Searching: Deterministic single-agent
Actually, this is optimization over time with discrete variables

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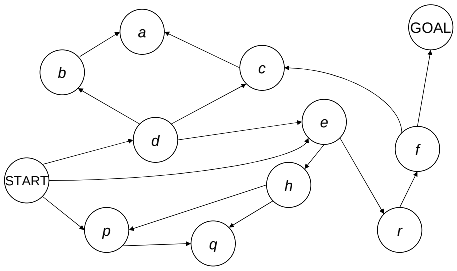
Overview

- Deterministic, discrete, single-agent, search problems
- Uninformed search
- Breadth First Search
- Optimality, Completeness, Time and Space complexity
- Search Trees
- Depth First Search
- Iterative Deepening
- Best First "Greedy" Search

A search problem



How do we get from S to G? And what's the smallest possible number of transitions?

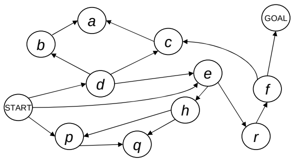
Formalizing a search problem

A search problem has five components:

Q , *S* , *G* , **succs** , **cost**

- *Q* is a finite set of states.
- $S \subseteq Q$ is a non-empty set of start states.
- $G \subseteq Q$ is a non-empty set of goal states.
- **succs** : $Q \rightarrow P(Q)$ is a function which takes a state as input and returns a set of states as output. **succs**(*s*) means "the set of states you can reach from *s* in one step".
- **cost** : $Q , Q \rightarrow$ *Positive Number* is a function which takes two states, *s* and *s*', as input. It returns the one-step cost of traveling from *s* to *s*'. The cost function is only defined when *s*' is a successor state of *s*.

Our Search Problem



Q = {START, *a* , *b* , *c* , *d* , *e* , *f* , *h* , *p* , *q* , *r* , GOAL}

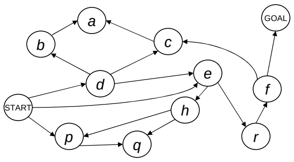
S = { START }

G = { GOAL }

succs(*b*) = { *a* }
succs(*e*) = { *h* , *r* }
succs(*a*) = NULL … etc.

cost(*s*,*s'*) = 1 for all transitions

Our Search Problem



Q = {START, *a* , *b* , *c* , *d* , *e* , *f* , *h* , *p* , *q* , *r* , GOAL}

S = { START }

G = { GOAL }

succs(*b*) = { *a* }
succs(*e*) = { *h* , *r* }
succs(*a*) = NULL … etc.

cost(*s*,*s'*) = 1 for all transitions

Why do we care? What problems are like this?

Search Problems

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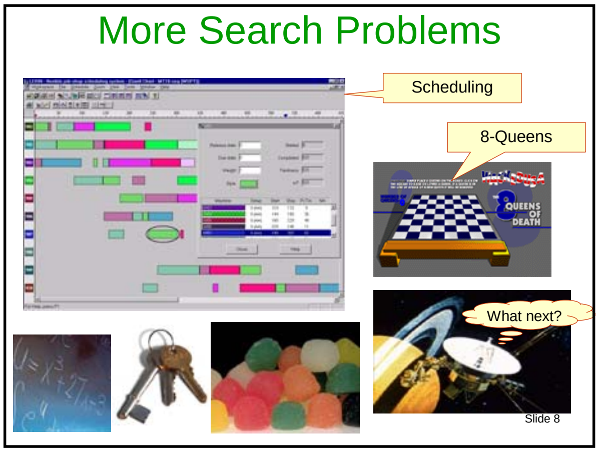

More Search Problems

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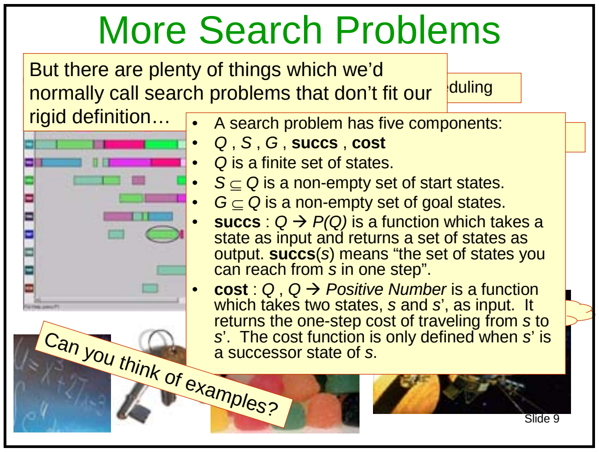

More Search Problems

But there are plenty of things which we'd normally call search problems that don't fit our rigid definition…

- A search problem has five components:
- *Q* , *S* , *G* , **succs** , **cost**
- *Q* is a finite set of states.
- *S* ⊆ *Q* is a non-empty set of start states.
- *G* ⊆ *Q* is a non-empty set of goal states.
- **succs** : *Q* → *P(Q)* is a function which takes a state as input and returns a set of states as output. **succs**(*s*) means "the set of states you can reach from *s* in one step".
- **cost** : *Q* , *Q* → *Positive Number* is a function which takes two states, *s* and *s*', as input. It returns the one-step cost of traveling from *s* to *s*'. The cost function is only defined when *s*' is a successor state of *s*.

Can you think of examples?

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Our definition excludes…

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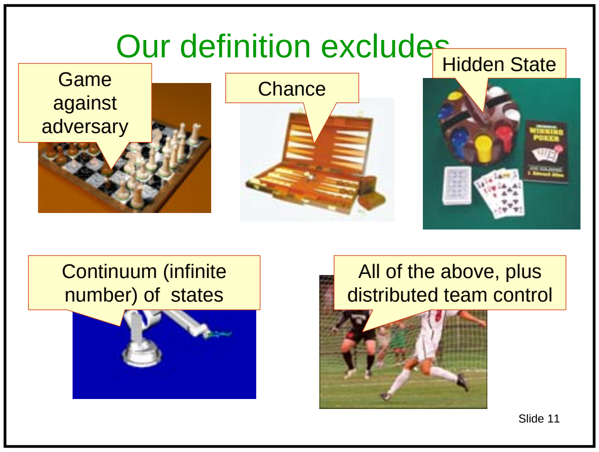

Our definition excludes…

Game against adversary

Chance

Hidden State

Continuum (infinite number) of states

All of the above, plus distributed team control

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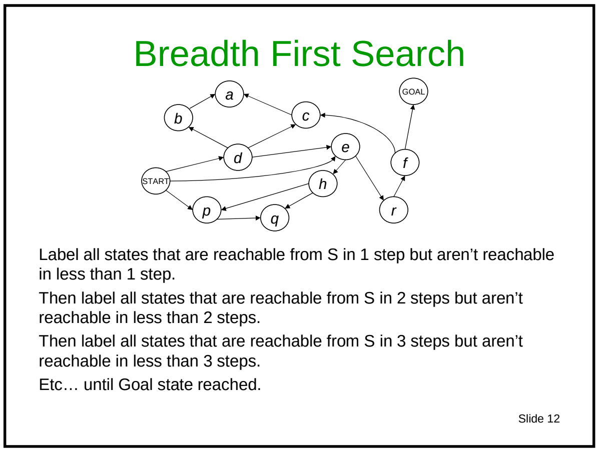

Breadth First Search

Label all states that are reachable from S in 1 step but aren't reachable in less than 1 step.

