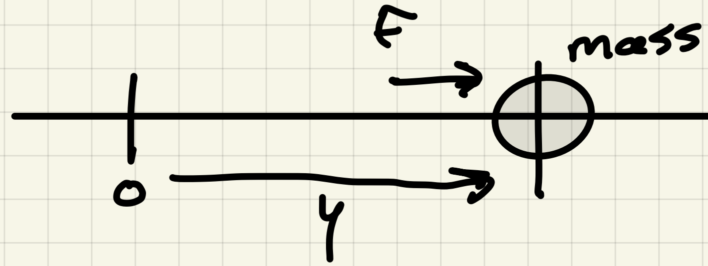
16219 Feb 4 2021

Bead on a wire



input: F

output: y

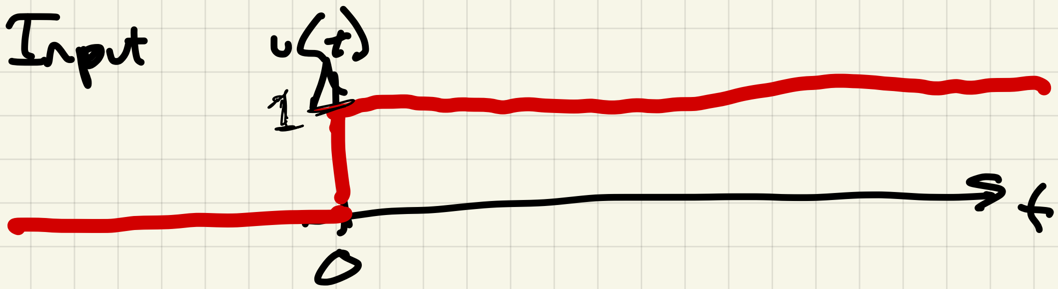
$$\xrightarrow{u} \boxed{\ddot{y} = \frac{1}{m} u} \xrightarrow{y}$$

$$\dot{y} = \frac{dy}{dt}$$

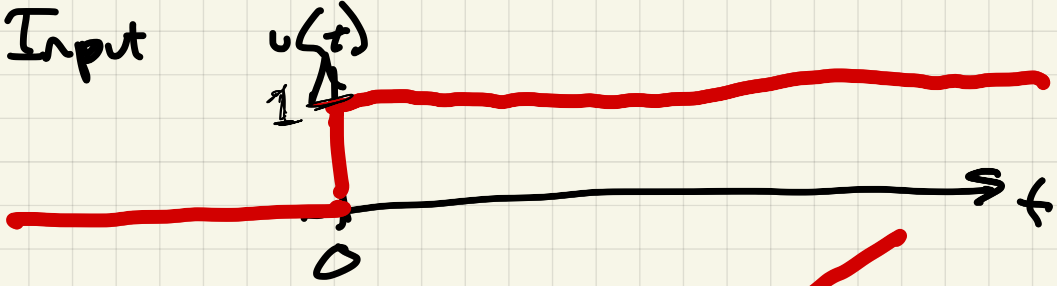
$$\ddot{y} = \frac{d^2 y}{dt^2}$$

$$F = ma \longrightarrow u = m\ddot{y} \longrightarrow \ddot{y} = \frac{1}{m} u$$

