16-299: Introduction to Feedback Control Systems

Spring 2021

Problem Set 4

due 22 April 2021

Please show all work and write clearly—all intermediate steps must be shown and legible. **Submit solutions as a zip file that contains written explanations of problem solutions and relevant MATLAB code.**

You are encouraged to work with other students to find the answers to these problems, however each student must submit his/her own *unique* set of solutions. If you do work with other students, please give them credit by listing their names at the end of your solutions.

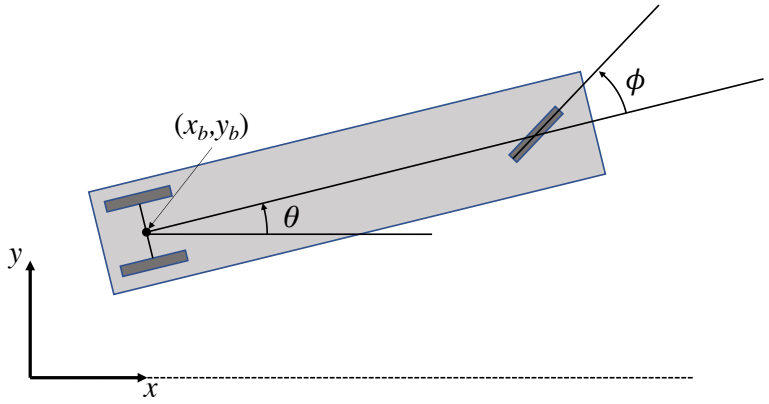

Figure 1: Buggy configuration.

The configuration of a buggy can be represented by the vector

$$x = \begin{bmatrix} x_b \\ y_b \\ \theta \\ \phi \end{bmatrix}$$

where (x_b, y_b) is the 2D position of the point at the center of the rear axle, θ is the angle of the buggy body relative to the x-axis, and ϕ is the angle of the front steering wheel relative to the body. Both angles are in radians. The input vector velocities are the forward speed of the buggy v_f and steering speed of the front wheel ω_ϕ. The kinematic equation of motion is

$$\dot{x} = \begin{bmatrix} v_f \cos\theta \\ v_f \sin\theta \\ \frac{v_f}{\ell} \tan\phi \\ \omega_\phi \end{bmatrix},$$

where v_f is the forward velocity of the cart. Assume that v_f is a constant velocity, $v_f = 10$ m/s and the wheel base ℓ is 1.6 m. Design a feedback controller $u(x)$ that will cause the cart to converge to and follow the line $y = 0$. You can use any method we have learned in this class – make sure to clearly explain the steps in the method you use. Demonstrate your controller through MATLAB simulation using a variety of initial conditions. Provide time series plots of the configuration variables, and explain how they show that your controller converges. (this is the only required problem, it is worth 100 points).

Extra credit (15 points) Use MATLAB plotting functions to animate the buggy under your controller and submit a movie that shows it converging from at least 3 different initial conditions.