Then label all states that are reachable from S in 2 steps but aren't reachable in less than 2 steps.

Then label all states that are reachable from S in 3 steps but aren't reachable in less than 3 steps.

Etc… until Goal state reached.

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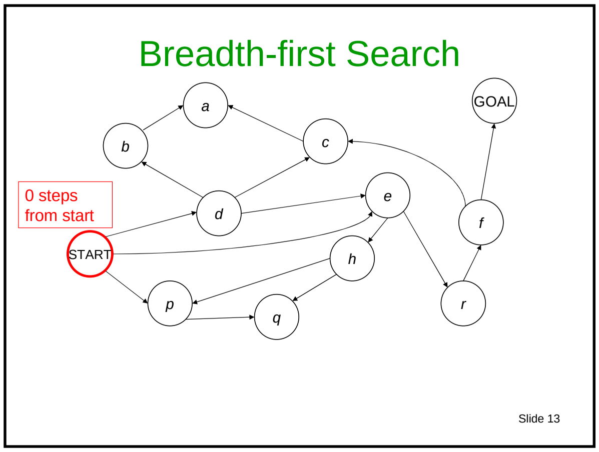

Breadth-first Search

0 steps from start

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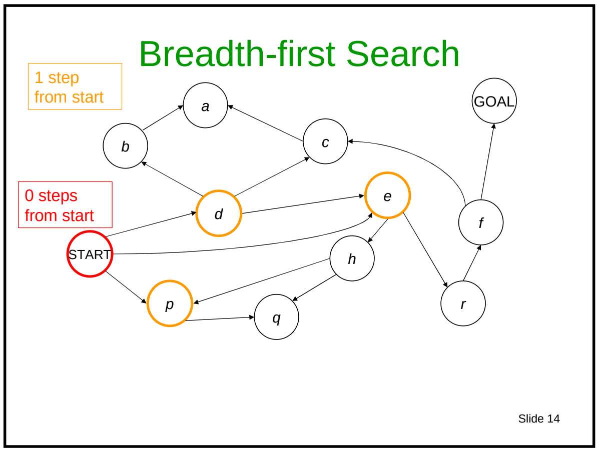

Breadth-first Search

1 step from start

0 steps from start

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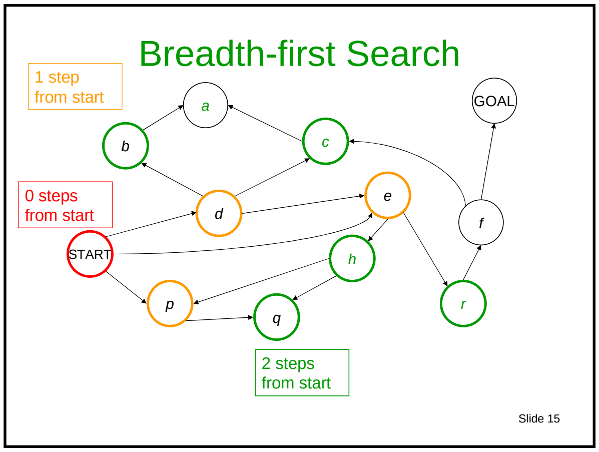


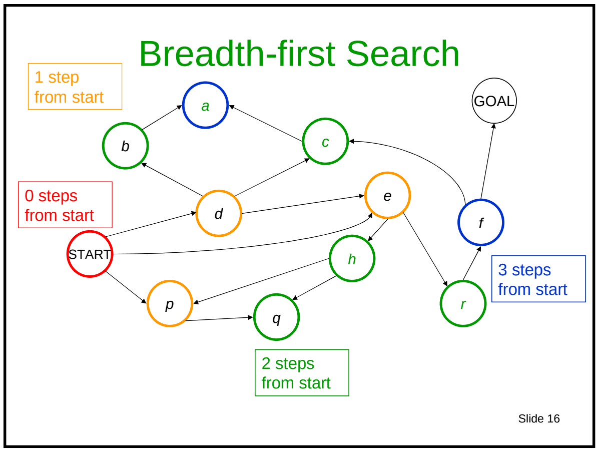
Breadth-first Search

1 step from start

0 steps from start

2 steps from start

Slide 15



Breadth-first Search

1 step from start

0 steps from start

3 steps from start

2 steps from start

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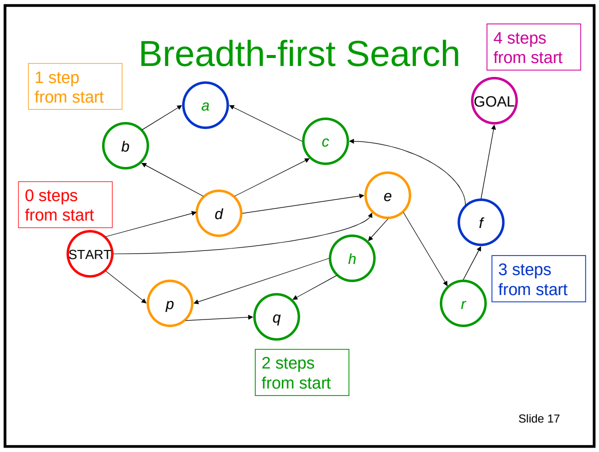

Breadth-first Search

4 steps from start

1 step from start

0 steps from start

3 steps from start

2 steps from start

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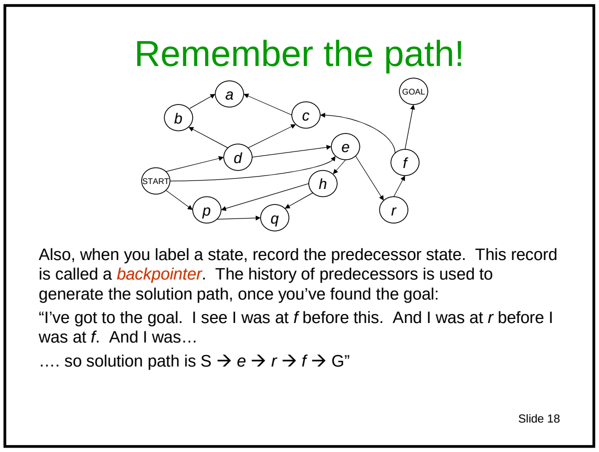

Remember the path!