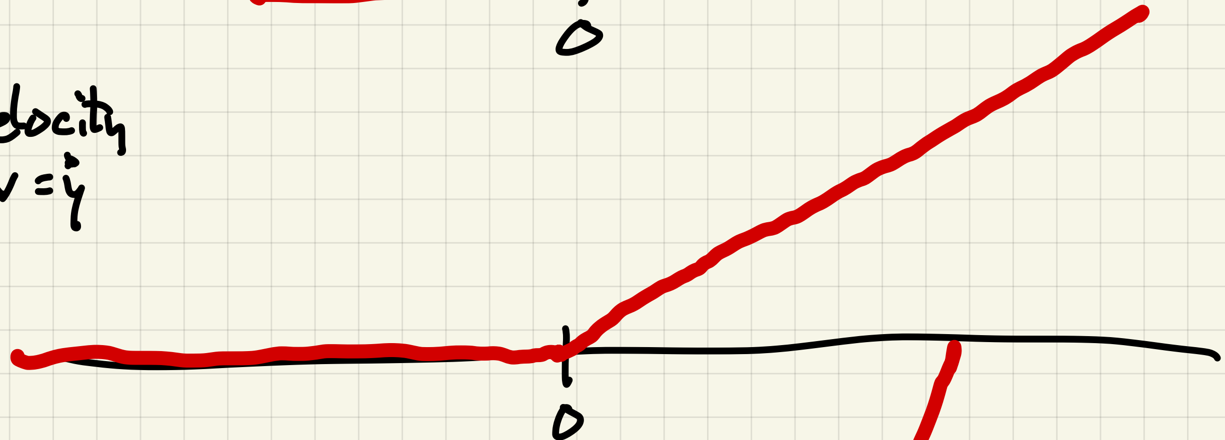
Step Input



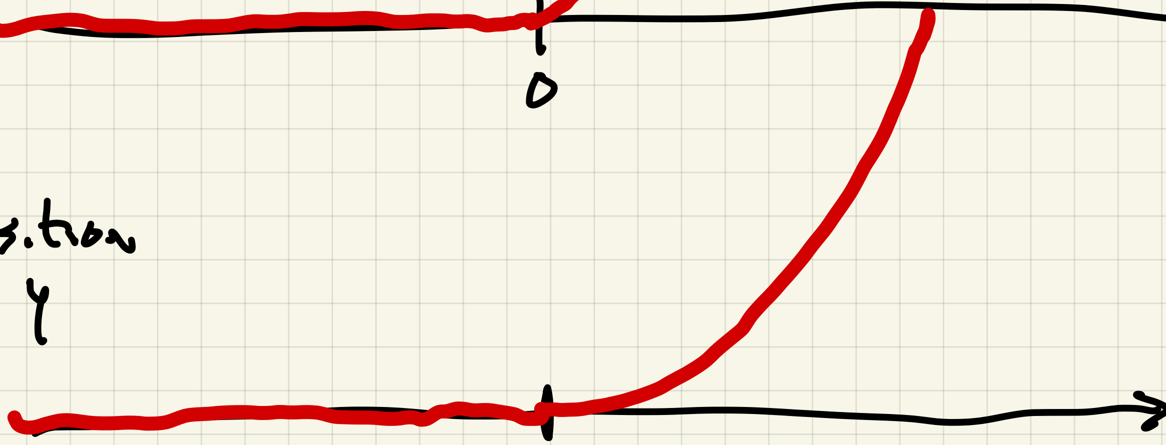
Step Input

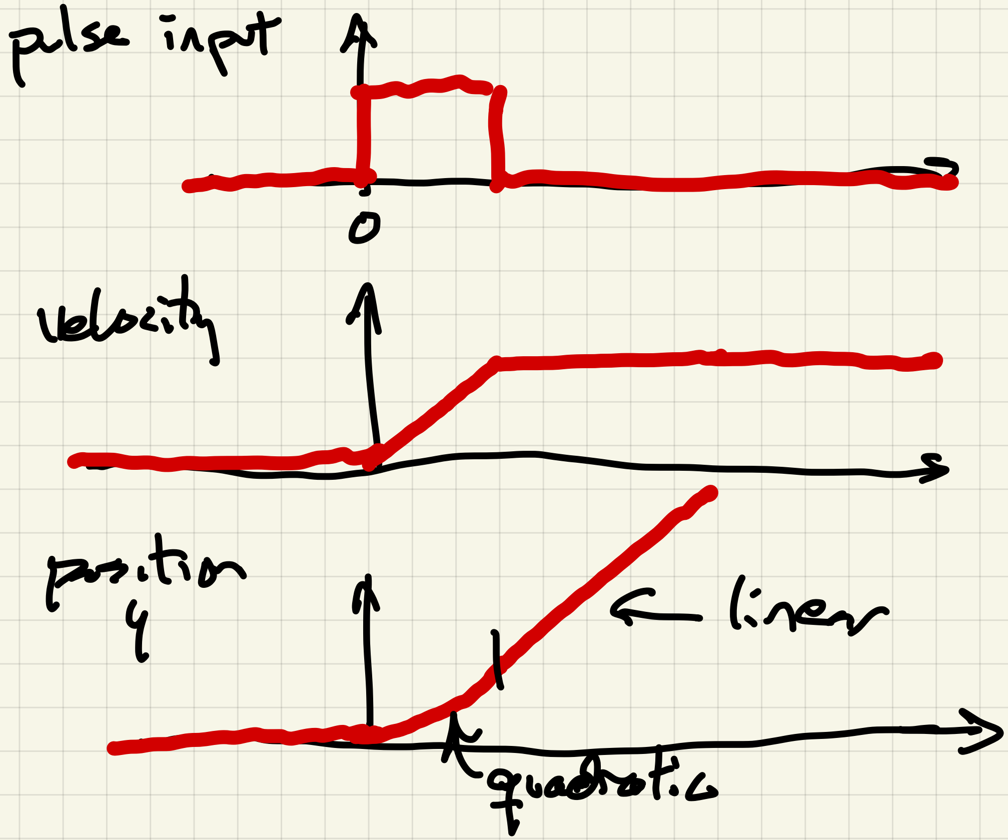


velocity
$v = \dot{y}$



position
y

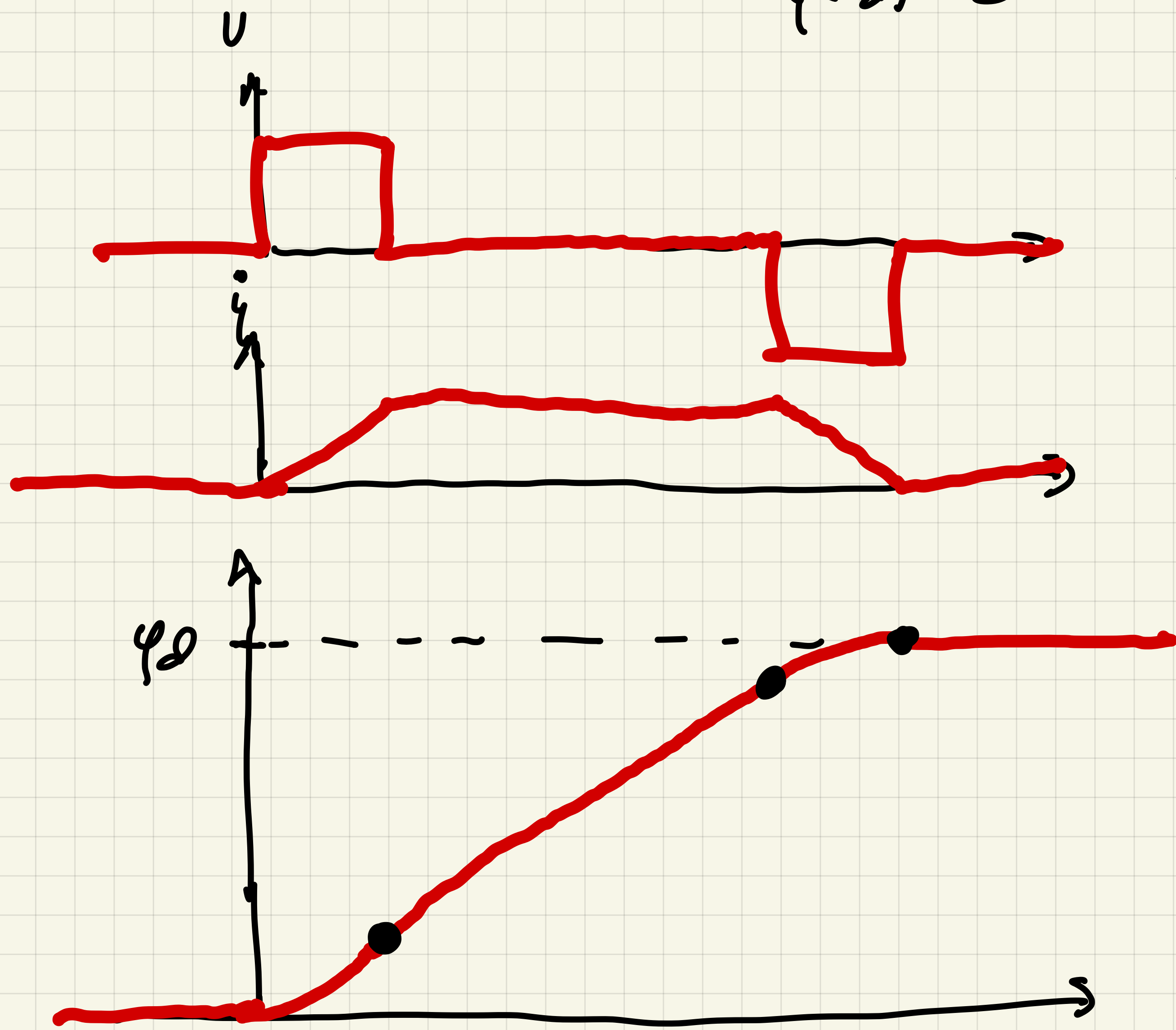



pulse input
0
velocity
position
y
linear
quadratic

sample problem. start at $y = \dot{y} = 0$

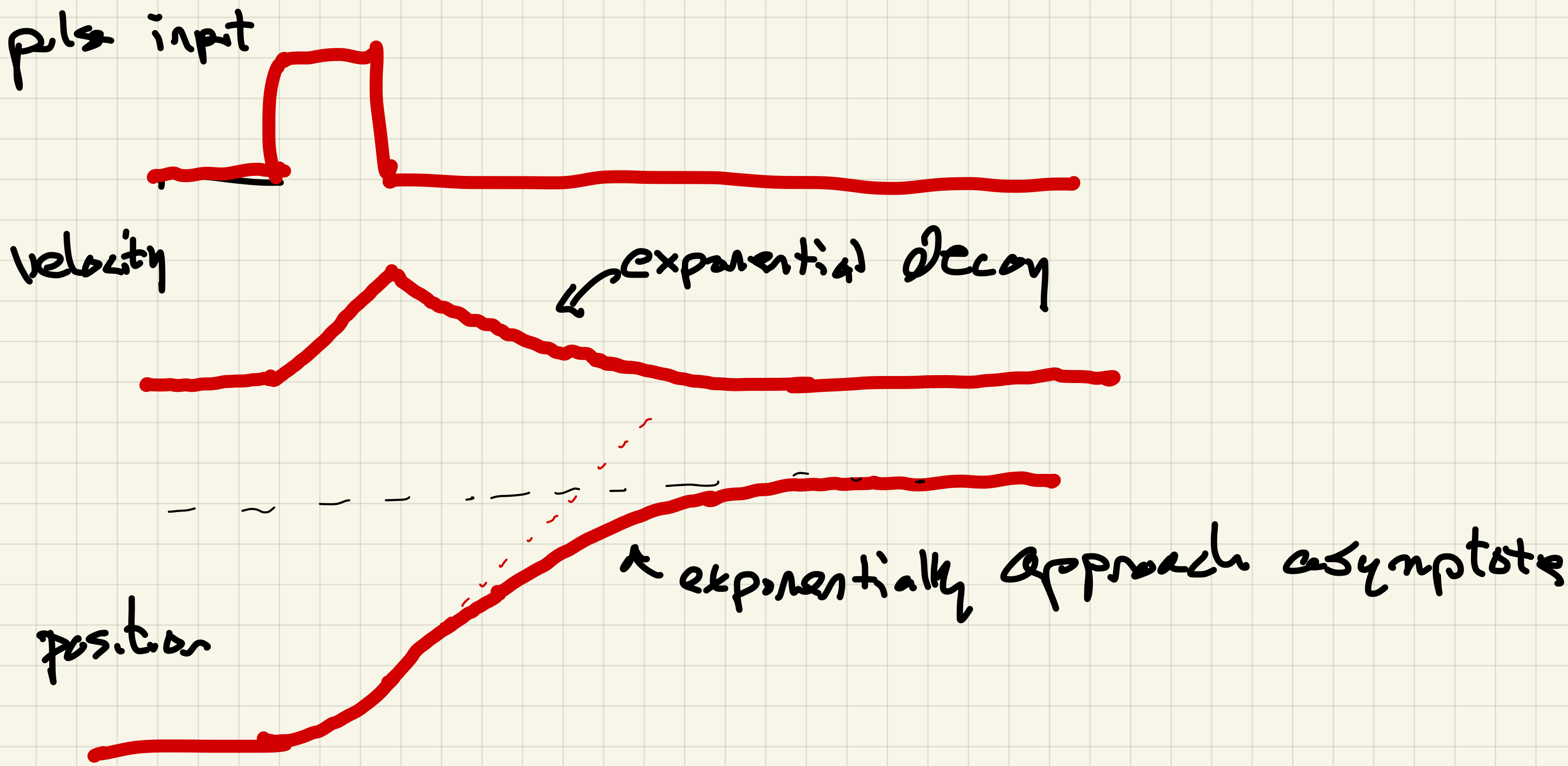
get to $y(t_d) = y_d$

$\dot{y}(t_d) = 0$



Linear
Spline
Parabolic
Blends
LSPB

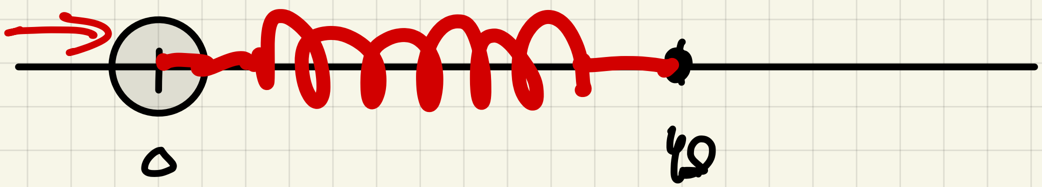
Suppose there is a small amount of friction on the wire

