Investigation of Combined Positive and Negative Muon Decay in a Scintillator

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Based on some classic experiments[1, 2, 3], the lifetime of the muon is investigated using a scintillation method, with coincidence counting. Passing muons from cosmic showers that are stopped in the scintillator decay and their decay times measured in time bins of width $w = 0.1009 \pm 0.0018\,\mu\text{s}$. However, due to the capture rate of negative muons in the scintillator, and the non-unity ratio between positive and negative muons, the decay curve is not a simple exponential, but a double exponential distribution. Having collected data over about 600 hours, a nonlinear fit was made and the value of the lifetime along with its numerical error due to the numerical method determined. Using a linear fit of the simple single-exponential approximation of the decay, the errors due to physical sources was also ascertained. These give the value of the muon lifetime in vacuum as $\tau_0 = 2.197 \pm 0.006 \pm 0.036\,\mu\text{s}$.

1. Introduction

Discovered by Neddermeyer and Anderson, and Street and Stevenson in 1937[4], the muons have become an important in high energy physics explaining the electroweak theory[5]. Their presence at sea level also demonstrates the validity of special relativity. Being created in the upper atmosphere, they speed towards the earth's surface near the speed of light as they decay with a lifetime much shorter than the time it takes for them to reach the surface. Yet their detection indicates the effects of time dilation such that their decay is slowed down as they travel at high speeds.

Muons are created when cosmic showers hit the upper atmosphere of the earth. When a high speed proton collides with a nucleus, it produces a myriad of mesons, some of them positive and negative pions. These pions subsequently decay to form the muons and neutrinos (see Figure 1.1)

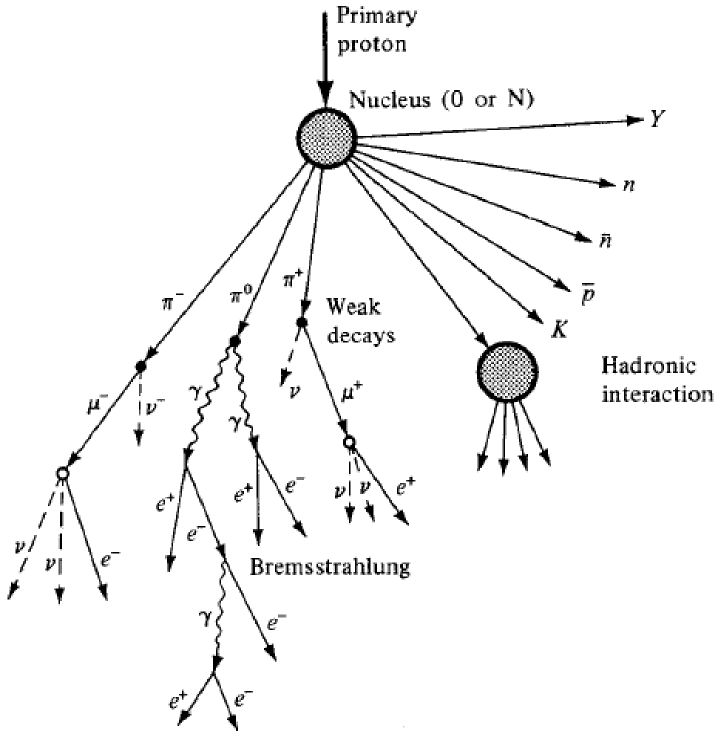

Figure 1.1: Cosmic ray shower as muon source in the atmosphere[6]

Muons can decay via a few paths, but almost all of them decay into an electron and two neutrinos[7, 8] according to Equation 1.1.

$$\begin{aligned} \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\ \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \end{aligned} \qquad (1.1)$$

The neutrinos are produced according to lepton number conservation. Due to the difference in the number of positive and negative pions in the atmosphere, the ratio of positive to negative muons is also not unity. As described by Dorman[9], its value depends on the momentum of the muons by the equation

$$\rho = N_+ / N_- = 1.268 \pm (0.008 + 0.0002p) \qquad (1.2)$$

where p is measured in GeV. For this experiment, since only muons that have stopped have their decays detected, it is assumed that $p = 0$ here.

