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THE DEVELOPMENT OF AN ADAPTIVE JACOBIAN METHOD FOR DYNAMIC CONSTRAINT HANDLING IN INVERSE KINEMATICS

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ABSTRACT

This paper presents inverse kinematics using a novel adaptive Jacobian method, capable of altering the structure of a device's Jacobian in response to a variety of dynamic constraints. These constraints are the result of a specific environment, but the adaptive Jacobian method can be applied to any task where the joints of a robot may become periodically inactive. This paper also summarizes the design of a redundant mechanism for payload delivery in a confined space. The redundant arm is capable of folding into a central stack for insertion through a limited access port, and then expanding to deliver the payload.

INTRODUCTION

The aeronautics industry requires riveting operations inside enclosed sections of aircraft assemblies which are already mostly constructed. Human labor is an unattractive option for this task, as the cramped ergonomic conditions make riveting both inefficient and difficult for the workers. Conventional robots are not an option, as they cannot fit through the limited access ports. Therefore, we are constructing a redundant mechanism that utilizes many degrees of freedom to reach inside of the structure and deliver a tool to key locations. See Figure 1.

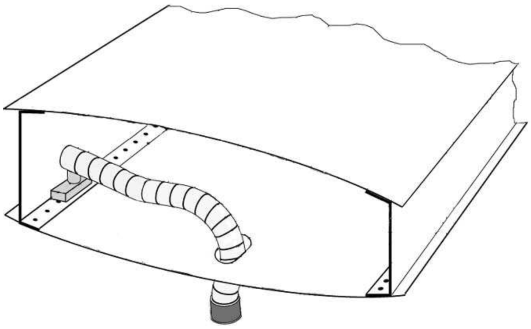

Figure 1. TOOL DELIVERY THROUGH LIMITED ACCESS.

For this work, the term redundant refers to a mechanism whose configuration space is higher dimension than its effector space. In other words, it has more internal degrees of freedom than are necessary to locate and orient the end effector in space. The same degrees of freedom that benefit the arm's performance also provide its major challenge: How to coordinate the internal degrees of freedom to move and position an end effector in a useful manner? This problem is often called the inverse-kinematics problem, and gains difficulty as the number of degrees of freedom increases.

To address this problem, we analyze the functional relationship between the robot's configuration space and effector space. In this paper, we define the configuration space, i.e. the set of robot configurations, as Q. We also define the effector space, i.e. the set of positions and orientations of the end effector, as E. The forward kinematic and inverse kinematic map are therefore denoted by

$$f : Q \to E \quad \text{and} \quad f^{-1} : E \to Q \tag{1}$$

respectively.

Often, it is appropriate to consider the mapping from a velocity in the configuration space to a velocity in the effector space, and vice versa. We relate them with the Jacobian and its inverse, which maps between the tangent space of the configuration space and to the tangent space of the effector space, both denoted as

$$J(q) : T_q Q \to T_x E \quad \text{and} \quad J(q)^{-1} : T_x E \to T_q Q, \tag{2}$$

where $x = f(q)$. In other words,

$$\dot{x} = J(q)\dot{q} \quad \text{and} \quad \dot{q} = J(q)^{-1}\dot{x}. \tag{3}$$

It is important to note here that J is a function of q. We omit writing $J(q)$ in this rest of this work for the sake of clarity.

In this application, links of the robot will unfold from a central stack once inserted into the enclosure. The mechanism will gain degrees of freedom as this occurs, thereby increasing the dimension of its configuration space. This also increases the size of its Jacobian matrix. In this paper, we define an adaptive Jacobian capable of handling the variable dimension configuration space. We also propose techniques that modify this adaptive Jacobian to apply dynamic constraints to the system.

REDUNDANT KINEMATICS

The Jacobian is an mxn matrix, where $m = \dim(E)$ and $n = \dim(Q)$. If you consider J as n column vectors,

$$J = [j_1 \quad j_2 \quad \dots \quad j_n], \tag{4}$$

$j_i\dot{\theta}_i$ represents the ith joint's contribution to the end effector's velocity. Given a robot in R^3, this column vector can be calculated as

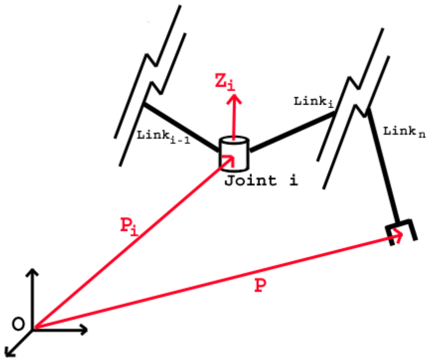

Figure 2. GEOMETRIC DETERMINATION OF j_i.