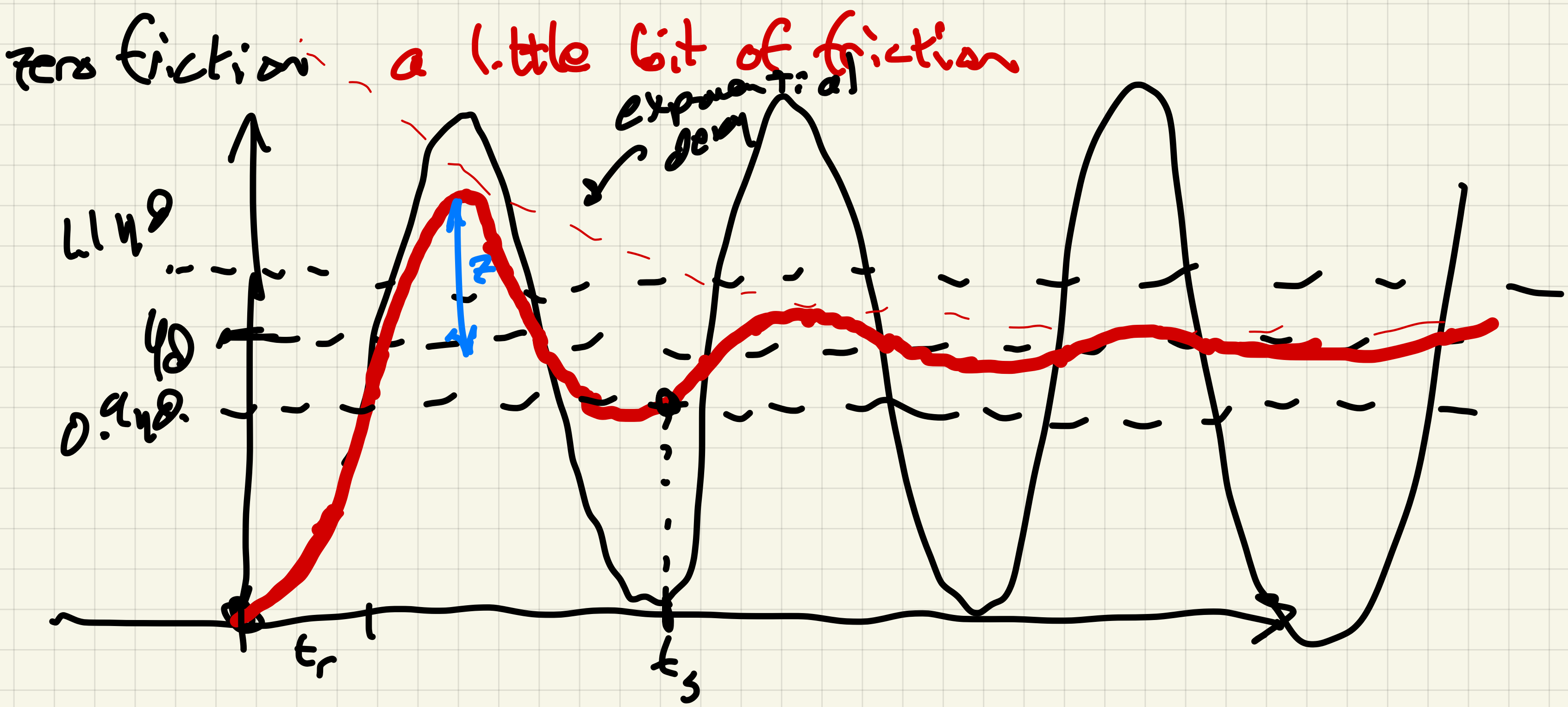


really hard to plan open loop.

closing the loop:

$u = F = k(y_d - y)$



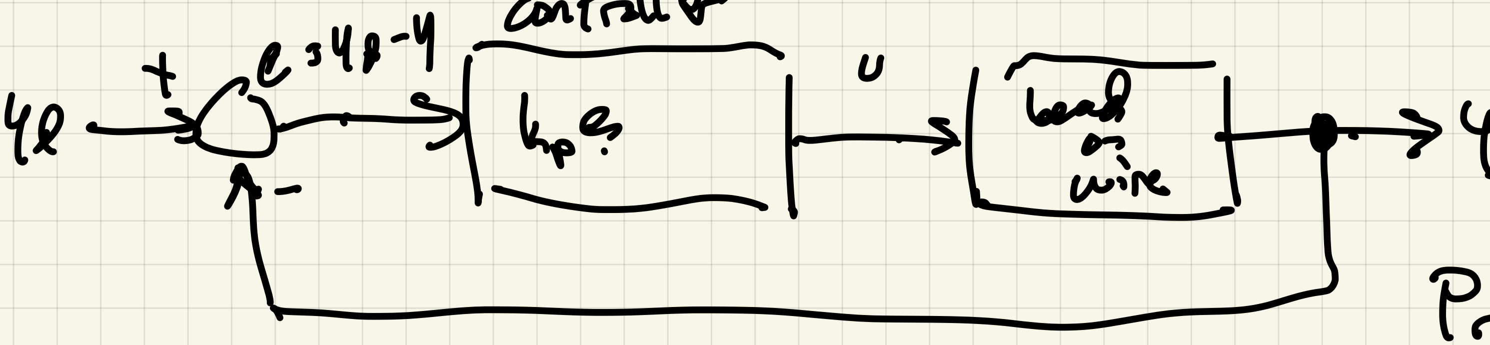


rise time t_r

10 % settling time t_s

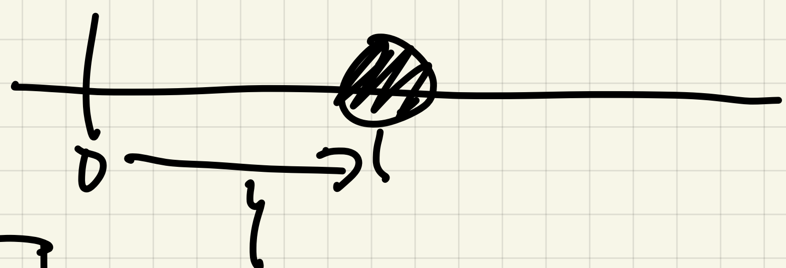
percent overshoot $100 \frac{z}{y_d}$

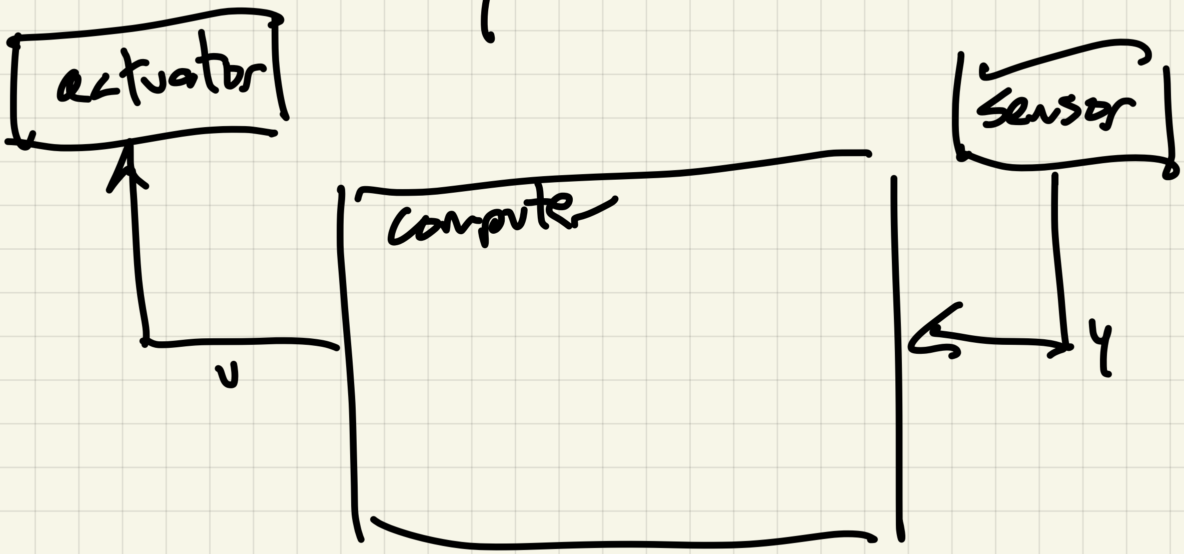
$u = k_p(y_d - y)$

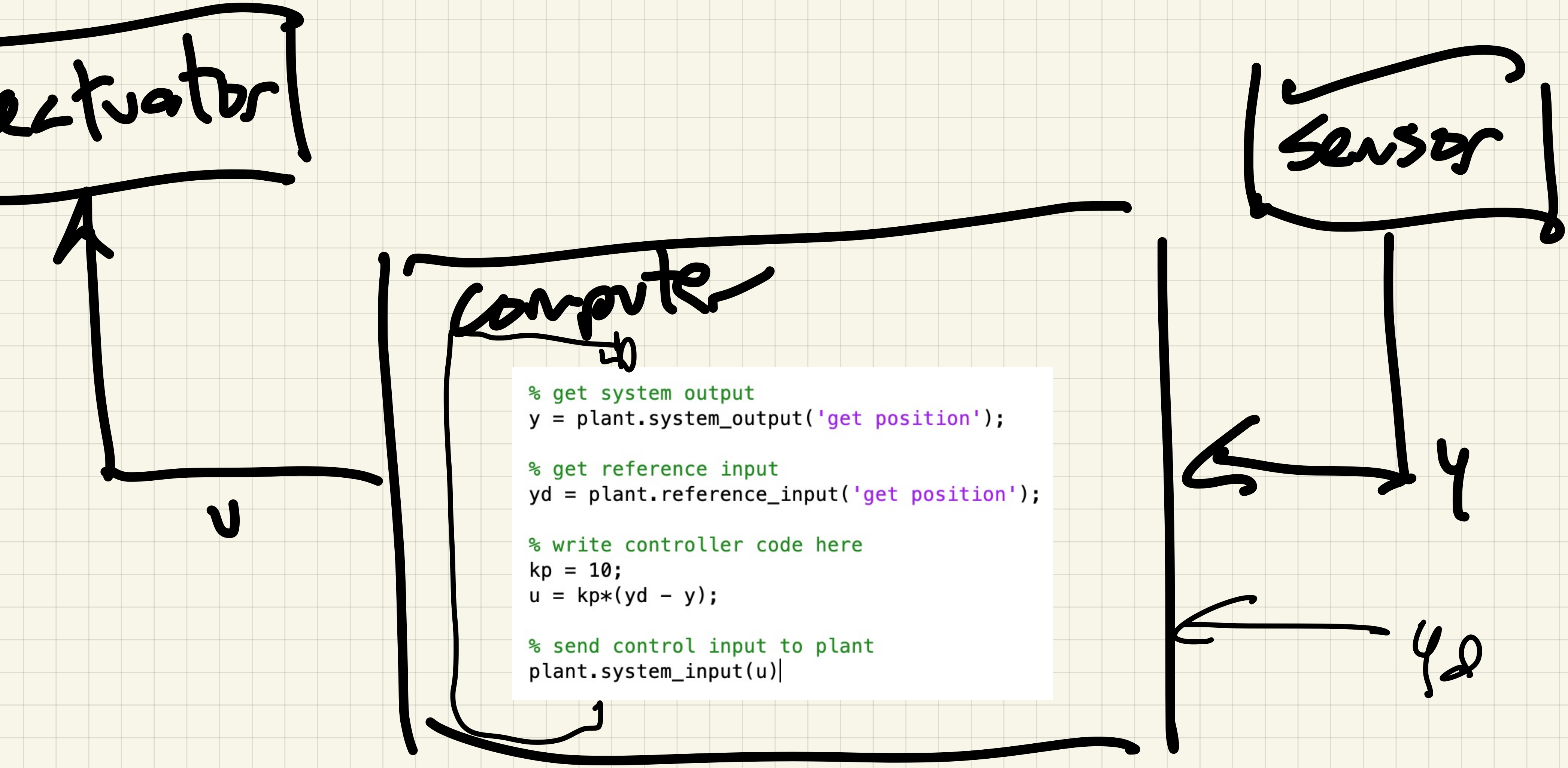


Proportional Control

P control

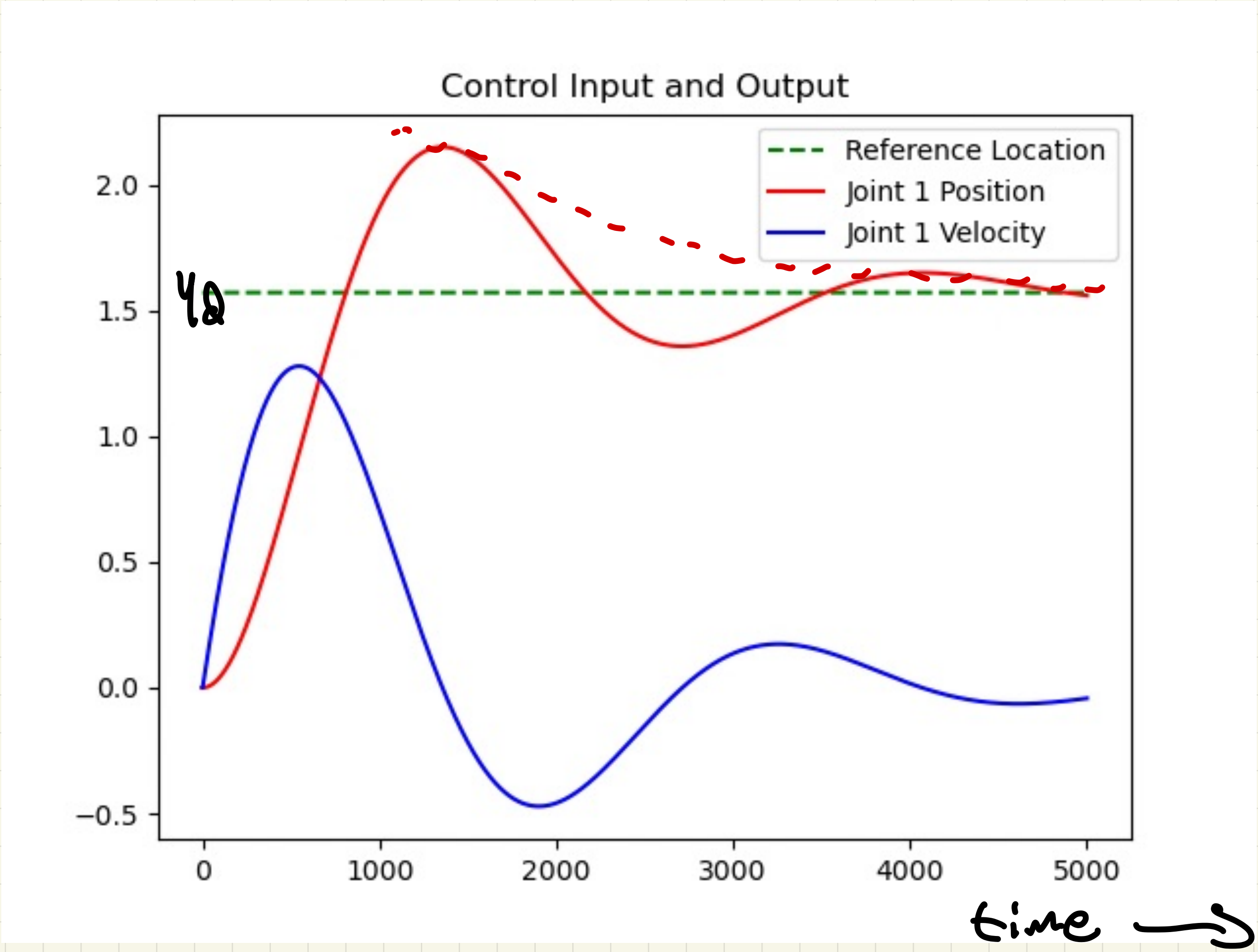


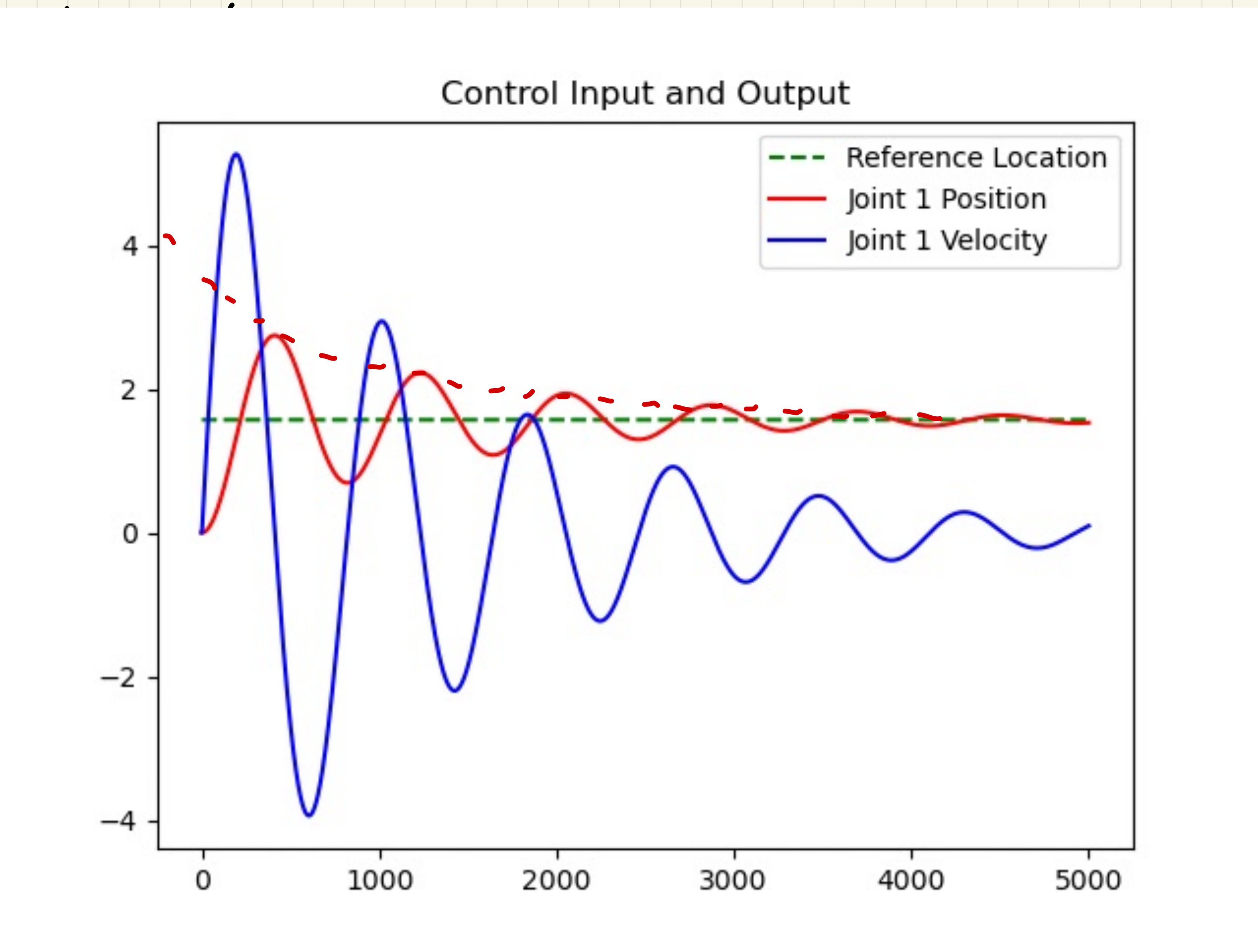