2. Description of Apparatus

The muons are detected using 9 flat plastic scintillators sheets, each connected to a photomultiplier tube (PMT) to detect the scintillation that occurs when the plastic is excited[10]. This happens when muons pass through the scintillator, or when a muon that has stopped in the scintillator undergoes decay and emits an electron. The photomultipliers (PMs) are powered by two synchronized Hewlett Packard Harrison 6516A DC Power Supply JIO, operating at 0-3500V and 0-5MA. They are connected to the LeCroy Research Systems, Model 121 Discriminator, that counts the occurrances of the aforementioned two events. When the scintillator is activated, the PMs send out a negative pulse, which is passed through a comparator and converted to a positive square pulse. This helps to distinguish an excitation from the background electronic noise intrinsic in the scintillator and circuit.

The circuit is set up to attempt to detect series of two pulses, using coincidence counting techniques, further elaborated by Leo[11] and Melissinoss[12]. When a muon passes through the scintillator, and ultimately generates the square pulse, Scaler 1, a set of LEDs, increments its count. After about 0.5µs, a timer, running off a 10MHz clock, is then started to wait for the detection of the second pulse This could be from the decay of a stopping muon or from the background. The timer increments at every clock cycle for up to 250 cycles. If the second pulse is detected, Scaler 2, another set of LEDs, increments its count, and a signal is sent to the serial interface of the IBM Personal Computer XT.

This signal tells during which of the 250 cycles the second pulse was detected. The observed duration after the first pulse, is thus divided into 250 time bins of roughly 0.1µs in width. If the pulse is due to a decay electron, that would contribute to the exponential curve, but if its due to another muon passing through the scintillator within 2.5µs of the first, or some other excitations, then that would contribute to the background.

Calibration is required for the voltage to the Pms, for reasons explained by Leo[10]. If the voltage is too low, the PMs would not be sensitive enough to detect all excitations in the scintillators. If the voltage is too high, then noise from the equipment would drastically increase the counts causing a serious systematic error. Measuring Scaler 1 counts with respect to PM voltage gives rise to Figure E.1. The insensitive part occurs before the plateau and the over-amplified portion after. The best operating voltage is at the middle of the plateau, which is estimated, from Figure E.2, to be 1.005V for the present set-up.

It should be noted that due to the charge of the negative muons, they can approach quite closely to the nuclei with appreciably large atomic numbers[5, 8, 13, 14, 15]. In the case of plastic scintillators which are mainly composed of carbon atoms, the negative muons can get close enough to the carbon nuclei to bond with them, forming boron nuclei and a neutrino as described in Equation 2.1.

$$\mu^{-} + C \rightarrow B + \nu_{\mu} \tag{2.1}$$

Eventually, the boron would decay into a carbon atom and electron which would contribute to the background. This does not happen for positive muons due to electrostatic repulsion from the nuclei.

3. Theory

Assuming that the rate of decay is constant, a single decay process is defined by

$$\frac{dN}{dt} = -\lambda N \tag{3.1}$$

where λ is the rate of decay, related to the lifetime by $\tau = 1/\lambda$, and N is the instantaneous number of particles . Integrating this with respect to time gives the relation

$$N = N_0 e^{-\lambda t} \tag{3.2}$$

where N_0 is the initial particle count. Within the time interval ($t, t+\Delta t$) the number of particles that decay are

$$N=\left|\int_t^{t+\Delta t}\frac{dN}{dt}dt\right|=\left|\left[N_0 e^{-\lambda t}\right]_t^{t+\Delta t}\right|$$
$$=N_0 e^{-\lambda t}\left(1-e^{-\lambda\Delta t}\right)\approx N_0 e^{-\lambda t}(1-1+\lambda\Delta t)$$
$$=\lambda N_0 e^{-\lambda t}\Delta t \tag{3.3}$$

Equation 3.3 holds for positive muons, but not for negative muons because of Equation 2.1. Due to this nuclear capture, there is a capture rate associated with negative muons, such that its decay becomes

$$N_-=\left(N_0 e^{-\lambda_c t}\right)e^{-\lambda t}=N_0 e^{-(\lambda+\lambda_c)t} \tag{3.4}$$

where λ_c is the capture rate due to muon loss through bonding. Mukhopadhyay et al combines results from several observations to arrive at the value[10]

$$\lambda_c=3.76\pm0.04\times10^4\,s^{-1} \tag{3.5}$$

Since the counts are contributed to by the decays of the positive and negative, and the background, its decay is

$$N(t)=N_{bg}+\left(\lambda_+ N_+ e^{-\lambda_+ t}+\lambda_- N_- e^{-\lambda_- t}\right)w \tag{3.6}$$