$$j_i = \begin{bmatrix} z_i \times (P - P_i) \\ z_i \end{bmatrix} \tag{5}$$

for a revolute joint, where P is a position vector of the end effector, P_i is a position vector of joint i, and z_i is the rotation axis of joint i. See Figure 2. Equivalently, the Jacobian can be defined as

$$J = \left[\frac{df}{d\theta}\right]. \tag{6}$$

A redundant mechanism will produce a non-square Jacobian, and there are several techniques used to approximate J^{-1} in this case. Here we use the Moore-Penrose pseudo-inverse. This is defined instantaneously as

$$J^{\dagger} = (JJ^T)^{-1}J^T, \tag{7}$$

if $(JJ^T)^{-1}$ exists.

The matrix $(JJ^T)^{-1}$ exists unless the mechanism is in a singular configuration. A singular configuration is defined as any configuration where the Jacobian matrix loses rank. This is important, because a serial manipulator in a singular configuration loses the ability to arbitrarily move its end effector.

At non-singular configurations, redundant mechanisms will have a null space, defined as

$$\text{null}(J) = \{\dot{q}_{null} \in T_q Q : J\dot{q}_{null} = 0\} \tag{8}$$

where

$$\dim(\mathrm{null}(J)) = n - \mathrm{rank}(J). \tag{9}$$

More intuitively, the null space is the result of having multiple configurations in Q that map to a single position in effector space. It follows that given an arbitrary curve in the workspace, $x(t)$, there will be many associated curves in the configuration space,

$$\{q(t) : f(q(t)) = x(t)\}. \tag{10}$$

The difficulty is finding a method that will choose a valid path from this set.

Baillieul [1] considers a modified version of Eqn. (3),

$$\dot{q}(t) = J^{\dagger}\dot{x}(t) + (I - J^{\dagger}J)v(t). \tag{11}$$

He states that for all curves in the configuration space that do not pass through a singularity, there is a continuous v that satisfies this equation. This implies that v can be used to select a specific curve from the set enumerated in Eqn. 10, but in actuality, finding a particular v is all but impossible without prior knowledge of its associated q.

Liegeois [2] proposes to approximate v by defining

$$v(t) = \alpha \frac{dg}{dq}(q(t)), \tag{12}$$

where $g(q(t))$ is any criterion to be maximized , and α is a scalar weighting function. By defining this criterion intelligently, it is possible to control some general properties of the inverse kinematics solution. As an example, Yoshikawa [3] incorporates a manipulability measure,

$$v(t) = M = \sqrt{\det(JJ^T)}. \tag{13}$$

The value M is zero at a mechanism's singularities, and increases with the distance from these configurations. By maximizing this value, Eqn. 11 will tend to produce solutions that avoid singular configurations.

For systems that must consider a large number of constraints which vary with both t and q, defining g becomes more difficult. If these constraints on the system cannot be predicted ahead of time, then there is no way to predict a useful g. This has led us to develop a strategy for handling dynamic, unpredictable constraints while producing inverse kinematics solutions.

THE ADAPTIVE JACOBIAN

Our task involves inserting a stack of folded joints through a limited access port, and then unfolding links from this stack to deliver a payload. Depending on the size of the enclosed space, only a portion of the device may fit inside at any given time.

To integrate this constraint into the inverse kinematics calculations, we propose an adaptive Jacobian method. This method will remove and restore parts of the Jacobian matrix as necessary to satisfy this constraint, and potentially a variety of others. While the Jacobian is generally non-linear, these inverse kinematic calculations are being linearized via Eqn. 7. This allows us to decompose J into specific parts, and consider the sum of all partial motions determined.

When the robot is only partially inserted, we consider a situation with k joints of the mechanism inserted into the device, where $k < n$. The typical calculation of

$$\Delta \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} j_1 j_2 \,..\, j_n \end{bmatrix}^{\dagger} \Delta x \tag{14}$$

becomes

$$\Delta \begin{bmatrix} q_{n-k+1} \\ q_{n-k+2} \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} j_{n-k+1} \; j_{n-k+2} \,..\, j_n \end{bmatrix}^{\dagger} \Delta x. \tag{15}$$

Entries in Δq and columns in J corresponding to the uninserted links have been removed from the calculation. We use entries $n-k+1$ to n rather than 1 to k, as the base links of the robot are still outside the enclosure. If the number of inserted links changes, their corresponding entries are simply restored or removed as necessary.