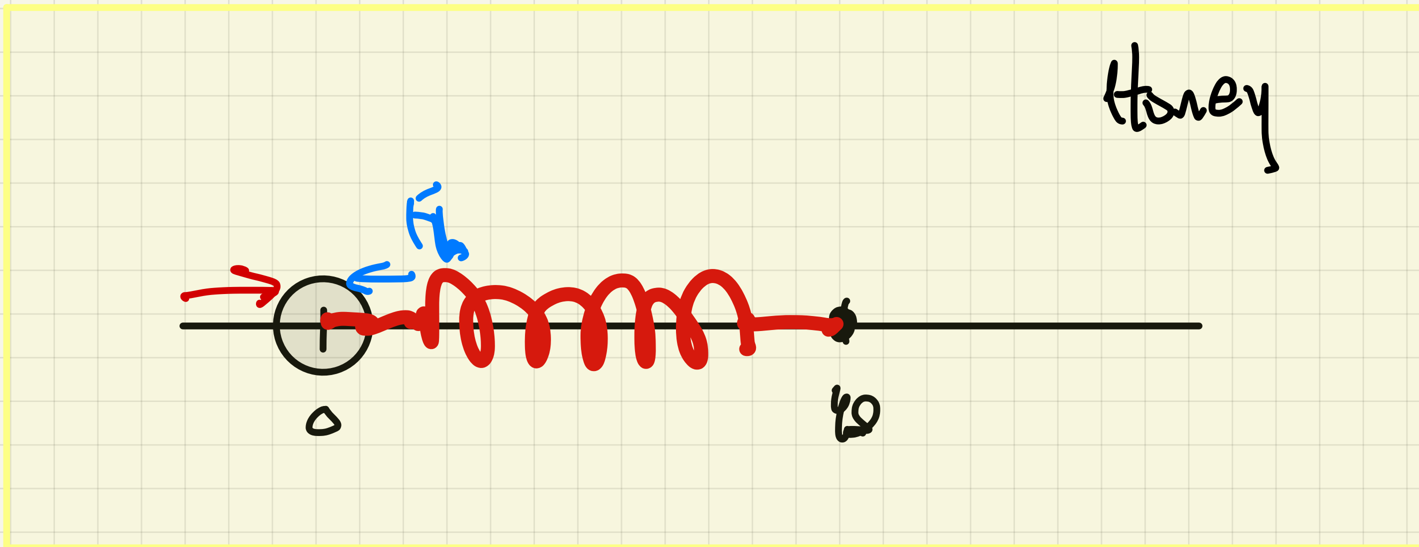



```
% get system output
y = plant.system_output('get position');

% get reference input
yd = plant.reference_input('get position');

% write controller code here
kp = 10;
u = kp*(yd - y);

% send control input to plant
plant.system_input(u)
```

P control with low gain k_p




Control Input and Output
Reference Location
Joint 1 Position
Joint 1 Velocity
4
2
0
−2
−4
0
1000
2000
3000
4000
5000