where there is a notational change of $w=\Delta t$; λ_+ and N_+ are the decay rate and number of positive muons, while λ_- and N_- are the decay rate and number of negative muons, respectively. It has been experimentally noted that positive and negative muons do not exist equally in the atmosphere and their ratio depends on their momentum and the altitude[6, 12]. Substituting Equations 1.2 into 3.6 gives

$$N(n)=N_{bg}+\frac{N_0 w}{1+\rho}e^{-\lambda_0 wn}\left[\lambda_0\left(\rho+e^{-\lambda_c wn}\right)+\lambda_c e^{-\lambda_c wn}\right] \tag{3.7}$$

where $\lambda_0=\lambda_+$ is the lifetime in free space, of which the positive muons maintain, and $\rho=N_+/N_-$ is the muon ratio. Here *wn* = *t*, considering time split into *n* time bins with width *w*.

If sufficient data is collected, it can be fitted to Equation 3.7 with parameters N_{bg}, N_0 and λ_0. Otherwise, the exponentials would have to be approximated to a single one. Starting with Equation 3.6, and taking Taylor expansions, one can derive Equation 3.8, the effective single exponential decay.

$$
\begin{aligned}
N_i - N_{bg} &= N_+ e^{-\lambda_0 t} + \frac{1}{\rho} N_- e^{-(\lambda_0+\lambda_c)t} \\
&= N_+ e^{-\lambda_0 t}\left(1+\frac{1}{\rho}e^{-\lambda_c t}\right) \\
&\approx N_+ e^{-\lambda_0 t}\left[1+\frac{1}{\rho}\left(1-\lambda_c t\right)\right] \\
&= N_+ e^{-\lambda_0 t}\left[\left(1+\frac{1}{\rho}\right)-\frac{1}{\rho}\lambda_c t\right] \\
&= N_+\left(1+\frac{1}{\rho}\right)e^{-\lambda_0 t}\left(1-\frac{\lambda_c/\rho}{\rho+1}t\right) \\
&\approx N_+\left(1+\frac{1}{\rho}\right)e^{-\lambda_0 t}e^{-\frac{\lambda_c}{1+\rho}t} \\
&= N_+\left(1+\frac{1}{\rho}\right)e^{-\left(\lambda_0+\frac{\lambda_c}{1+\rho}\right)t}
\end{aligned} \tag{3.8}
$$

Thus the effective lifetime is $\tau_{eff}=1/\lambda_{eff}$, where $\lambda_{eff}=\lambda_0+\dfrac{\lambda_c}{1+\rho}$, from which the lifetime in empty space can be estimated. Equation 3.8 can be rewritten in linear form as

$$\ln(N_i')=\ln(N_0')-\lambda_{eff}t \tag{3.9}$$

where $N_i'=N_i-N_{bg}$ and $N_0'=N_+(1+1/\rho)$.

4. Data

Data was collected over about 600 hours with 4 samples, their durations noted along with the Scaler 1 and 2 counts at the end of each run. The times between scintillator pulses are recorded in text files labeled by dates, as shown in Table 4.1. Printouts of the data files are in Appendix B.

Due to overflow, the scaler 2 counts are lower than expected, but the actual value can be retrieved by knowing the overflow rate. However, this is not important, as the values can be obtained from summing the counts recorded in each data file. Since Scaler 2 has only 4 digits, it has a significant overflow rate, which when accounted for reveals a discrepancy between the recorded second pulses by the computer and by the circuit, possibly due to some circuitry unreliability. In each data file, the muon count during decay is distributed into 255 time bins of about 0.1μs.

Run	**1**	**2**	**3**	**4**
Duration (s)	423399 ± 1	153504 ± 1	1036965 ± 1	536912 ± 1
Scaler 1 reading, S_1	10032301	3614150	25081541	12881573
Scaler 2 reading	8491	6302	6976	9851
Scaler 2 count, S_2	*45580*	*16262*	*116645*	*59670*
Data file	`oct12.txt`	`oct14.txt`	`oct26.txt`	`nov01.txt`