Also, when you label a state, record the predecessor state. This record is called a *backpointer*. The history of predecessors is used to generate the solution path, once you've found the goal:

"I've got to the goal. I see I was at *f* before this. And I was at *r* before I was at *f*. And I was…

…. so solution path is S → *e* → *r* → *f* → G"

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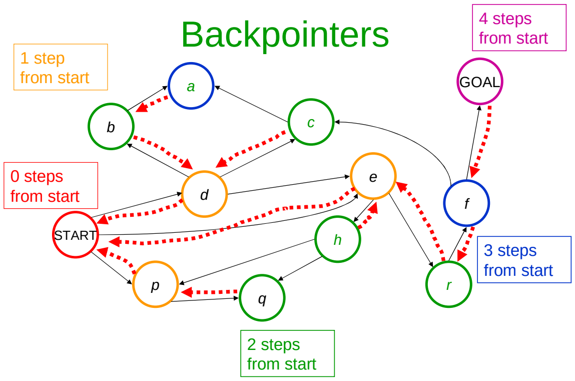
Backpointers



1 step from start

0 steps from start

4 steps from start

3 steps from start

2 steps from start

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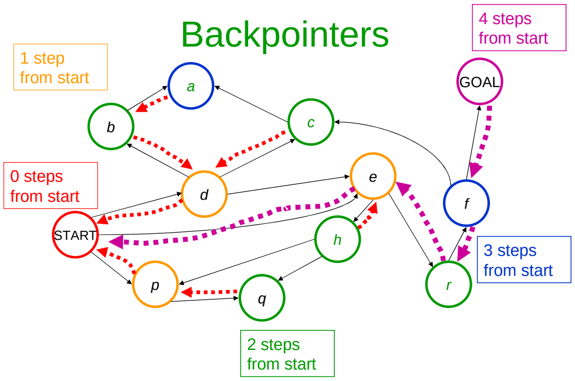
Backpointers



1 step from start

0 steps from start

4 steps from start

3 steps from start

2 steps from start

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Starting Breadth First Search

For any state *s* that we've labeled, we'll remember:

•*previous*(*s*) as the previous state on a shortest path from START state to *s*.

On the *k*th iteration of the algorithm we'll begin with V_k defined as the set of those states for which the shortest path from the start costs exactly *k steps*

Then, during that iteration, we'll compute V_{k+1}, defined as the set of those states for which the shortest path from the start costs exactly *k+1 steps*

We begin with *k = 0*, V_0 = {START} and we'll define, *previous*(*START*) = *NULL*

Then we'll add in things one step from the START into V_1. And we'll keep going.

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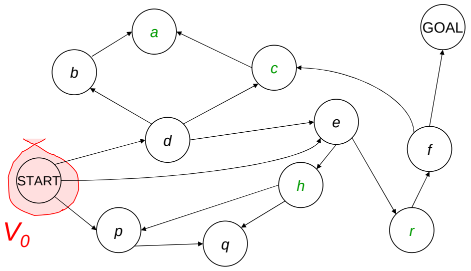
BFS



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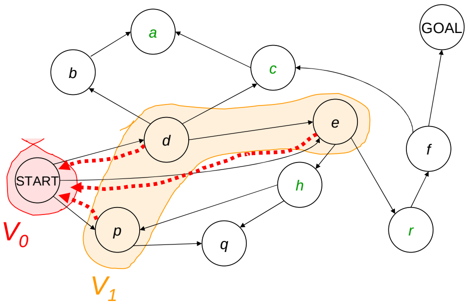
BFS



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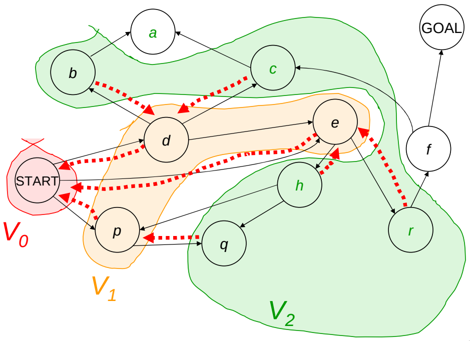
BFS



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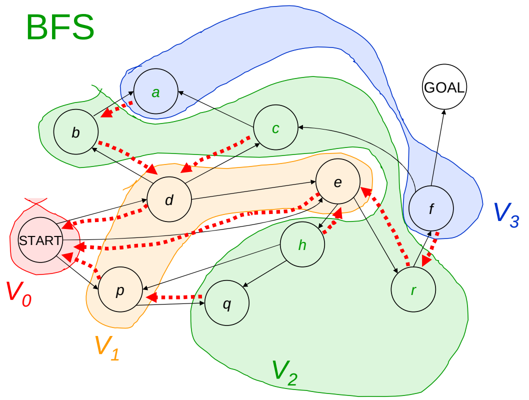
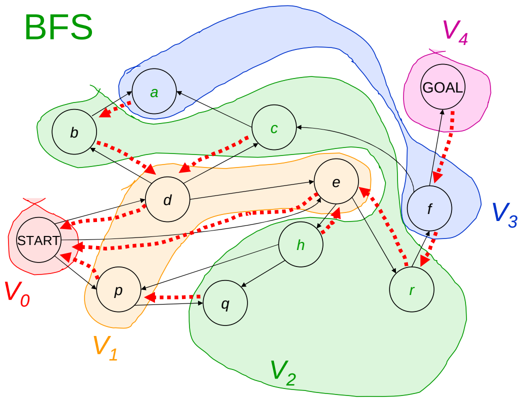
BFS



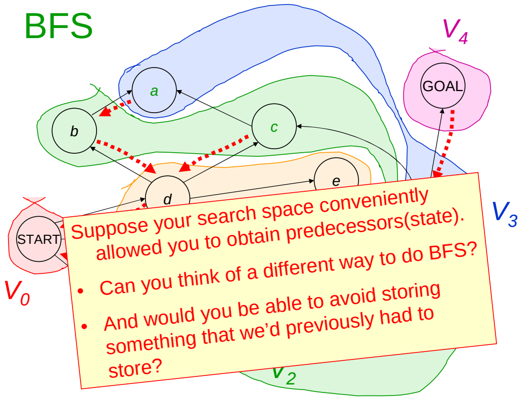
BFS



Breadth First Search

```
$V_0$ := $S$ (the set of start states)
previous(START) := NIL
$k$ := 0
while (no goal state is in $V_k$ and $V_k$ is not empty) do
    $V_{k+1}$ := empty set
    For each state $s$ in $V_k$
        For each state $s'$ in succs($s$)
            If $s'$ has not already been labeled
                Set previous($s'$) := $s$
                Add $s'$ into $V_{k+1}$
    $k$ := $k$+1
If $V_k$ is empty signal FAILURE
Else build the solution path thus: Let $S_i$ be the $i$th state in the shortest path. Define $S_k$ = GOAL, and forall $i <= k$, define $S_{i-1}$ = previous($S_i$).
```

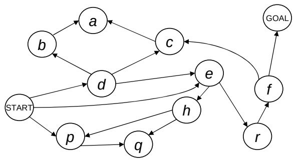
BFS



Suppose your search space conveniently allowed you to obtain predecessors(state).