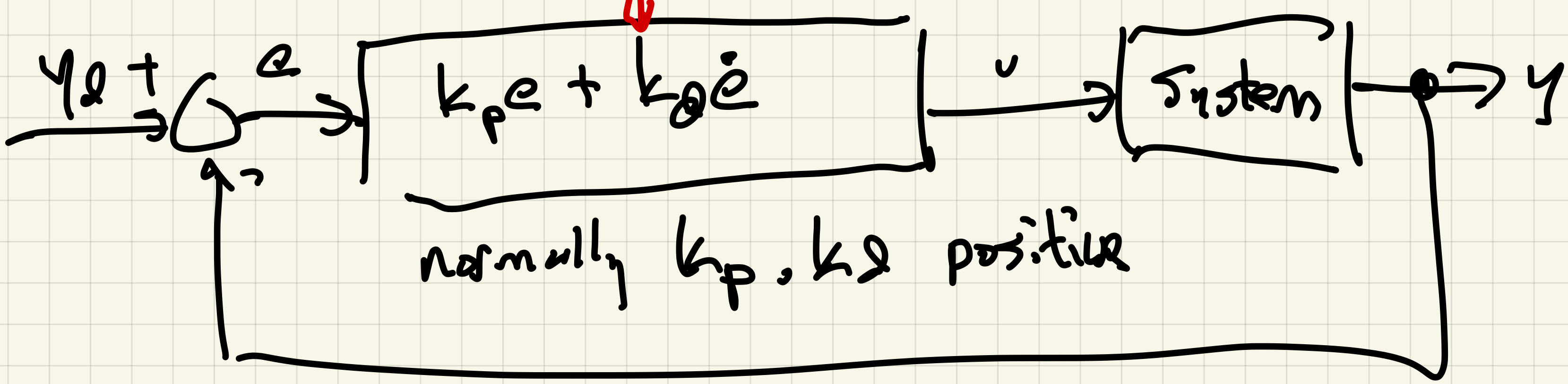


Honey gives viscous damping

$$F_{honey} = -\gamma v$$

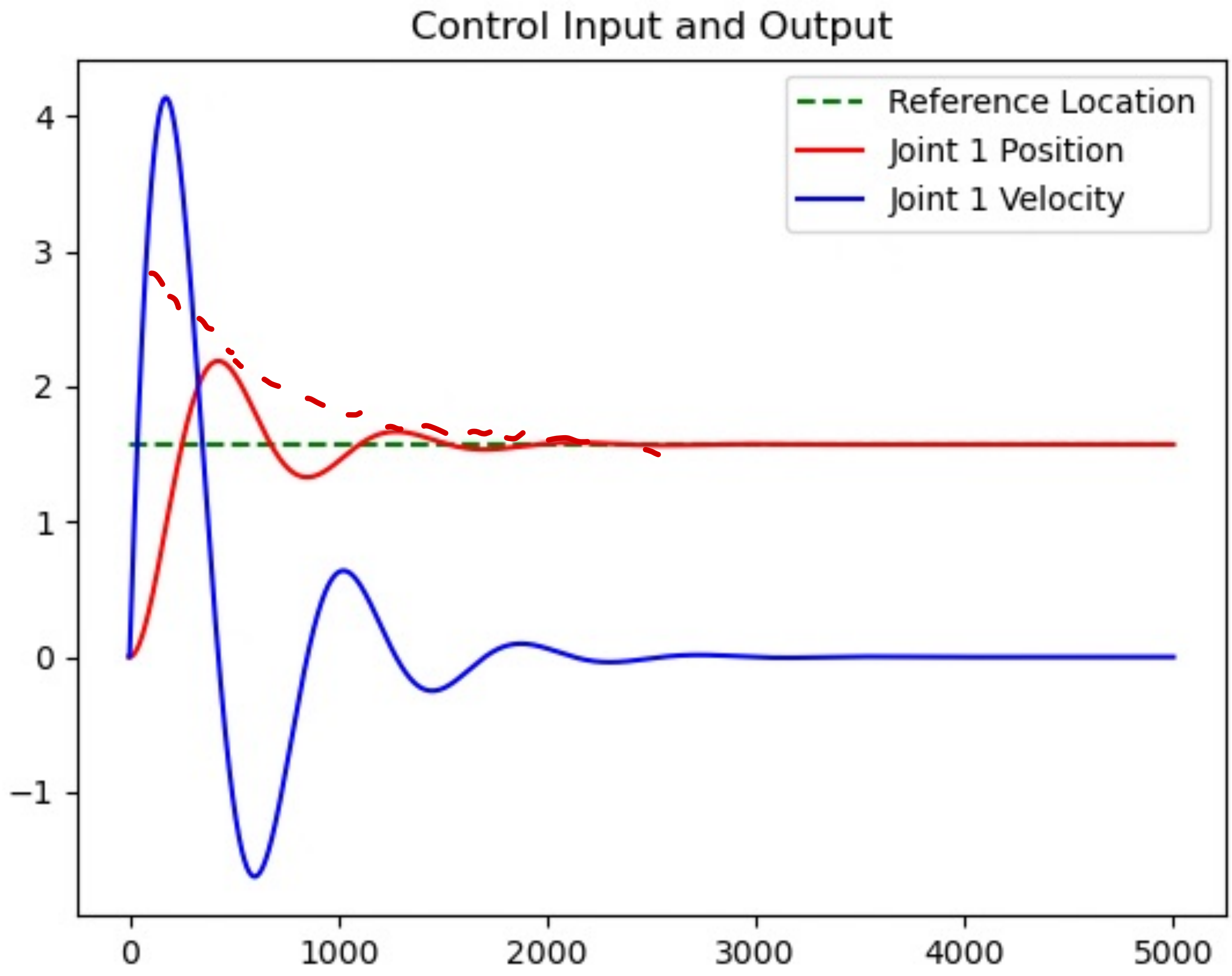
Proportional-Derivative control

PD control

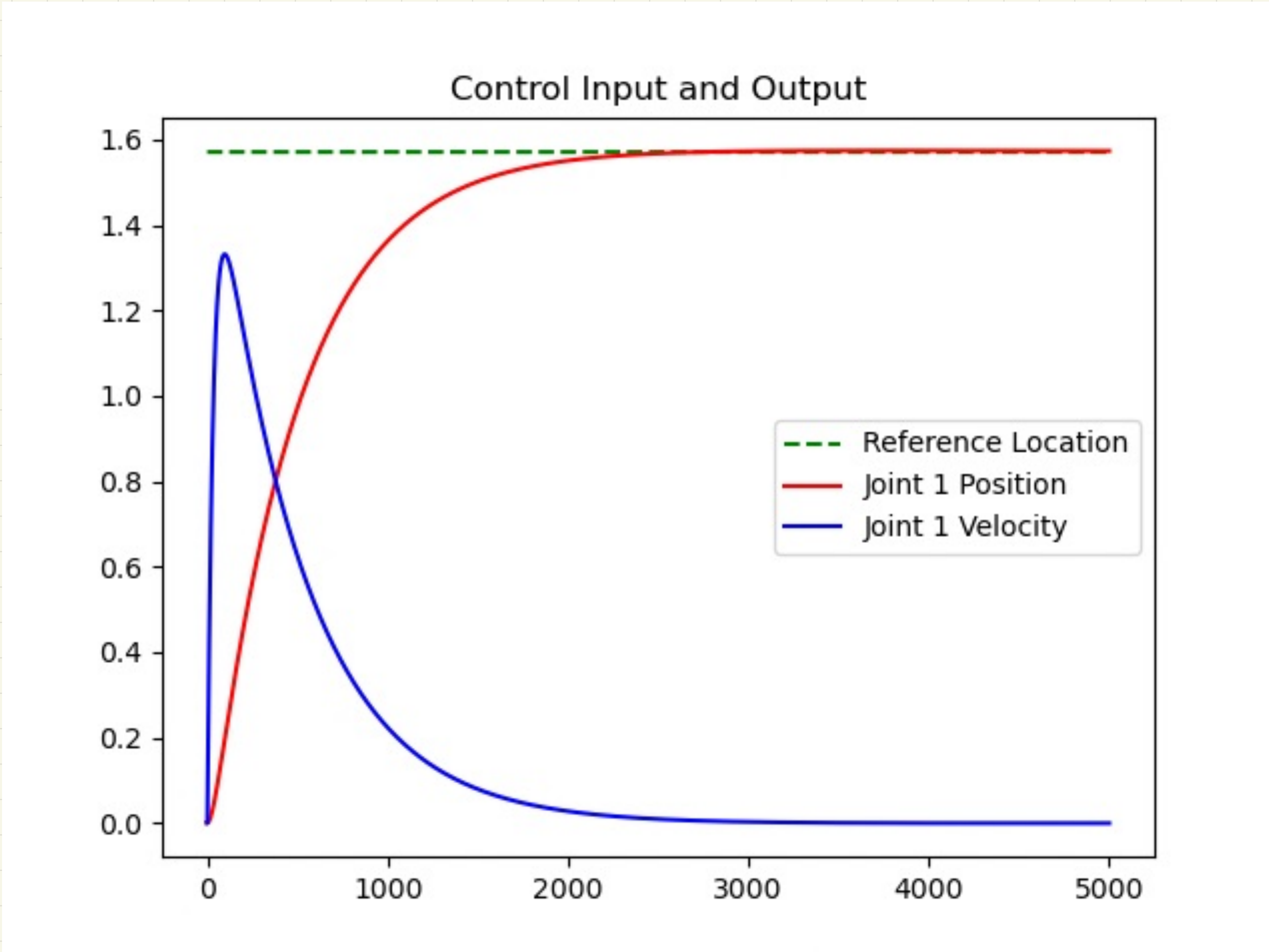


normally k_p, k_d positive

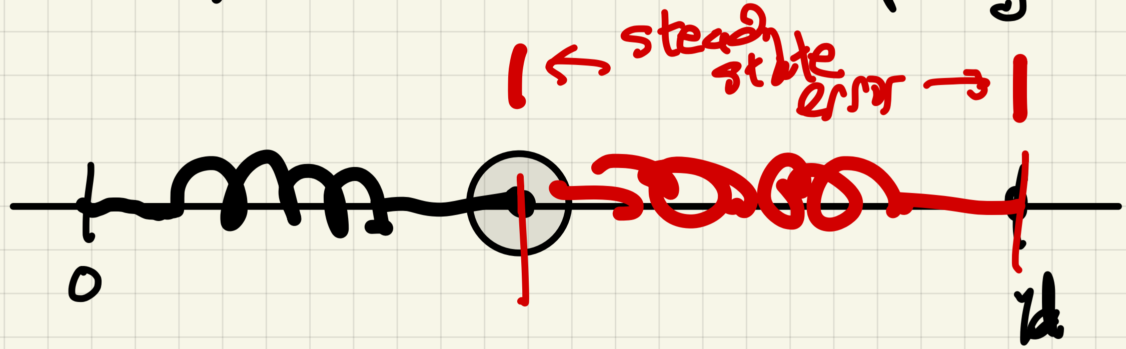
