16-299 Lecture 11: ARMA models, low pass filtering

What is the problem?

We want to predict (or smooth) a signal which we don't know much about, and which is noisy.

Moving averages

One way to reduce the noise in a signal is to average adjacent values to estimate a corrected center value. This "window" slides along, estimating an entire new signal. In real time filtering, the value estimated is on the leading edge of the filter:

$$o_k = w_0 * i_k + w_1 * i_{k-1} + w_2 * i_{k-2} + \cdots \tag{1}$$

The crudest window (a "boxcar" or "rectangular" filter), which has all the same weights, favors frequencies that fit an odd number of half periods into the window. This causes ripples in the frequency response of the filter.

It is possible to design moving average filters by choosing a pattern of multipliers or weights that 1) favor the center or leading value and put less weight on the values far away from the center or leading value, and 2) avoids ripples.

Moving average filters are called "finite impulse response (FIR) filters" because their impulse response is the pattern of weights, and there are a finite number of weights.

Autoregressive models

In autoregressive models past values of the output are used to predict the next output:

$$o_k = w_0 * i_k + v_1 * o_{k-1} + v_2 * o_{k-2} + \cdots \tag{2}$$

Autoregressive models are also called "infinite impulse response (IIR) filters", because a single input takes an infinite number of iterations (samples) to die out to zero.

AutoRegressive Moving Average (ARMA) models

ARMA models combine moving average and autoregressive models.

$$
\begin{aligned}
o_k = {} & w_0 * i_k + w_1 * i_{k-1} + w_2 * i_{k-2} + \cdots \\
& + v_1 * o_{k-1} + v_2 * o_{k-2} + \cdots
\end{aligned}
\tag{3}
$$