- Can you think of a different way to do BFS?
- And would you be able to avoid storing something that we'd previously had to store?

Another way: Work back



Label all states that can reach G in 1 step but can't reach it in less than 1 step.

Label all states that can reach G in 2 steps but can't reach it in less than 2 steps.

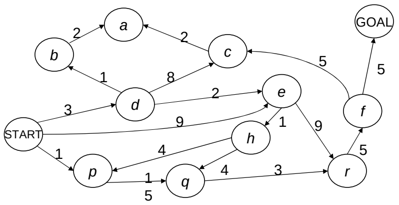
Etc. … until start is reached.

"number of steps to goal" labels determine the shortest path. Don't need extra bookkeeping info.

Breadth First Details

- It is fine for there to be more than one goal state.
- It is fine for there to be more than one start state.
- This algorithm works forwards from the start. Any algorithm which works forwards from the start is said to be *forward chaining*.
- You can also work backwards from the goal. This algorithm is very similar to Dijkstra's algorithm.
- Any algorithm which works backwards from the goal is said to be *backward chaining*.
- Backward versus forward. Which is better? Checkmate example

Costs on transitions



Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

We will quickly review an algorithm which does find the least-cost path. On the *k*th iteration, for any state *S*, write *g*(*s*) as the least-cost path to *S* in *k* or fewer steps.

Least Cost Breadth First

V_k = the set of states which can be reached in exactly *k* steps, and for which the least-cost *k*-step path is less cost than any path of length less than *k*. In other words, V_k = the set of states whose values changed on the previous iteration.

```
V_0 := S (the set of start states)
previous(START) := NIL
g(START) = 0
k := 0
while (V_k is not empty) do
        V_{k+1} := empty set
        For each state s in V_k
                For each state s' in succs(s)
                        If s' has not already been labeled
                        OR if g(s) + Cost(s,s') < g(s')
                                Set previous(s') := s
                                Set g(s') := g(s) + Cost(s,s')
                                Add s' into V_{k+1}
        k := k+1
If GOAL not labeled, exit signaling FAILURE
Else build the solution path thus:  Let S_k be the kth state in the shortest path.
Define S_k = GOAL, and forall i <= k, define S_{i-1} = previous(S_i).
```

Uniform-Cost Search

- A conceptually simple BFS approach when there are costs on transitions
- It uses priority queues

Priority Queue Refresher

DETOUR

A priority queue is a data structure in which you can insert and retrieve (thing, value) pairs with the following operations:

Init-PriQueue(PQ)	initializes the PQ to be empty.
Insert-PriQueue(PQ, thing, value)	inserts *(thing, value)* into the queue.
Pop-least(PQ)	returns the *(thing, value)* pair with the lowest value, and removes it from the queue.

Priority Queue Refresher

DETOUR

A priority queue is a data structure in which you can insert and retrieve *(thing, value)* pairs with the following operations:

For more details, see Knuth or Sedgwick or basically any book with the word "algorithms" prominently appearing in the title.

Init-PriQueue(PQ)	initializes the PQ to be empty.
Insert-PriQueue(PQ, thing, value)	inserts *(thing, value)* into the queue.
Pop-least(PQ)	returns the *(thing, value)* pair with the lowest value, and removes it from the queue.

Priority Queues can be implemented in such a way that the cost of the insert and pop operations are

Very cheap (though not absolutely, incredibly cheap!)

O(*log*(*number of things in priority queue*))

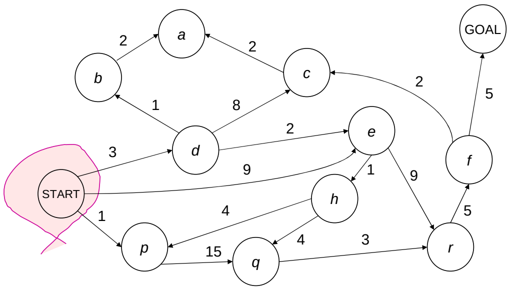
Uniform-Cost Search

- A conceptually simple BFS approach when there are costs on transitions
- It uses a priority queue

PQ = Set of states that have been expanded or are awaiting expansion

Priority of state *s* = *g(s)* = cost of getting to *s* using path implied by backpointers.

Starting UCS



PQ = { (S,0) }

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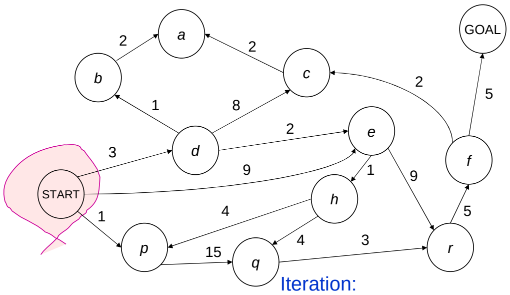
UCS Iterations



Iteration:
1. Pop least-cost state from PQ
2. Add successors

PQ = { (S,0) }

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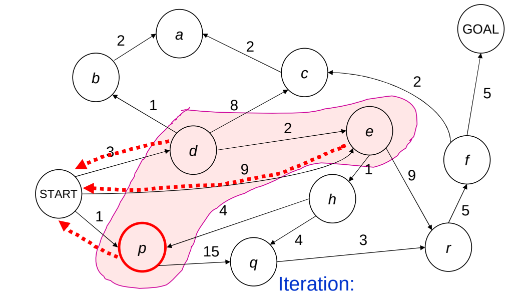
UCS Iterations



PQ = { (p,1), (d,3) , (e,9) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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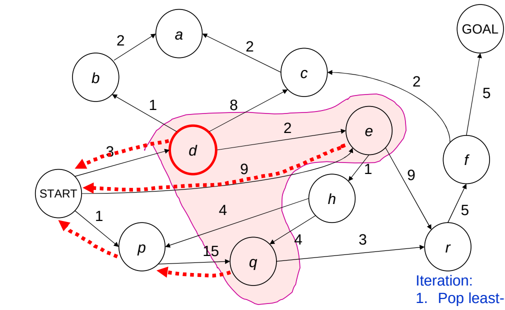
UCS Iterations