Table 4.1: Data collected

5. Data Analysis

To work out the lifetime with respect to time, t, the time bin independent variable, n, has to be scaled with the bin width, w, which has to be determined. This is be calculated by counting the number of scintillator excitations in the background of the decay. It is assumed that this background is contributed to by the flux of muons passing through the scintillator $F = S_1/T$, where S_1 is the Scaler 1 count after the duration, T, of an experiment run. Fitting the line $T=(1/F)S_1$ to Table 4.1, and finding the slope, using the technique of weighted least squares described in Appendix A, gets $F=24.3396$ Hz, with negligible error since the error in T is extremely small. Choosing time bins 160 to 190, for which to consider the background contribution, sets the bin count $n_B = 31$, and the excitation count $N_{bg} = 3985$. Each time a muon passes through the scintillator, a gate opens to wait for the second PM pulse, either due to the decay electron or background muon. In the total duration T, this happens S_1 number of times. In the time $t = n_B w$, there should be Ft muons passing through the scintillator, where n_B is the number of bins considered for the background. So, for the whole experiment, the number of counts due to background is $N_B=F\,t\,S_1=F^2 n_B w/T$. Rearranging would give the expression for the width of the time bins

$$w = \frac{N_B}{F^2 T n_B} \tag{4.1}$$

With 238157 counts during decay, the data is of a high enough quality to attempt a nonlinear fit[17] to the explicit form of Equation 3.8, with the substitution $\tau_0 = 1/\lambda_0$

$$N(n) = N_{bg} + \frac{N_0 w}{1+\rho} e^{-\frac{wn}{\tau_0}} \left[\frac{1}{\tau_0} \left(\rho + e^{-\frac{wn}{\tau_0}} \right) + \lambda_c e^{-\lambda_c wn} \right] \tag{4.2}$$

This is done via the method of maximum likelihood estimation[16, 18, 19]. Assuming that each measurement of N_n follows a Gaussian distribution, this leads to determining the least squares of $N(n)$ by minimizing

$$M = \sum_n \frac{[N_n^{(obs)} - N(n)]^2}{\sigma^2_{N_n^{(obs)}}} \tag{4.3}$$

where $N_n^{(obs)}$ is the observed count in each time bin, $N(n)$ is the calculated count at each time bin, n, according to Equation 3.8, and $\sigma_{N_n^{(obs)}}$ is the standard error in each measurement of $N_n^{(obs)}$. Conceptually, this would require taking the partial derivatives of the unknowns, N_{bg} , N_0, λ_0 , with respect to M, setting them to 0, and solving as in Equation 4.4.

$$\frac{\partial M}{\partial N_{bg}} = \frac{\partial M}{\partial N_0} = \frac{\partial M}{\partial \lambda_0} = 0 \tag{4.4}$$

Subsequently, the errors in the three parameters can be determined, by finding the inverse Hessian matrix[16, 18, 19]. However, given the nonlinearity of $N(n)$, numerical methods[20] have to be employed to determine the best fit. Using the Matlab curve-fitting implementation of the Trust-Region algorithm[21] for robust least squares[22] the three parameters are found to be

$$\begin{gathered} N_{bg} = 123 \pm 9 \\ N_0 = (2.949 \pm 0.006) \times 10^5 \\ \tau_0 = 2.197 \pm 0.006\, \mu\text{s} \end{gathered} \tag{4.5}$$

The standard deviations in these results are due to the numerical method.

