Kalman Filter, Unscented Kalman Filter
Model Predictive Control (Receding Horizon Control).

Dynamic Programming

What is it?
Curse of dimensionality
Ways to reach the curse of dimensionality:
Random actions
Bubbles
Function approximation