PD with low K_d

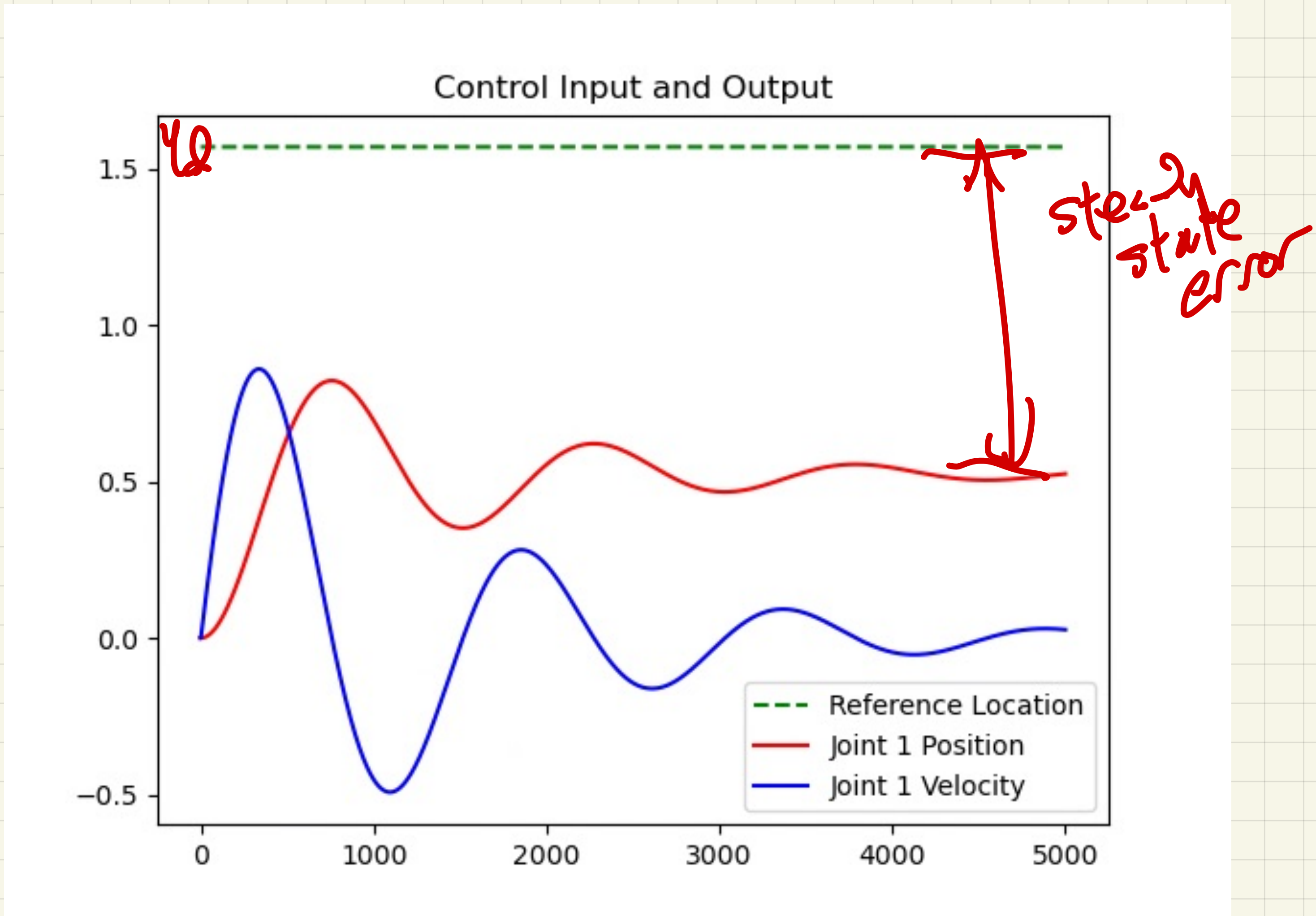


higher kd

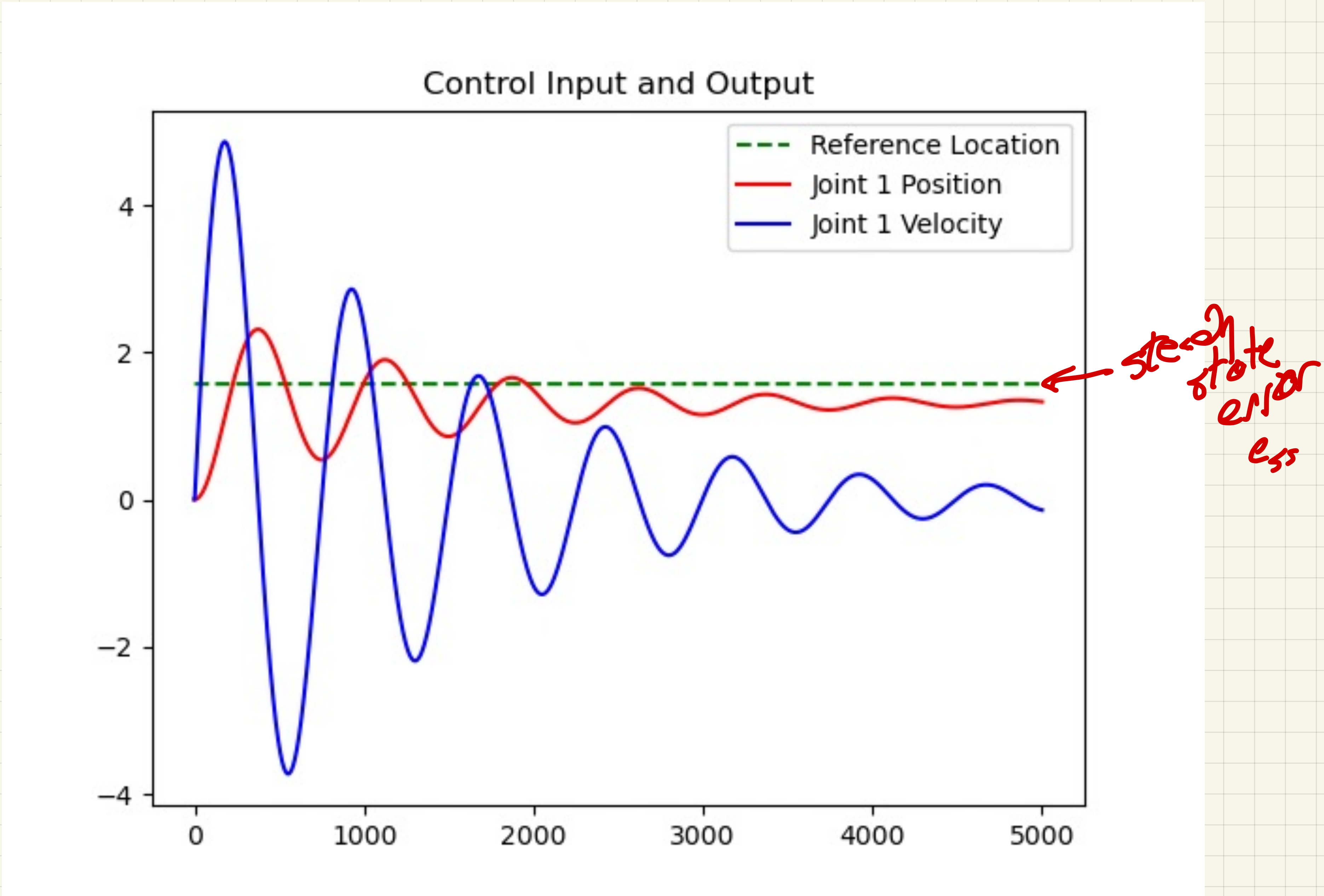

Control Input and Output
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000
Reference Location
Joint 1 Position
Joint 1 Velocity

What if system has its own spring?