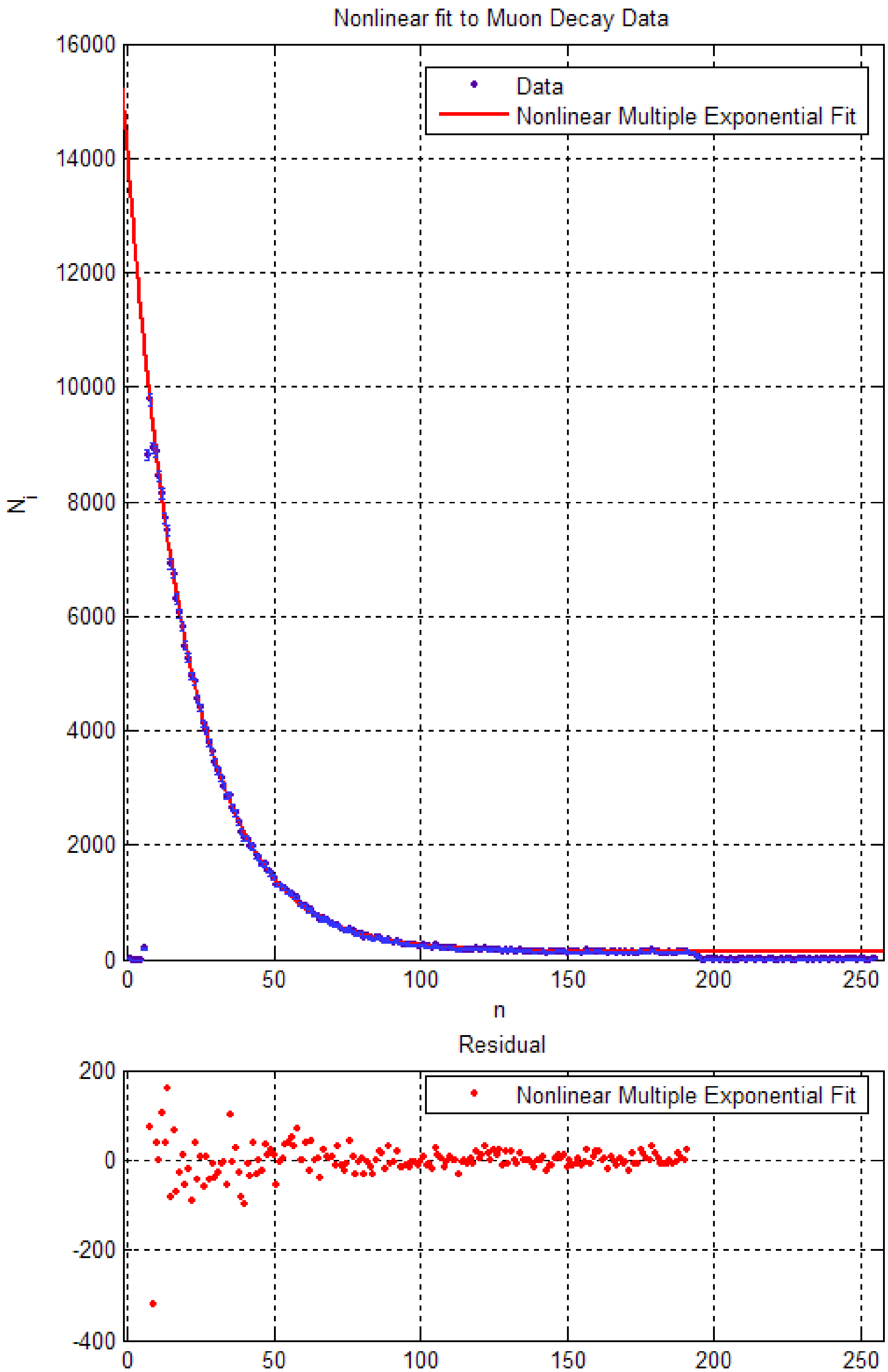

Figure 4.1: Data points plotted and fitted against Equation 4.2, along with the residual. Note the initial low counts at the beginning as the timing gate opens and the tapering to 0 after 190 bins as the gate closes. Because of the relatively small errors of the counts, the error bars are hard to see.

The error of the lifetime due to the physical system would still need to be worked out using quadrature of error propagation. Developing from Equation 4.1, the error in the width of the time bin is

$$\begin{aligned}\delta w &= \sqrt{\left(\frac{\partial w}{\partial T}\delta T\right)^2 + \left(\frac{\partial w}{\partial N_B}\delta N_B\right)^2 + \left(\frac{\partial w}{\partial F}\delta F\right)^2} \\ &\approx \sqrt{\left(\frac{\partial w}{\partial T}\delta T\right)^2 + \left(\frac{\partial w}{\partial N_B}\delta N_B\right)^2} \\ &= \frac{1}{n_B}\sqrt{\left(-\frac{N_B}{F^2 T^2}\delta T\right)^2 + \left(\frac{1}{F^2 T}\delta N_B\right)^2} \qquad (4.6)\end{aligned}$$

Due to the difficulty in working out the error propagation for the nonlinear Equation 4.2, its approximation, Equation 3.10 is referred to instead, in terms of time bins