PQ = { (d,3) , (e,9) , (q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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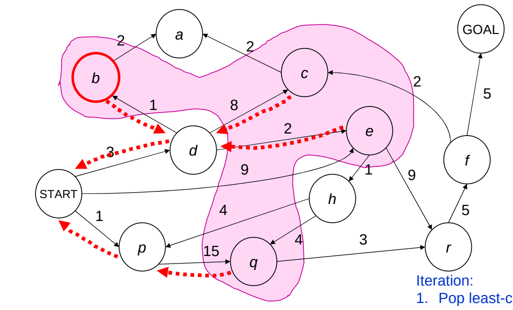
UCS Iterations



PQ = { (b,4) , (e,5) , (c,11) , (q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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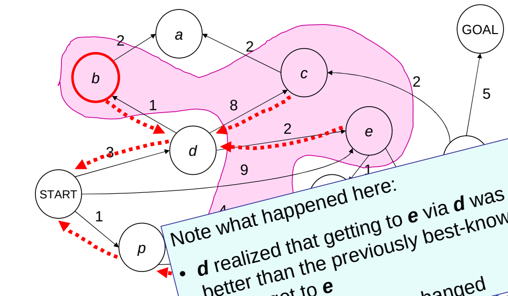
UCS Iterations



Note what happened here:
- ***d*** realized that getting to ***e*** via ***d*** was better than the previously best-known way to get to ***e***
- and so ***e***'s priority was changed

PQ = { (b,4) , (e,5) *) }*

east-cost state from PQ
2. Add successors

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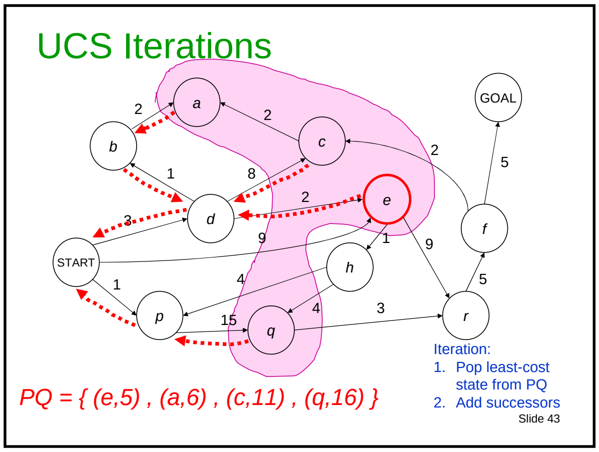

UCS Iterations

PQ = { (e,5) , (a,6) , (c,11) , (q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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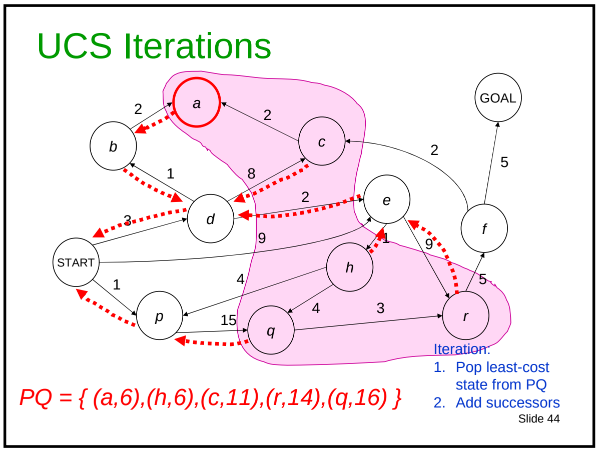

UCS Iterations

PQ = { (a,6),(h,6),(c,11),(r,14),(q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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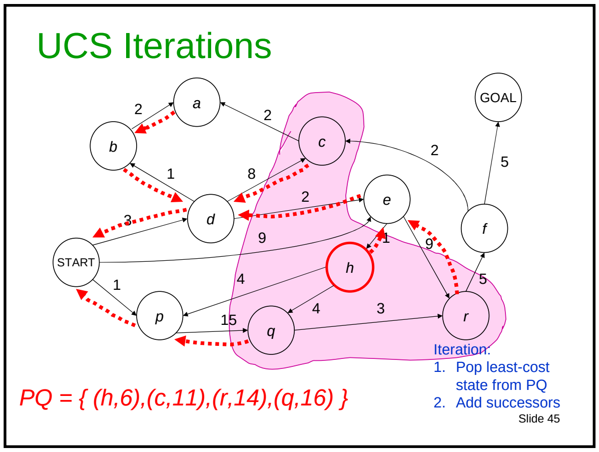
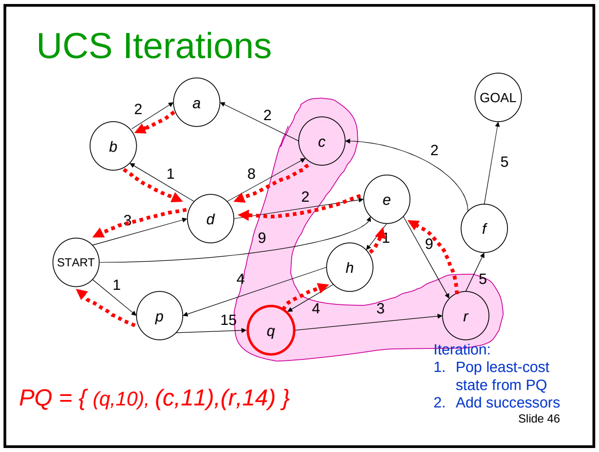


UCS Iterations

PQ = { (h,6),(c,11),(r,14),(q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Slide 45



UCS Iterations

PQ = { (q,10), (c,11),(r,14) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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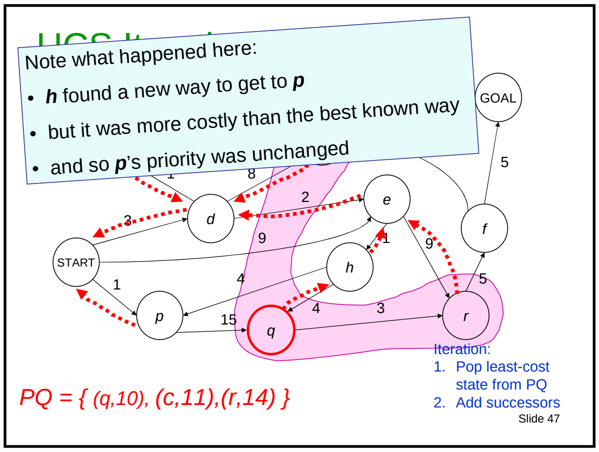

UCS Iterations

Note what happened here:

- ***h*** found a new way to get to ***p***
- but it was more costly than the best known way
- and so ***p***'s priority was unchanged