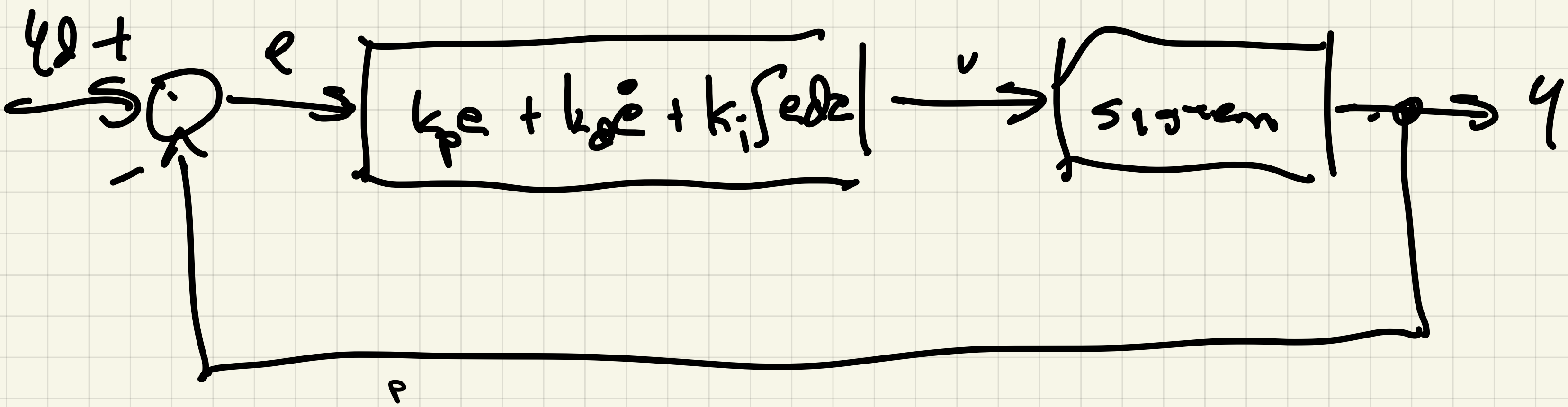


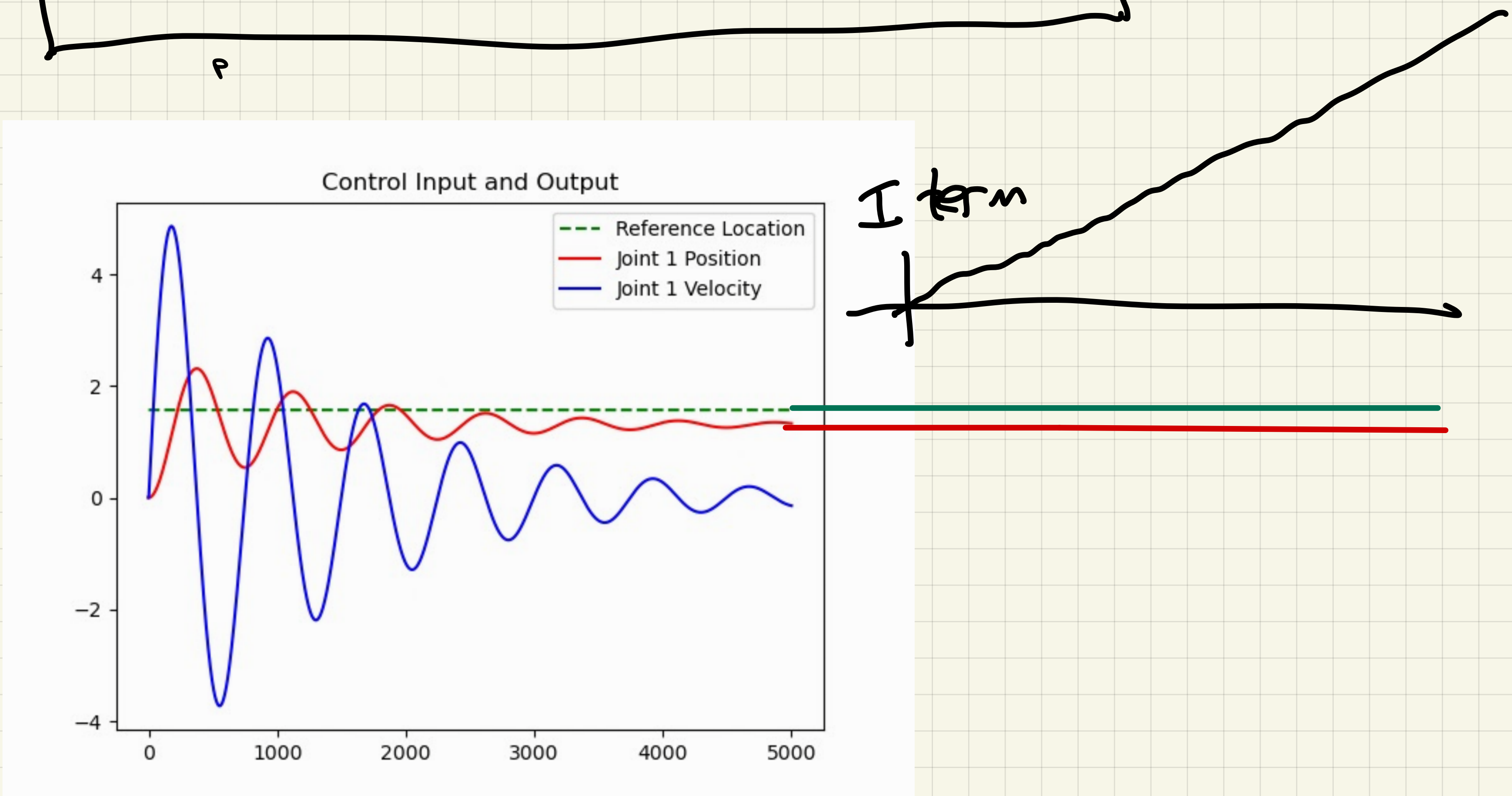
higher k_p



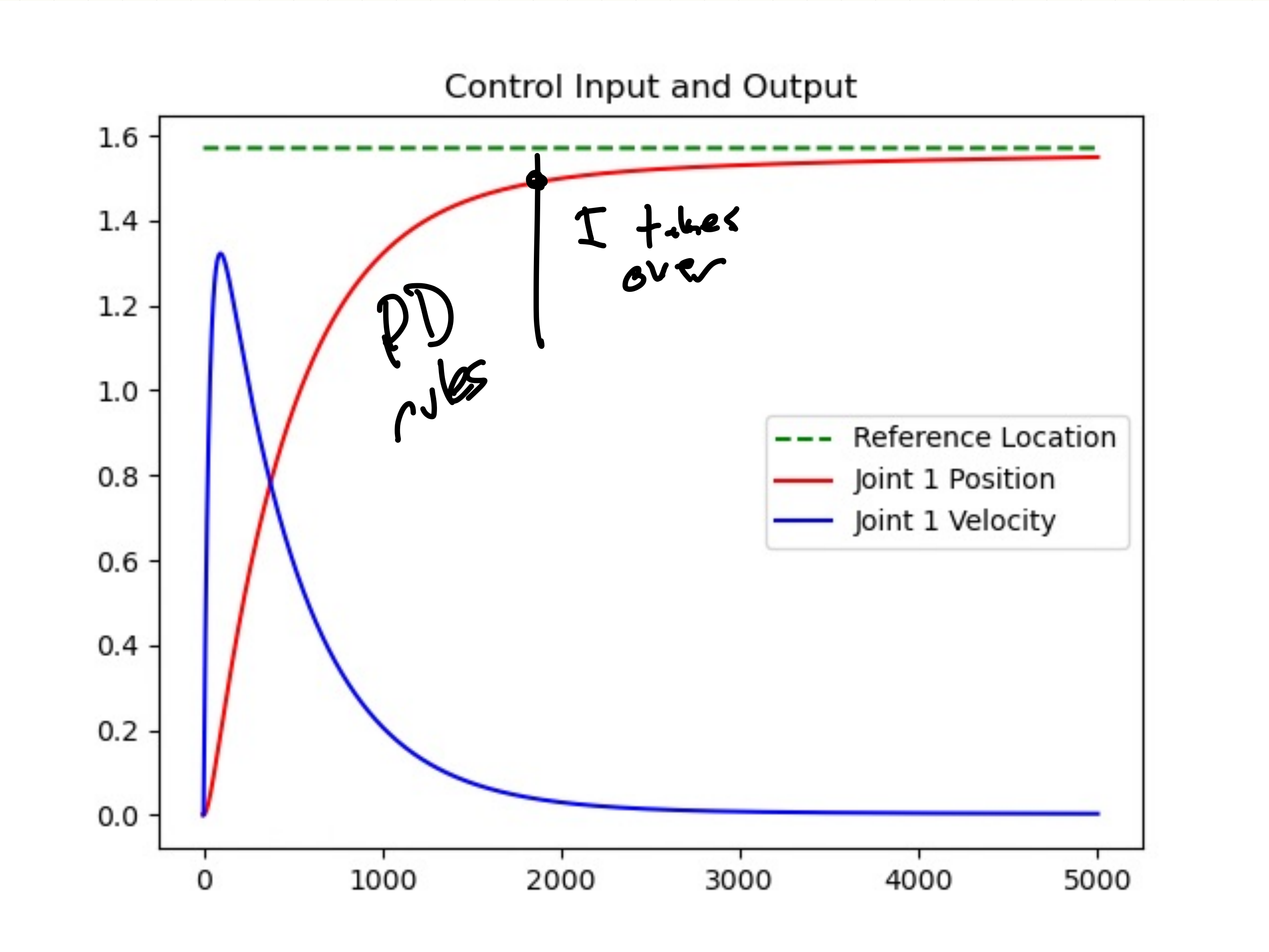
Control Input and Output
Reference Location
Joint 1 Position
Joint 1 Velocity
4
2
0
−2
−4
0
1000
2000
3000
4000
5000
steady state error
e_{ss}

Integral Term

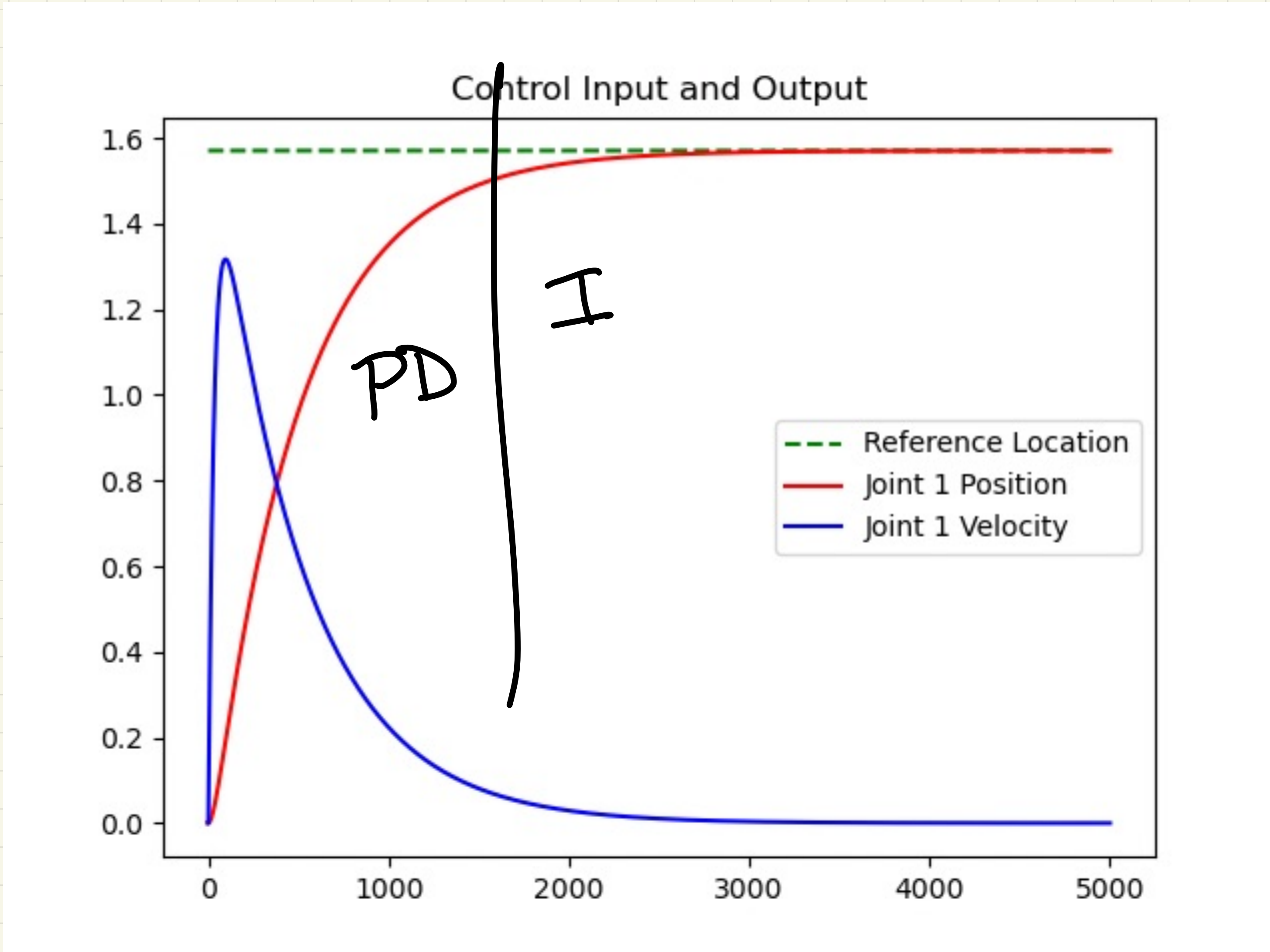




Very very small I term added



Bigger k_i;


Control Input and Output
PD
I
Reference Location
Joint 1 Position
Joint 1 Velocity
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
5000

even bigger k_i