

# 16-299: Introduction to Feedback Control Systems

Spring 2021

## Problem Set 5

due 10 May 2021

Please show all work and write clearly—all intermediate steps must be shown and legible. **Submit solutions as a zip file that contains written explanations of problem solutions and relevant MATLAB code.**

You are encouraged to work with other students to find the answers to these problems, however each student must submit his/her own *unique* set of solutions. If you do work with other students, please give them credit by listing their names at the end of your solutions.

The sequence of problems in this set will implement a Kalman filter pose estimate for the differential drive robot. The configuration of the robot can be represented by the vector

$$q = \begin{bmatrix} x_r \\ y_r \\ \theta \end{bmatrix}$$

where  $(x_r, y_r)$  is the 2D position in meters of the point midway between the two wheels, and  $\theta$  is the angle in radians of the body relative to the  $x$ -axis. The input vector is

$$u = \begin{bmatrix} v_f \\ \omega \end{bmatrix},$$

where  $v_f$  is the forward speed of the body in meters per second, and  $\omega$  angular velocity of the body in radians per second. The kinematic discrete time equation of motion is

$$q[k+1] = \begin{bmatrix} q_1[k] + T(u_1[k] + v_1[k]) \cos q[3] \\ q_2[k] + T(u_1[k] + v_1[k]) \sin q[3] \\ u_2[k] + v_2[k] \end{bmatrix},$$

where  $T = 0.01$  seconds is the rate at which the input is applied, and  $v[k]$  is assumed to zero-mean white Gaussian noise with constant covariance matrix  $V \in \mathbb{R}^{2 \times 2}$ .

The robot receives GPS measurements once every 10 timesteps, where the output equation is

$$y[k] = \begin{bmatrix} q_1[k] \\ q_2[k] \end{bmatrix} + w[k],$$

where  $w[k]$  is assumed to zero-mean white Gaussian noise with constant covariance matrix  $W \in \mathbb{R}^{2 \times 2}$ .

1. (15 points) Find the linearized approximation of the system as a function of the current state. In particular, your approximation should look like

$$q[k+1] \approx F(q[k], u[k])q[k] + G(q[k])u[k] + \Gamma(q[k])v[k],$$

where  $F(q[k])$  is a  $3 \times 3$  matrix, and  $G(q[k])$  and  $\Gamma(q[k])$  are both  $3 \times 2$ .

2. (20 points) The file `calibrate.mat` contains data generated during a run where an expensive ground truth system was available that could measure the full state of the robot. This data can be used to estimate the noise parameters, namely the noise on the input and on the GPS. The variables in `calibrate.mat` are:

- `t_groundtruth`: a vector of times associated with the input signals.
- `q_groundtruth`: a matrix whose columns are the robot states at the times associated the times in `t_groundtruth`. These states are assumed to be perfect.
- `u`: a matrix whose columns are the robot inputs at the times associated with `t_groundtruth`. This signal is the signal applied to the robot, it does *not* include the noise terms in  $v[k]$ .
- `t_y`: a vector of times associated with the GPS measurement.
- `y`: a matrix whose columns are the noisy GPS measurements taken at the times in `t_y`.

Using this information, estimate the process covariance  $V$  and measurement covariance  $W$ . You may want to start with  $W$  since computing it is the simpler of the two.

3. The file `kfData.mat` contains another data set that you can use to demonstrate the effectiveness of a Kalman filter. The variables it contains are

- `t`: a vector of times associated with the input signals.
- `u`: a matrix whose columns are the robot inputs at the times associated with `t`. This signal is the signal applied to the robot, it does *not* include the noise terms in  $v[k]$ .
- `t_y`: a vector of times associated with the GPS measurement.
- `y`: a matrix whose columns are the noisy GPS measurements taken at the times in `t_y`.
- `q_groundtruth`: a matrix whose columns are the robot states at the times associated the times in `t`. These states are assumed to be perfect. Note that this should not be used to help you solve the problem, it is meant for comparison only.

Use an initial estimate

$$\hat{q}[1] = \begin{bmatrix} 0.355 \\ -1.590 \\ 0.682 \end{bmatrix}$$

and assume that the initial covariance matrix is

$$P[1 | 1] = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 0.154 \end{bmatrix}$$

- (20 points) Write a dead reckoner, i.e., make an extended Kalman filter that ignores the GPS signal and only does prediction steps. Plot  $y_r$  vs.  $x_r$  for your dead reckoner on the same plot with  $y_r$  vs.  $x_r$  for the ground truth.
- (30 points) Write a full extended Kalman filter. Make a plot that contains the following:
  - a trajectory plot of  $y_r$  vs.  $x_r$  for the ground truth.
  - a scatter plot or  $y_r$  vs.  $x_r$  from the GPS measurements.
  - a trajectory plot of  $y_r$  vs.  $x_r$  for your EKF.
- (15 points) Rerun your EKF, only this time, scale  $W$  down by a factor of 10. Generate all of the same plots. Explain why the difference between this plot and the prior one makes sense.