It may be the case that the number of joints inserted is small, $k < m$. If this situation occurs, the dimension of Δx

must be reduced by removing $m-k$ principle velocity constraints on the end effector. The rows of J corresponding to those entries are removed as well.

We can now expand the concepts used by the adaptive Jacobian method to incorporate a second type of constraint. Consider a mechanism operating in the enclosed space with an $m \times n$ Jacobian matrix. It is determined that the configuration of one joint must be changed so the robot can be partially removed from the work area.

This joint is now assigned a velocity towards the desired configuration, $\dot{q}_i$. The contribution of this joint velocity towards end effector motion is defined as

$$\Delta x_i = j_i \Delta q_i. \tag{16}$$

Note that Δx_i is not to be confused with the ith component of an effector space motion. The end effector is still required to operate with a task motion, Δx. In order to compensate for Δx_i, we define an adjusted motion term

$$\Delta x_{adj} = \Delta x_i - \Delta x. \tag{17}$$

A modified version of Eqn. 14 is used to produce the motion of the rest of the active joints, compensated for Δx_i,

$$\Delta \begin{bmatrix} q_1 \\ \vdots \\ q_{i-1} \\ q_{i+1} \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} j_1 & .. & j_{i-1} & j_{i+1} & .. & j_n \end{bmatrix}^{\dagger} \Delta x_{adj}. \tag{18}$$

To be more general,

$$\Delta x_{const} = J_{const} \Delta q_{const}, \tag{19}$$

where J_{const} consists of all the column vectors of the original J that are being used to satisfy additional constraints, and Δq_{const} contains the associated joint motions. Equation 17 changes slightly to

$$\Delta x_{adj} = \Delta x_{const} - \Delta x. \tag{20}$$

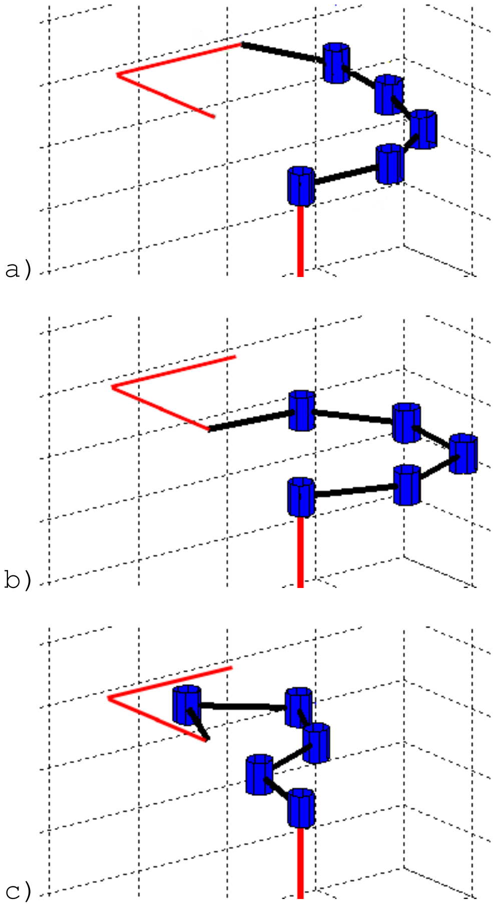

Figure 3. SIMULATING THE ADAPTIVE JACOBIAN [4]

Using a five link planar, revolute mechanism as an example, Figure 3a shows the arm in an initial configuration. Figure 3b shows the end configuration of the robot after the end effector was constrained to an L-shaped path. Figure 3c shows the end configuration of the robot after following the same L-shaped path, but with an additional constraint applied via adaptive Jacobian methods. Specifically, the velocity of the base joint was designated.

MECHANISM DEVELOPMENT

Task Specification, Difficulties

The motivation behind the mechanical design portion of this work is to design a robot arm capable of delivering a 40lb tool payload to a series of designated points. These points are located inside an enclosed space where only one small (10” by 18”) access port is provided. The target points may be up to 7’ away from this access port.

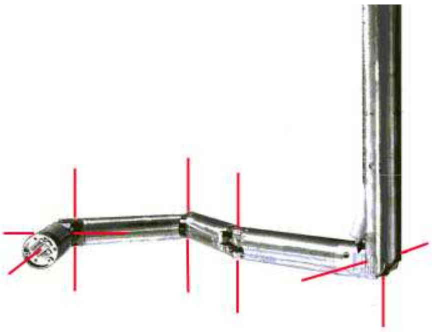

Figure 4. THE LDUA WITH JOINT AXES INDICATED.