PQ = { (q,10), (c,11),(r,14) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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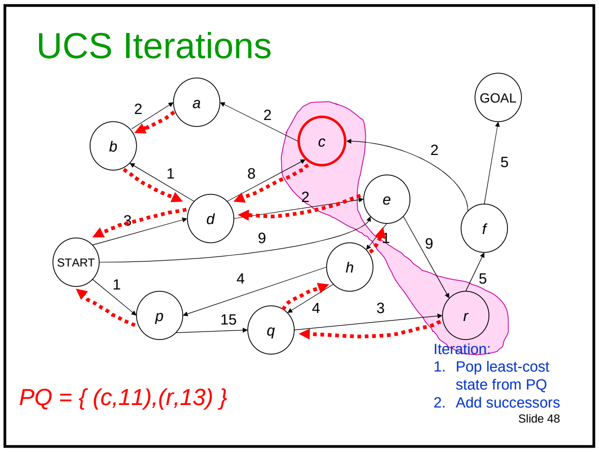

UCS Iterations

PQ = { (c,11),(r,13) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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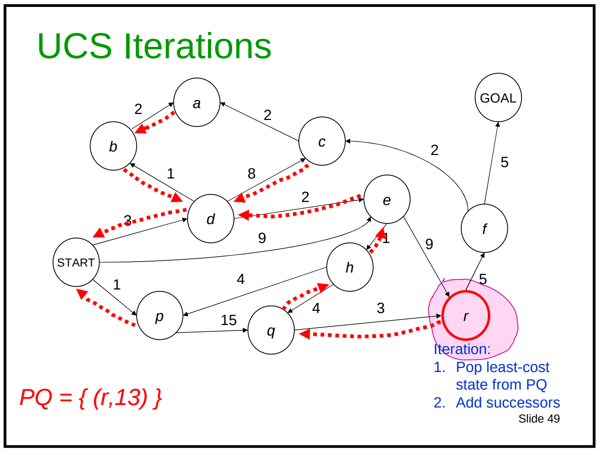
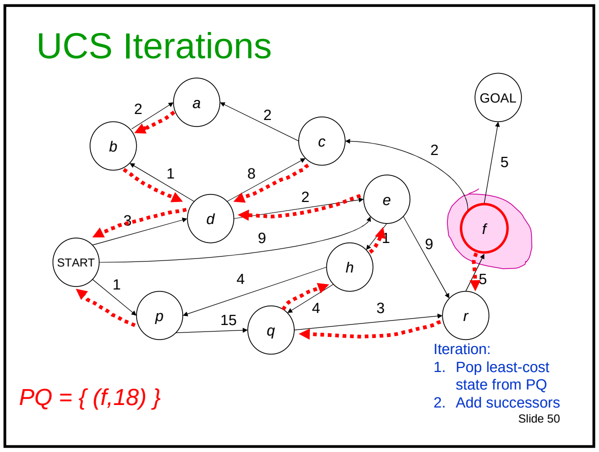


UCS Iterations

PQ = { (r,13) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Slide 49

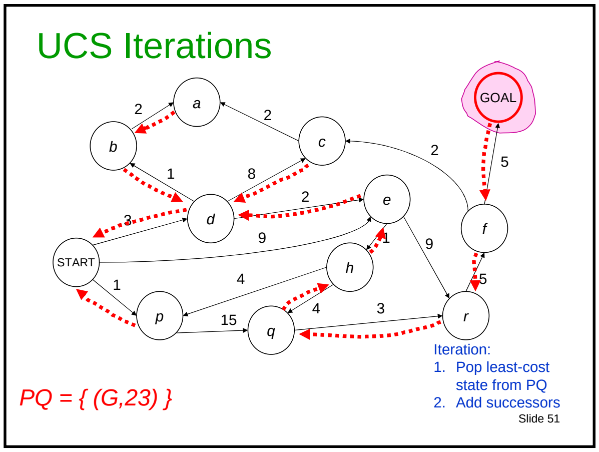


UCS Iterations

PQ = { (f,18) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Slide 50

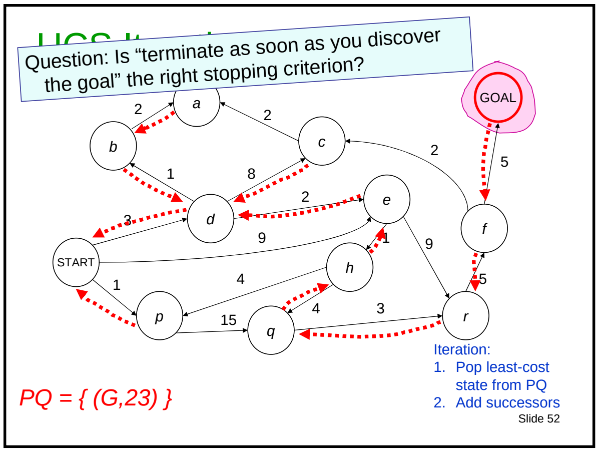


UCS Iterations

PQ = { (G,23) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Slide 51

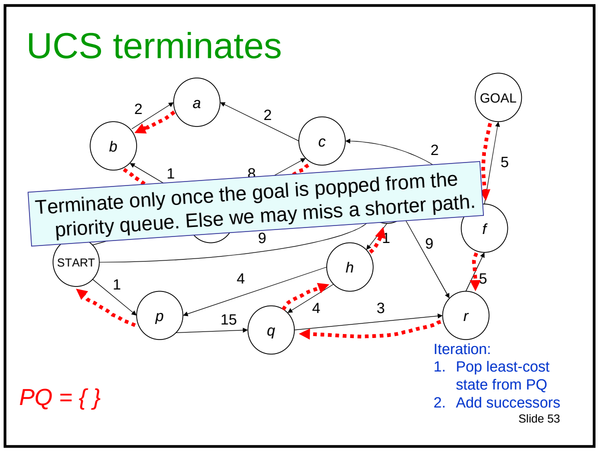


Question: Is "terminate as soon as you discover the goal" the right stopping criterion?

PQ = { (G,23) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Slide 52



UCS terminates

Terminate only once the goal is popped from the priority queue. Else we may miss a shorter path.

PQ = { }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

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Judging a search algorithm

- Completeness: is the algorithm guaranteed to find a solution if a solution exists?
- Guaranteed to find optimal? (will it find the least cost path?)
- Algorithmic time complexity
- Space complexity (memory use)

Variables:

N	number of states in the problem
B	the average branching factor (the average number of successors) (*B*>1)
L	the length of the path from start to goal with the shortest number of steps

How would we judge our algorithms?