$$\ln(N_i') = \ln(N_0') - (\lambda_{eff} w) n \qquad (4.7)$$

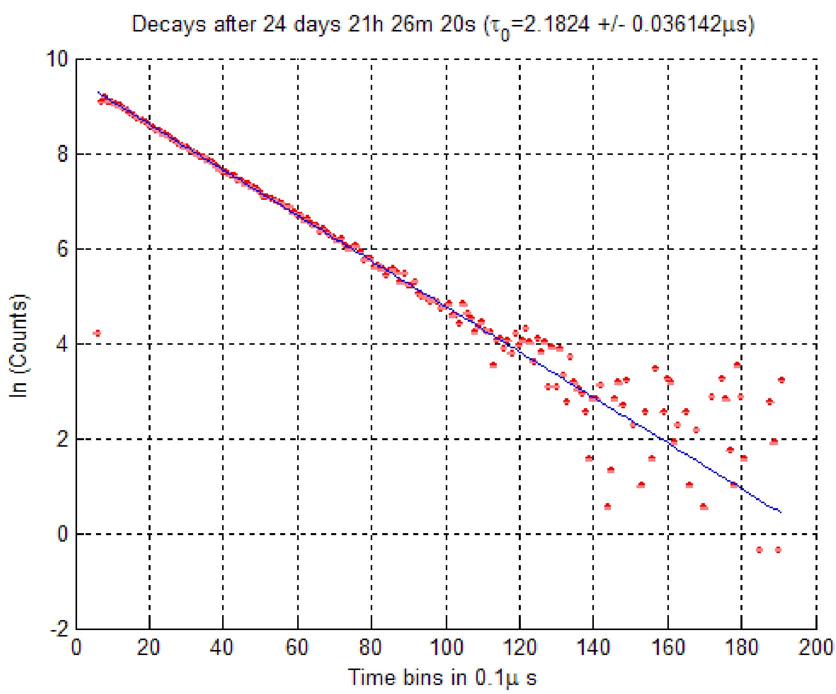

Figure 4.2: Linear fit to the single exponential approximation according to Equation 4.7. Also hardly visible error bars here.

Noting that N_i' follows a Poisson distribution, its error is $\delta N_i' = \sqrt{N_i'}$. Using Equation A.3 to find the error in the slope of Equation 4.7, $\delta B = \delta(-\lambda_{eff} w)$, and applying quadrature gives the physical error in

τ_{eff} as

$$\delta \tau_{eff} = \sqrt{\left(\frac{\delta w}{B}\right)^2 + \left(\frac{w}{B^2}\delta B\right)^2} \tag{4.8}$$

The error in $\lambda_0 = \lambda_{eff} - \lambda_c / (1+\rho)$ is

$$\begin{aligned}\delta \lambda_0 &= \sqrt{\left(\frac{\partial \lambda_0}{\partial \lambda_{eff}} \delta \lambda_{eff}\right)^2 + \left(\frac{\partial \lambda_0}{\partial \lambda_c} \delta \lambda_c\right)^2 + \left(\frac{\partial \lambda_0}{\partial \rho} \delta \rho\right)^2} \\ &= \sqrt{\left(\delta \lambda_{eff}\right)^2 + \left(-\frac{1}{1+\rho} \delta \lambda_c\right)^2 + \left(-\frac{\lambda_c}{(1+\rho)^2} \delta r\right)^2}\end{aligned} \tag{4.9}$$

giving the physical error in τ_0 as

$$\delta \tau_0 = \frac{1}{\lambda_0^2} \delta \lambda_0 = 0.036\,\mu\text{s} \tag{4.10}$$

Combining this physical error with the result in (4.5) gives

$$\begin{aligned}\tau_0 &= \overline{\tau}_0 \pm \sigma_{\tau_0} \pm \delta \tau_0 \\ &= 2.197 \pm 0.006 \pm 0.036\,\mu\text{s}\end{aligned} \tag{4.11}$$

which is within 1.9% relative error.

A chi-square test[23] is performed to ascertain the goodness-of-fit of the parameters to the data. The test statistic is calculated to be

$$c = \sum_n \frac{\left(N_n^{(obs)} - N(n)\right)^2}{N(n)} = 186.7549 \tag{4.12}$$

Since bins 8 to 191 were used to fit the curve, 184 bins were considered. There are three parameters, so the degrees of freedom is $\nu = n - k - 1 = 184 - 3 - 1 = 180$. The chi-square probability is then

$$P\left(X^2_{1-\alpha,\,180} > c\right) \approx 65.05\% \tag{4.11}$$

which is quite low, meaning that the double exponential fit is very good and so are the estimated parameters.

6. Conclusion

The lifetime determined in this experiment, $\tau_0 = 2.197 \pm 0.0006 \pm 0.0036\,\mu\text{s}$, has a relative error of under 2%, making it quite precise. Moreover, it in very good agreement with the accepted value $\tau_0 = 2.19703 \pm 0.00004\,\mu\text{s}$ [7], substantiating its accuracy. However, there were some uncertainties which were not settled. The exact chemical composition of the scintillator was not known, so the detailed capture rate of the negative muons could be inaccurate. Also, the positive/negative muon ratio was not measured at the lab, so its exact value is also unknown, resulting in a reliance on a ratio[9] that is not necessarily valid.

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