16-299 Introduction to Feedback Control Systems
Spring 2021
Midterm Exam

Open note, open book, open internet.
You may use any resource except for other humans.
Intermediate steps must be legible and answers must be explained to receive credit.
There is a **24-hour time limit** on the exam,
Once you start it in Canvas you will have 24 hours to submit.
Your solution must be submitted as a single pdf file to Canvas no later than
8:30am EDT on Friday May 14.

1. (20 points) Consider a system with the transfer function
$$H(s) = \frac{5s+25}{s^2+5s+25},$$
i.e., $Y(s) = H(s)U(s)$. An input
$$u(t) = \frac{3}{\sqrt{2}}\sin(5t)$$
has been applied to this system for a long enough time that any transients have died out. What is $y(t)$?

2. Consider the LTI ODE
$$\frac{d^2}{dt^2}y(t) + 2\frac{dy}{dt}(t) + 5y(t) = 6\frac{du}{dt}(t) - 2u(t).$$
Answer the following questions. Please justify all of your answers.
   (a) (5 points) What are the poles and zeros of the system?
   (b) (5 points) What is the stability of the system?
   (c) (10 points) Will the step response of the system converge to a steady state value? If so, what value will it converge to?

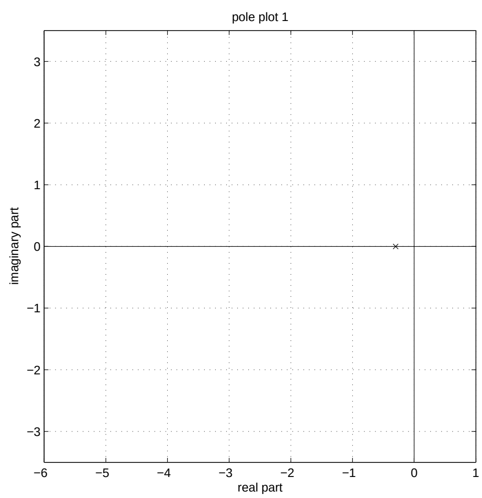
3. Consider a system with the transfer function
$$G(s) = \frac{5}{s^2+3s+10}.$$
The objective is to design a closed loop system that has a step response rise time $t_r < 0.4$ seconds and a percent overshoot $M_p < 5\%$.
   (a) (10 points) Sketch the "good" region of the complex plane, i.e., the region of the complex plane that corresponds to pole locations that will satisfy both of the step response design objectives. Please label your sketch well enough so that it is possible to tell exactly where the boundaries are.
   (b) (10 points) Consider the system under closed loop P control. Does there exist a k_p that satisfies the rise time constraint? Does there exist a k_p that satisfies the percent overshoot constraint? Does there exist a k_p that simultaneously satisfies both? Explain your answers.

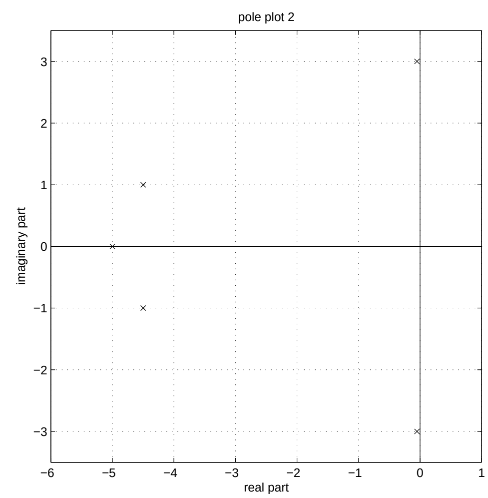
4. (20 points) Suppose you have two independent pose sensors for a vehicle. The first one provides the (North,East) position and a heading θ. The second one only provides (North, East) position. In other words:

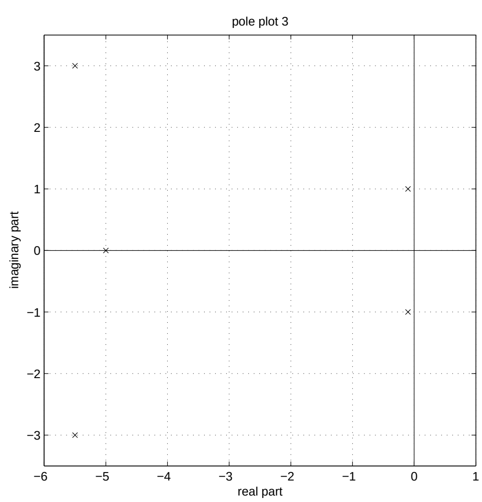
$$y_1 = \begin{bmatrix} \text{North} \\ \text{East} \\ \theta \end{bmatrix} \qquad y_2 = \begin{bmatrix} \text{North} \\ \text{East} \end{bmatrix}.$$