Slide 54

Judging a search algorithm

N	number of states in the problem
B	the average branching factor (the average number of successors) (*B*>1)
L	the length of the path from start to goal with the shortest number of steps
Q	the average size of the priority queue

Algorithm		Complete	Optimal	Time	Space
BFS	Breadth First Search				
LCBFS	Least Cost BFS				
UCS	Uniform Cost Search				

Slide 55

Judging a search algorithm

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UCS	Uniform Cost Search	Y	Y	$O(log(Q) * min(N,B^L))$	$O(min(N,B^L))$

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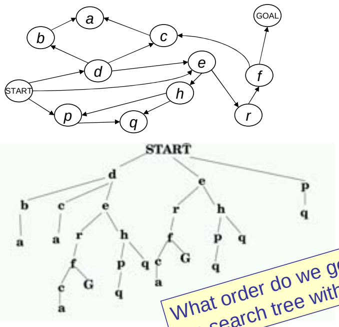
Search Tree Representation



What order do we go through the search tree with BFS?

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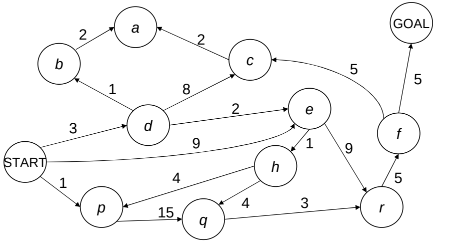
Depth First Search



An alternative to BFS. Always expand from the most-recently-expanded node, if it has any untried successors. Else backup to the previous node on the current path.

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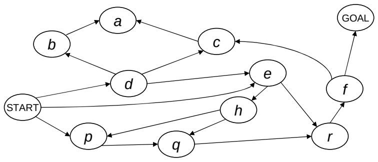
DFS in action

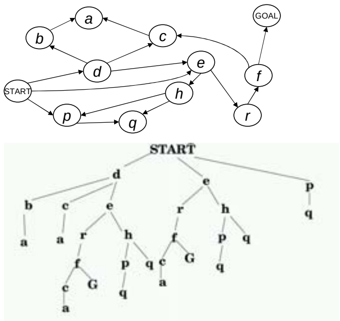
START
START *d*
START *d b*
START *d b a*
START *d c*
START *d c a*
START *d e*
START *d e r*
START *d e r f*
START *d e r f c*
START *d e r f c a*
START *d e r f* GOAL



Slide 59

DFS Search tree traversal

Can you draw in the order in which the search-tree nodes are visited?



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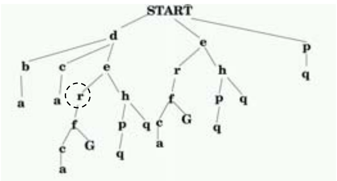
DFS Algorithm

We use a data structure we'll call a Path to represent the , er, path from the START to the current state.

E.G. Path P = <START, d, e, r >

Along with each node on the path, we must remember which successors we still have available to expand. E.G. at the following point, we'll have



P = <START (expand=e , p) ,
d (expand = NULL) ,
e (expand = h) ,
r (expand = f) >

DFS Algorithm

```
Let P = <START (expand = succs(START))>
While (P not empty and top(P) not a goal)
        if expand of top(P) is empty
        then
                remove top(P) ("pop the stack")
        else
                let s be a member of expand of top(P)
                remove s from expand of top(P)
                make a new item on the top of path P:
                        s (expand = succs(s))
If P is empty
        return FAILURE
Else
        return the path consisting of states in P
```

This algorithm can be written neatly with recursion, i.e. using the program stack to implement P.

Judging a search algorithm

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DFS	Depth First Search				

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UCS	Uniform Cost Search	Y	Y	*O(log(Q) * min(N,B^L))*	*O(min(N,B^L))*
DFS	Depth First Search	N	N	*N/A*	*N/A*

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DFS**	Depth First Search				

Assuming Acyclic Search Space

Judging a search algorithm

N	number of states in the problem
B	the average branching factor (the average number of successors) (*B*>1)
L	the length of the path from start to goal with the shortest number of steps
LMAX	Length of longest path from start to anywhere
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UCS	Uniform Cost Search	Y	Y	*O(log(Q) * min(N,B^L))*	*O(min(N,B^L))*
DFS**	Depth First Search	Y	N	*O(B^{LMAX})*	*O(LMAX)*

Assuming Acyclic Search Space

Questions to ponder

- How would you prevent DFS from looping?
- How could you force it to give an optimal solution?

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Answer 1:

PC-DFS (Path Checking DFS):

Answer 2:

MEMDFS (Memoizing DFS):

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- How would you prevent DFS from looping?
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Don’t recurse on a state if that state is already in the current path

Answer 2:

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Remember all states expanded so far. Never expand anything twice.

Questions to ponder

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PCDFS	Path Check DFS				
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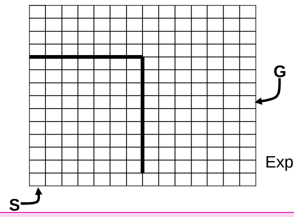
Maze example

Imagine states are cells in a maze, you can move N, E, S, W. What would plain DFS do, assuming it always expanded the E successor first, then N, then W, then S?



Expansion order E, N, W, S

Other questions: What would BFS do?
What would PCDFS do?
What would MEMDFS do?

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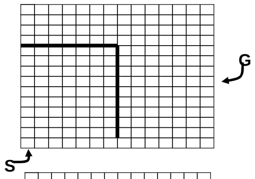
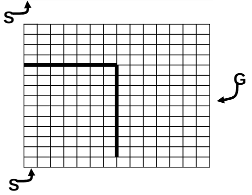
Two other DFS examples



Order: N, E, S, W?



Order: N, E, S, W

with loops prevented

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Forward DFSearch or Backward DFSearch

If you have a predecessors() function as well as a successors() function you can begin at the goal and depth-first-search backwards until you hit a start.

Why/When might this be a good idea?

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Invent An Algorithm Time!

Here's a way to dramatically decrease costs sometimes. Bidirectional Search. Can you guess what this algorithm is, and why it can be a huge cost-saver?

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PCDFS	Path Check DFS	Y	N	$O(B^{LMAX})$	$O(LMAX)$
MEMDFS	Memoizing DFS	Y	N	$O(\min(N,B^{LMAX}))$	$O(\min(N,B^{LMAX}))$
BIBFS	Bidirection BF Search				

Slide 78

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BIBFS	Bidirection BF Search	Y	All trans same cost	$O(min(N,2B^{L/2}))$	$O(min(N,2B^{L/2}))$

Iterative Deepening

Iterative deepening is a simple algorithm which uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up any path of length 2)
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

….and so on until success

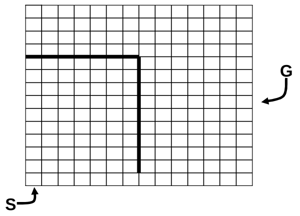
Cost is

$O(b^1 + b^2 + b^3 + b^4 \ldots + b^L) = O(b^L)$

Can be much better than regular DFS. But cost can be much greater than the number of states.

Maze example

Imagine states are cells in a maze, you can move N, E, S, W. What would **Iterative Deepening** do, assuming it always expanded the E successor first, then N, then W, then S?



