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Hierarchical Segmentation of Surfaces Embedded in $\mathbb{R}^3$ for Auto-Body Painting*

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***Abstract*— Complete automation of trajectory planning tools for material deposition/removal applications has become increasingly necessary to reduce the "concept-to-consumer" timeline in product development. Segmentation of a complex automotive surfaces into topologically simple surfaces remains a barrier in the automation of trajectory generation. In this paper, we develop a novel hierarchical procedure to segment a complex automotive surface into geometrically as well as topologically simple components.**

***Index Terms*— segmentation, automation, coverage, trajectory planning, spray painting.**

I. Introduction and Prior Work

Robots are widely used for spray-painting in the automotive industry. Owing to a lack of fully automated trajectory generation tools, paint specialists often require 3 to 5 months to produce the coverage trajectories for a new automobile model. This programming time is a critical bottleneck in the "concept-to-consumer" timeline for bringing a new automobile to the market. Automating the process of path planning will help the paint specialists significantly reduce this programming time by offering them reasonable guidelines for effective paths.

As a first step towards fully automating the task of trajectory generation, we have studied the paint distribution coming out a typical spray gun mechanism such as an electrostatic rotating bell (ESRB) atomizer, and its often complex interaction with the non-planar geometry of the automotive surface [5]. We have also developed algorithms for automated trajectory generation on "simple surfaces" (surfaces that are geodesically convex and diffeomorphic to a disk) that are *a priori* known (using a surface CAD model) [2], [3]. In this prior work, we first select a seed pass termed a *start curve* on the target surface, and then generate additional passes in the coverage trajectory by successively offsetting the start curve along geodesics orthogonal to the start curve (see Fig. 1(a)). By using concepts from differential geometry and standard optimization tools, we can determine the location and orientation of the start curve, the spacing between passes and the end-effector velocity along the passes in order to simultaneously optimize the uniformity of paint deposition, cycle time and paint waste.

Our prior trajectory planning procedures are limited to topologically simple surfaces that approximate individual automobile body components such as doors, fenders, or hoods. However, an automobile body has complex geometry and contains many "holes." An obvious strategy to ensure complete coverage of the automobile surface is to partition it into a collection of simple components. Then, by covering each simple cell with trajectories generated by our existing coverage procedures, the entire surface is guaranteed to be covered. Apart from guaranteeing complete coverage of the surface, there are several other reasons that justify the segmentation of the target surface into "simple" components. First, a good choice of surface segmentation yields a significant reduction in paint waste by avoiding painting over the holes. Second, the number of turns in the coverage trajectory can be minimized by selecting an appropriate combination of segmentation and path orientation in each cell, and thus reducing the number of passes in the coverage trajectory reduces the process cycle time. Finally, segmenting the surface into multiple cells can ensure that the quality of path (e.g. geodesic curvature of the passes) in each cell meets the required standards, thereby ensuring uniform paint deposition in each cell.

In this paper, we assume that as the spray gun follows the coverage trajectory in a particular cell, the resultant paint deposition in neighboring cells is negligible. Although not realistic, this assumption helps us to address the underlying issues in surface segmentation. Our future work will address the problem of stitching cells together, and planning paths across the local cell boundaries. Also, we assume that the dynamic constraints on the motion of the robotic end-effector do not have a significant impact on the segmentation procedure itself and are dealt with in our trajectory planning procedures.

The main contribution of this work is the development of an hierarchical surface segmentation procedure that attempts to satisfy our requirements for the surface segmentation,

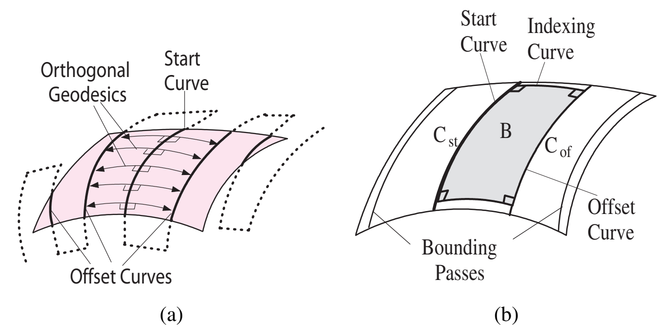

Fig. 1. (a) Our algorithms generate a coverage path on arbitrary simple surfaces by selecting a start curve and offsetting it sideways within the surface to generate new passes. (b) Application of the Gauss-Bonnet theorem to the region bounded between the start curve and the offset curve.

