# Intrinsic Tactile Sensing for Mobile Manipulaton

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# Outline

1. Project overview
2. Invertibility of dynamical systems
3. Modeling deformable structure as a dynamical system
4. Inverting the dynamical system to get tactile data.

1 and 2 by Matt today.
3 and 4 by Sidd in a couple of weeks.

# What is *Intrinsic Tactile Sensing*?

Problem: Given

- Cantilevered beam of known shape,
- Single contact applied at tip,
- Sensor embedded at base, giving $f$ and $\tau$,

find contact location.

Solution:

- Line of force is given by Plucker coordinates $(\tau, f)$. Equivalently line has direction $f$ and passes through point $\tau \times f / f \cdot f$.
- Intersect line of force with known beam shape.

# Can we generalize Intrinsic Tactile Sensing?

- From serial mechanism to parallel?
- From static to dynamic?
- From one sensor to distributed sensors?
- To include other information, e.g. motor models?

# Applications

- Instrumented bumper:
  - Collision detection,
  - Closed loop pushing,
  - Compliant navigation,
- Instrumented wheels and suspension:
  - Monitor wheel contact stick versus slip,
  - Closed-loop manipulation (dungbeetle mode),
  - Terrain mapping,
  - More precise odometry,
  - Navigation using terrain map,
  - More intelligent control of wheels and active suspension,

# Our proposal

1. Build a mobipulator with
   - Encoders and strain gauges all over the place,
   - Motor models,
   - Model of dynamic shape deformation of chassis,
   - Contact models,
2. Develop a general framework that takes all this info and maintains an estimate of robot configuration, terrain, task state, etc
3. Let the mobipulator map and clean up the mlab. Then make us some refreshments.

# Version 1

- New mobipulator. Compliance lumped with sensor. Rigid front end, rigid back end, joined by compliant force/torque sensor.
- Reconfigures to place force/torque sensor between front end and bumper?

# What is the general framework?

We are presently following an idea of Bicchi et al:

1. Develop dynamical system model.
   - Let $q$ be mechanical configuration, i.e. shape, of deformable structure;
   - Let $x = (q, \dot{q})$ be mechanical state of structure;
   - Let $u$ be applied forces;
   - Let $y$ be sensor output;
   - For small deformations frame LTI dynamical system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

2. Invert the system, to map known outputs (sensor readings) to unknown inputs (forces).

# Part 2: Invertibility of dynamical systems

Let $S$ be a system with state $x(t)$, input $u(t)$, output $y(t)$. All signals are defined for $t \in [0, \infty)$. Assume initial state is zero: $x(0) = 0$. Then $S$ is a function mapping input signals to output signals.

$S$ is *invertible* if the mapping of input signals to output signals has an inverse. That is, if every possible output signal determines a unique input signal.

Questions:

- What is relation to observability and controllability?
- Can invertibility of LTI systems be phrased in terms of system structure $A$, $B$, $C$, $D$?
- How does one actually construct an inverse?

# Invertibility of LTI dynamical systems

Define system $S = (A, B, C, D)$ as before, let $\mathbb{L}[\cdot]$ be Laplace transform, let $G(s)$ be transfer function:

$$G(s) = C(sI_n - A)^{-1}B + D$$

Then

$$\mathbb{L}[y(t)] = G(s)\mathbb{L}[u(t)]$$

System $\hat{S} = (\hat{A}, \hat{B}, \hat{C}, \hat{D})$ is an inverse if the corresponding transfer function $\hat{G}(s)$ gives

$$\hat{G}(s)G(s) = I_m$$

So LTI system is invertible iff transfer function has left inverse. We need dimension of output at least as large as dimension of input $m$. $S$ is invertible iff $G(s)$ has rank $m$ over the field of rational functions

$\ldots$

# $L$-integral inverse

But there is a problem with inverse as defined: we will get acausal inverses that way. If $S$ integrates, $\hat{S}$ has to differentiate. If $S$ has a delay in it, $\hat{S}$ has to predict the future. We have to tolerate the delay, so we need a less restrictive notion of inverse.
If

$$\hat{G}(s)G(s) = \frac{1}{s^L} I_m$$

then $\hat{S}$ is an $L$-*integral inverse* of $S$.
Smallest such $L$ is the *inherent integration* $L_0$ of an invertible system.

# A Big Matrix

We're switching to discrete time. It's more or less the same thing. Let $u_{[0,i]}$ be first $i$ inputs stacked up in one big $(i+1)m$-vector. Let $y_{[0,i]}$ be first $i$ outputs stacked up in one big $(i+1)r$-vector. Then the linear transform is

$$y_{[0,i]} = M_i u_{[0,i]}$$

where $M$ is on the board.
Theorem: first column (i.e. first $m$ columns) is full rank iff $i$-integral inverse exists.
Theorem: If $L$-integral inverse exists, $L$-integral inverse exists with $L \leq n$.

To determine inherent integration, start with $M_1$, $M_2$, etc. until first column is full rank. If you get past $n$, give up.

# Related concepts: controllability

- *Pointwise state controllability.* (Also known as controllability.) Can you find $u(t)$ to steer system from given initial state and time to goal state and time?
- *Functional state controllability.* Can you find $u(t)$ to steer state along desired trajectory $x(t)$?
- *Pointwise output controllability.* Steering the output rather than the state.
- *F*unctional output controllability.

$\ldots$

# Related concepts: observability

(I lost my notes, so I'm just making this up.)

- *Pointwise state observability.* (Also known as observability.) Given output and input over some interval of time, can you determine the initial state?
- *Functional state observability.* Given output and input over some interval of time, can you determine the state over some (smaller?) interval of time?
- *Pointwise input observability.* Given initial state and output over some time interval can you determine input?
- *Functional input observability.* Given initial state and output over some time interval can you determine input over some (smaller?) time interval?

# Functional reproducibility; invertibility

- *Functional reproducibility.*
  Given an arbitrary output function, does corresponding input exist?
  The same as functional output controllability.
- *Invertibility.*
  Given an arbitrary output function, is corresponding input function determined?
  The same as functional input observability.

# Pointwise versus functional controllability

- Obviously functional implies pointwise. Does pointwise imply functional?
- What about for LTI systems?

# How to build the inverse system

I sure hope I'm out of time by now!!!!