Expansion order E, N, W, S

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PCDFS	Path Check DFS	Y	N	$O(B^{LMAX})$	$O(LMAX)$
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ID	Iterative Deepening				

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ID	Iterative Deepening	Y	if all transitions same cost	$O(B^L)$	$O(L)$

Best First "Greedy" Search

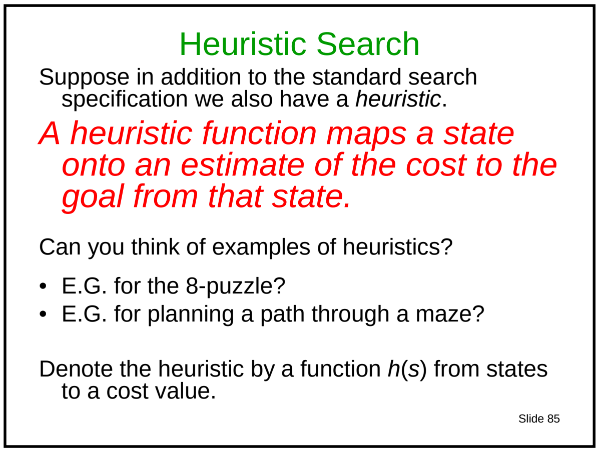
Needs a *heuristic*. A heuristic function maps a state onto an estimate of the cost to the goal from that state.

Can you think of examples of heuristics?

E.G. for the 8-puzzle?

E.G. for planning a path through a maze?

Denote the heuristic by a function *h*(*s*) from states to a cost value.



Heuristic Search

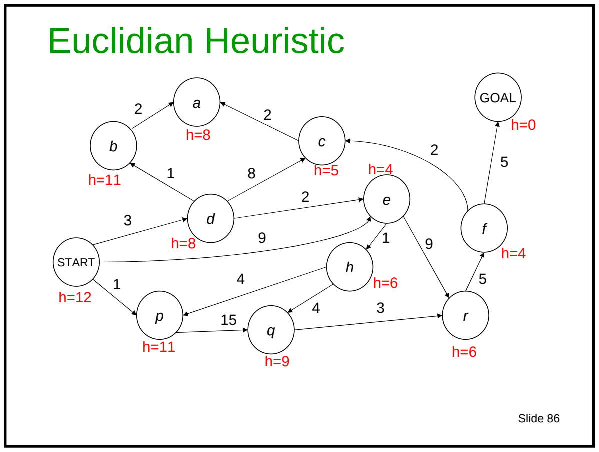
Suppose in addition to the standard search specification we also have a *heuristic*.

A heuristic function maps a state onto an estimate of the cost to the goal from that state.

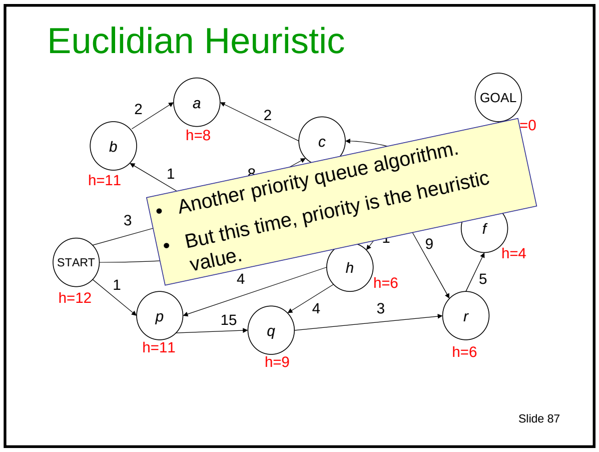
Can you think of examples of heuristics?

- E.G. for the 8-puzzle?
- E.G. for planning a path through a maze?

Denote the heuristic by a function *h*(*s*) from states to a cost value.

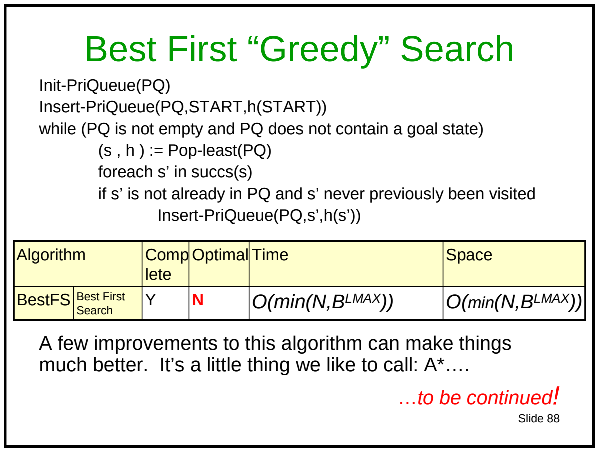


Euclidian Heuristic



Euclidian Heuristic

- Another priority queue algorithm.
- But this time, priority is the heuristic value.



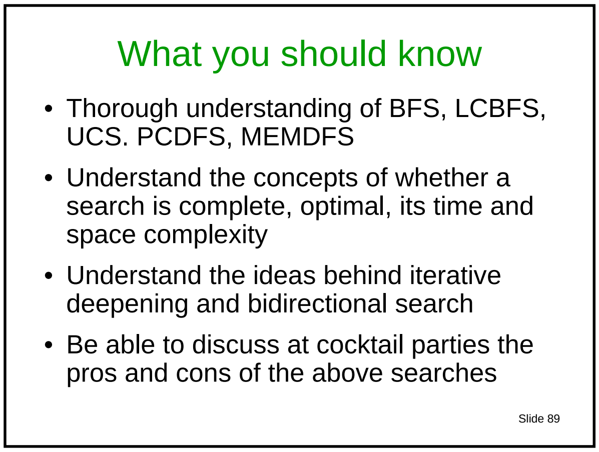
Best First “Greedy” Search

```
Init-PriQueue(PQ)
Insert-PriQueue(PQ,START,h(START))
while (PQ is not empty and PQ does not contain a goal state)
        (s , h ) := Pop-least(PQ)
        foreach s’ in succs(s)
        if s’ is not already in PQ and s’ never previously been visited
                Insert-PriQueue(PQ,s’,h(s’))
```

Algorithm		Complete	Optimal	Time	Space
BestFS	Best First Search	Y	N	$O(min(N,B^{LMAX}))$	$O(min(N,B^{LMAX}))$

A few improvements to this algorithm can make things much better. It’s a little thing we like to call: A*….

…to be continued!



What you should know

- Thorough understanding of BFS, LCBFS, UCS. PCDFS, MEMDFS
- Understand the concepts of whether a search is complete, optimal, its time and space complexity
- Understand the ideas behind iterative deepening and bidirectional search
- Be able to discuss at cocktail parties the pros and cons of the above searches