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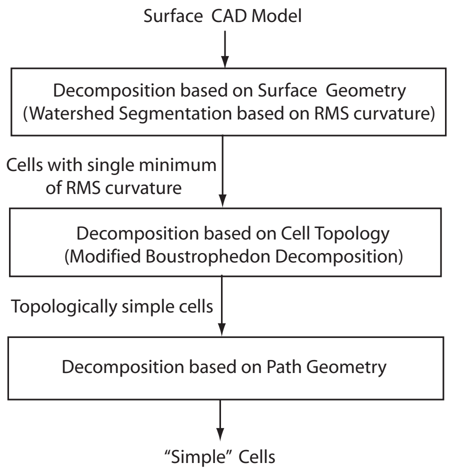

Fig. 2. Block diagram of hierarchical surface segmentation.

by ensuring that the resultant cells are geometrically and topologically simple (see Fig. 2). In Section III, we describe the first step in our segmentation hierarchy that partitions the target surface into geometrically simple cells based on the geometrical features of the target surface such as root mean square curvature. Next, in Section IV, we discuss a topological decomposition procedure that partitions each geometrically simple cell into topologically simple (i.e., diffeomorphic to a disk) components. Finally, to ensure that the quality of paths produced in each cell meets uniformity requirements, we segment each cell on-the-fly based on geodesic curvature of the passes during the trajectory generation process by successive offsetting the start curve. These segmentation procedures are discussed in Section V.

II. Related Work

Several researchers have developed strategies for segmentation of planar environments. Our prior work partitions a planar surface into *topologically monotonic* cells by using critical points of a Morse function [1]. By topologically monotonic cell, we mean that the connectivity of any level set of the selected Morse function remains the same in the cell. Then, by tracing out the level sets of the Morse function with monotonically varying values, each cell can be covered with simple motions.

For cellular decomposition procedures, the number of resultant cells, as well as the number of turns in the coverage path depend on the spatial orientation (or inclination) of the passes. In order to minimize the cycle time, it is desired to minimize the number of turns in the coverage path. For direction-parallel CNC milling applications, Held [6] attempts to select an appropriate orientation for passes to minimize the number of cells, thus minimizing the number of tool retractions and the number of turns in coverage path. Huang [7] develops a procedure to minimize the number of turns in a coverage path for planar polygonal surfaces based on minimizing the cell "heights" along the directions orthogonal to passes. His approach allows the direction of passes to vary from cell to cell and employs dynamic programming to find a cycle time optimal segmentation. Sheng et al. [11] develop procedures that segment a planar surface or a planar projection of a curved surface to optimize a weighted sum of the process cycle time and a cost measure that evaluates the quality of resultant cells in terms of convexity properties. Although these research efforts work well for approximately planar surfaces, the scope of their utility for a complete automobile surface that has typically high surface curvature is limited.

There have been a few attempts to partition a non-planar surface based on the geometrical properties of target surface. Vincze et al. [13] extract cavities (regions with negative curvature), ribs (parallel narrow regions with high curvature), and elementary surface geometries such as planes or cylinders using range sensor data. Each region is painted by generating trajectories using pre-defined strategies for each region. Their procedure is mainly oriented towards *a priori* unknown surfaces and is limited to surfaces that can be segmented into pre-defined parts. Kim and Sarma [8] use vector fields to generate coverage trajectories on a class of simple surfaces and discuss a possible formulation of surface segmentation based on a principle that minimizes cycle time. They do not pursue the details of surface segmentation techniques.

Note that the prior research in surface segmentation for material deposition applications either focus on topological decomposition of planar surfaces or elementary segmentation based on surface geometry of non-planar surfaces. None of the prior research efforts considers surface segmentation based on geometric qualities of passes such as geodesic curvature. The algorithms we develop in this paper focus on all three aspects of segmentation: surface geometry, surface topology, and path geometry. For segmentation based on surface geometry, we borrow the concept of watershed segmentation of surfaces from computer graphics literature [9], [10], and use it in the context of surface segmentation for material deposition applications.