Head space upon entering the enclosed space is typically limited, varying from 8” to 3’.

A long, flexible robot would be ideal in this situation. In most cases, long and flexible implies a redundant mechanism. Such a device would be able to continuously feed through the sharp turn required upon entering the access port, yet still reach the furthest target zones.

For any long, uniform robot, the overturning moment at the base joint increases with the square of the device’s length, thus very quickly becoming the dominant force on the actuators and other structural members. In order to increase a device’s reach, it is important to minimize this moment. The difficulty of delivering an end effector of significant mass is obvious.

Current Redundant Mechanisms for Confined Space Tasks

The Biorobotics Laboratory has developed several joints specifically for use in hyper-redundant mechanisms. Shammas [5] gives a brief overview of them, as well as some issues associated with designing compact joints. Design metrics for these joints typically favor minimizing diameter and maximizing range of motion, which makes them less than ideal for load bearing tasks.

The Light Duty Utility Arm (LDUA), developed by SPAR Aerospace for the Department of Energy, is a 7 degree of freedom arm designed for cleaning tasks in waste tanks. These tanks are only accessible through small vertical pipe through limited access. It features a 13.5 ft horizontal reach from the point of insertion, with the capacity to carry a 50 lb end-effector [6]. Joints proximal to the base are oriented such that their actuators do not have to overcome the effects of gravity. See Figure 4. This gives the device impressive reach and load bearing capabilities, but the required headspace for insertion is a significant fraction of the mechanism’s length.

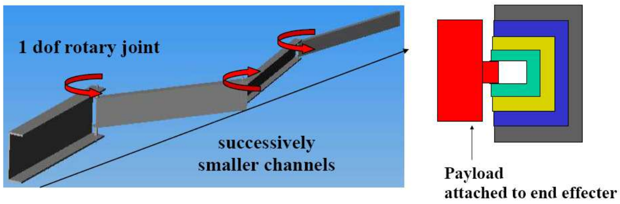

Figure 5. DEPLOYMENT MECHANISM AS PICTURED IN [6].

Roy [7] has also considered a design for the enclosed space assembly task. The design utilizes tapered C-channel links that are capable of nesting together for insertion through the access port. Utilizing a similar joint configuration relative to gravity as the LDUA, this design is able to incorporate lightweight shape memory alloy actuators. See Figure 5.

New Redundant Mechanism Design

Our initial design concept proposed the incorporation of a bracing mechanism into the end effector. This bracing device would be used to steady the tool during riveting operations. However, further investigation made it obvious that scaling current designs would not produce devices able to both navigate the entrance and support the load out to the target zones.

Without resorting to inchworm-like motion, it is difficult to use a bracing mechanism to extend a device’s reach. A bracing device rigid enough to support significant forces from the arm would not be compliant enough allow links of undetermined configuration to be fed through. Features in the enclosed space prevent the bracing mechanism from being able translate at the bracing point on the walls.

Designing a new manipulator was chosen as a more viable alternative. The concept consists of a series of revolute joints configured in such a way that all the links can fold into a central stack. This concept resembling the folding rules used frequently by bricklayers. As seen in other designs discussed above, the joints of our device are configured so that the actuators are not used to support the dominant loads on the structure. Box links are used in order to have both torsional and bending stiffness, which precludes any sort of nesting.

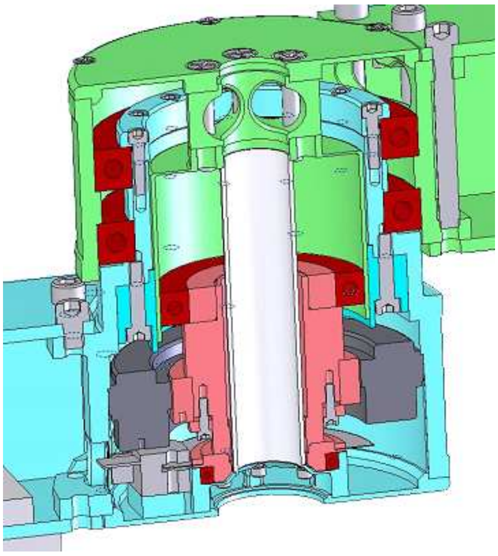

Figure 6. SECTION VIEW OF JOINT DESIGN.