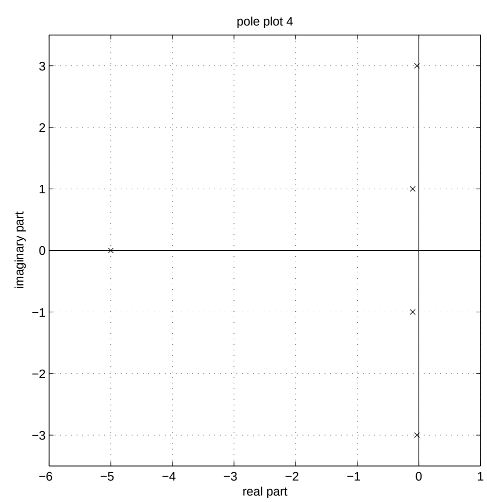
The two sensors are corrupted by noise signals that are zero mean Gaussian random variables with covariance matrices $\Sigma_1 \in \mathbb{R}^{3\times3}$ for the first sensor and $\Sigma_2 \in \mathbb{R}^{2\times2}$ for the second sensor. Suppose you receive y_1 and y_2 at the same time. What is your best estimate for the actual (North, East) position at that time? What is the covariance associated with that estimate?

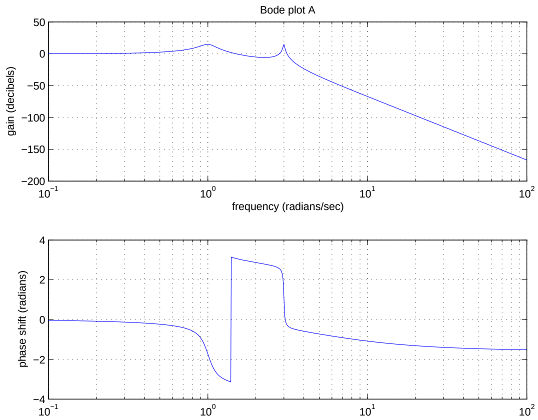
5. (20) The following pages contain pole location plots and Bode plots for 4 different systems. For each pole plot, identify which Bode plot goes with it, and explain your answer.

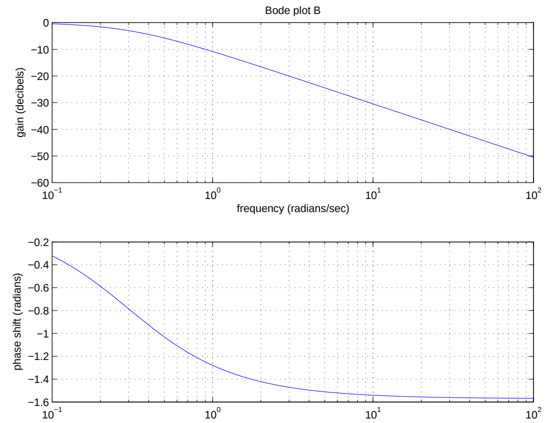


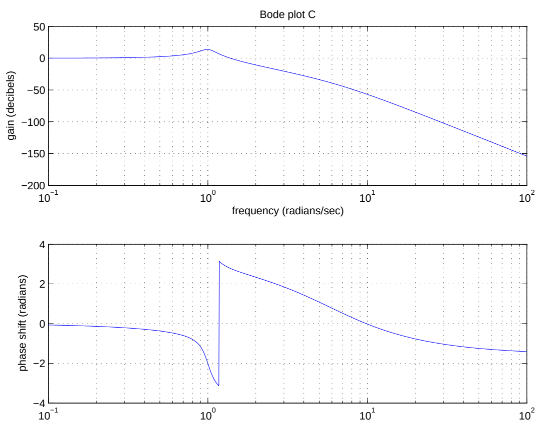


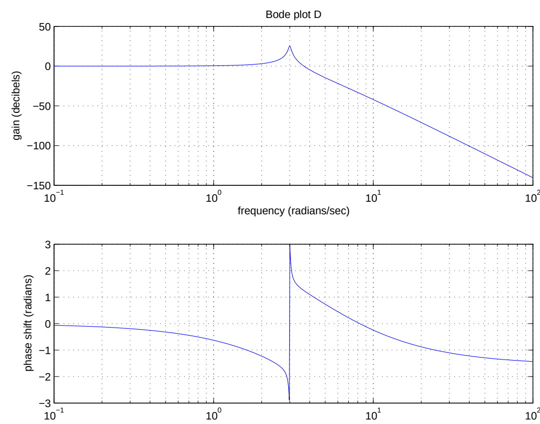





Bode plot A
gain (decibels)
frequency (radians/sec)
phase shift (radians)


Bode plot B
gain (decibels)
frequency (radians/sec)
phase shift (radians)


Bode plot C
gain (decibels)
frequency (radians/sec)
phase shift (radians)


Bode plot D
gain (decibels)
frequency (radians/sec)
phase shift (radians)