In order to fit as many links as possible into available headspace, the joints need to have a small vertical dimension. This conflicts with the need for a joint able to resist a large overturning moment, as a small vertical dimension would decrease the strength of any bearing assemblies contained within. In order to satisfy both requirements, these joints utilize a number of high strength parts with large diameter/height aspect ratios. This incudes a harmonic drive mated to a direct drive style brushless DC motor. See Figure 6.

For deployment, all or part of the stack would be lifted up through the access port by an external device. See Figure 7. Depending on the vertical clearance of the particular confined space, the number of links that will fit into the space will vary. The external positioner will contain at least three degrees of freedom; one linear to lift the stack vertically, and two rotational to tilt the arm for accessing certain target zones.

The arm will use a tapered link design to decrease the overall weight of the system. There will be 2-3 small joints, and the remaining joints proximal to the base will be slightly larger and stronger. The specific number of smaller links in the robot will be determined after load bearing experiments on the prototype joint. Having a finer resolution in joint sizes would be ideal for minimizing actuator weight, but increases design time and manufacturing cost prohibitively.

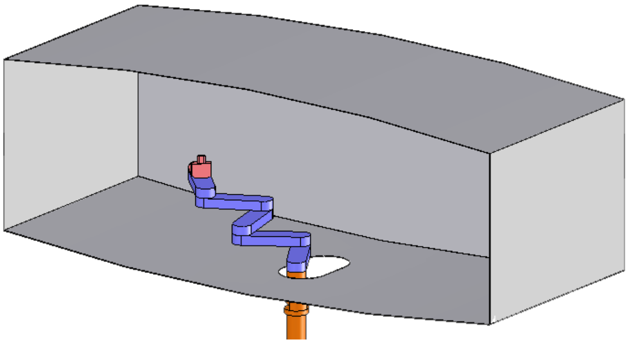

Figure 7. STACKABLE DESIGN, FULLY INSERTED.

The prototype joint has been built, and preliminary testing indicates capabilities nominally within our design parameters. See Figure 8. Planned tests include stiffness measurements, torque capabilities, and life testing.

The end effector itself will be attached to the final joint via 2-3 additional points of actuation. This degree of freedom will provide fine adjustment of the tool, as well as bulk motion for folding and accessing target zones. The specific configuration of these joints will be determined, pending information on target zone locations.

CONCLUSION

Certain robot applications may apply a number of dynamic constraints to a system. These constraints can vary with time and configuration, or they may not vary predictably at all. In that case, it becomes difficult to apply traditional inverse kinematics techniques and still calculate a viable solution. This is particularly true in the case of a system where the dimension of the configuration space is both variable and unpredictable.

The steep physical performance requirements specified in our particular application could not be handled by an algorithm alone, requiring a specialized mechanical design. Rather than attempting to extend the capabilities of a current design with bracing devices, we have opted for a modified joint configuration. The arm will be constructed with joint axes configured to lessen the overturning moment felt by the actuators. This reduces the resultant weight and power consumption of the arm by an order of magnitude.

This end result is a 5 link planar, revolute arm. This simplified configuration may make certain kinematic calculations easier, but the resulting decrease in the manipula-

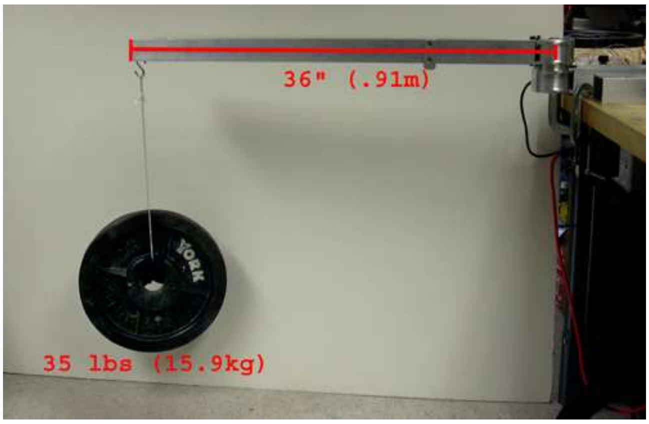

Figure 8. PRELIMINARY LOAD TESTING.

tor's degree of freedom density can reduce the arm's ability to position and orient the end effector. This is especially true given the special constraints of our workspace. The adaptive Jacobian method was developed as a method of handling the inverse kinematics of such a system.

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