III. Segmentation Based on Surface Geometry

A. Motivation behind Geometrical Decomposition

An essential requirement to ensure uniform paint deposition on the target surface is that the passes in the coverage path should have minimal geodesic curvature. On surfaces with non-zero Gaussian curvature, the offsets of a geodesic are not geodesics in general. In [2], we examine the effect of surface Gaussian curvature, K, on the geodesic curvature, κ_g, of an offset curve. We apply the Gauss-Bonnet theorem to a region, B, bounded between the start curve, C_{st}, and its offset curve, C_{of}, (see Fig. 1(b)) and arrive at

$$\int_{C_{\mathrm{of}}} \kappa_g = \int_B K + \int_{C_{\mathrm{st}}} \kappa_g. \tag{1}$$

Fig. 3(a) illustrates the consequences of Eq. 1 on a cuboidal surface with three facets. Here, the start curve (shown as a dark bold curve) is a geodesic. As the offset passes cross region of high Gaussian curvature (near the "vertex" region), the geodesic curvature of the passes increase significantly resulting

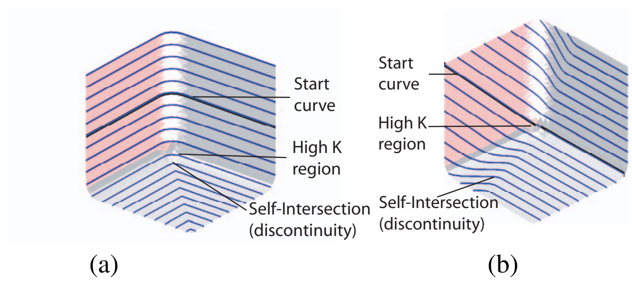

Fig. 3. (a) The geodesic curvature of offset passes increases as the passes sweep past a region of high Gaussian curvature. (b) If the start curve (shown as a dark thick curve) splits the surface into two regions with equal integrals of Gaussian curvature, the geodesic curvature of the passes is minimized.

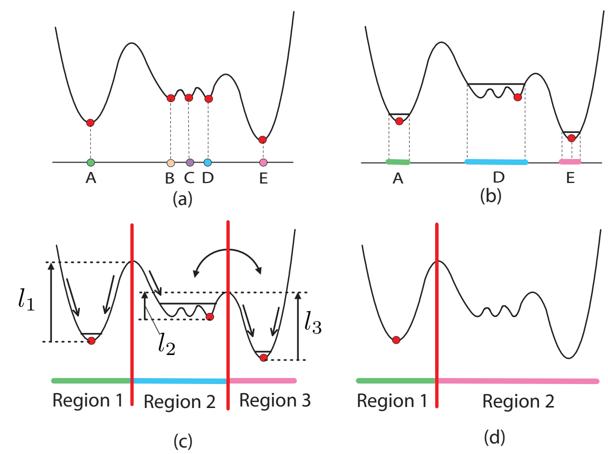

Fig. 4. Progress of watershed algorithm: (a) Minima computation (b) Expansion of minima (c) Descent to local minima (d) Region merging based on watershed height.

in self-intersections (not shown in the Fig.). Removal of closed loops from the self-intersections leave the passes with tangent discontinuities as depicted in Fig. 3(a). The appearance of self-intersections or tangent discontinuities on the passes in a coverage trajectory is highly detrimental to the uniformity of resultant paint deposition.

In [2], we develop a procedure to select the optimum position of the start curve in order to minimize the geodesic curvature of resulting offset passes. Our algorithm selects the start curve as a curve that splits the surface into two parts with equal integrals of Gaussian curvature. Fig. 3(b) shows the geodesic start curve that also divides the integral of Gaussian curvature of the cuboidal surface equally. Although such a choice of start curve ensures minimization of geodesic curvature of the offset passes, the offset passes may ultimately develop self-intersections (see Fig. 3(b)).

The example of the two choices of start curves on the cuboidal surface, when put in the context of Eq. 1 illustrate an important fact about regions of high Gaussian curvature and the quality of the path: *When the regions of high (magnitude) Gaussian curvature appear closer to the start curve, the offset passes develop geodesic curvature early and the quality of the coverage path deteriorates. On the other hand, when the regions of high Gaussian curvature are confined near the surface boundary, the possibility of offset passes developing high geodesic curvature reduces, and thus the quality of the coverage path improves.*

Unfortunately, a path planner does not have a direct control over the target surface geometry. Therefore, in order to optimize the quality of the coverage path (measured in terms of geodesic curvature of the passes), we seek a segmentation of the target surface such that the regions of high Gaussian curvature are confined near the boundaries of the cells. We rely on the watershed segmentation algorithm [9], [10] commonly used in image processing and pattern recognition to produce such segmentations.

B. Watershed Algorithm

By applying the concept of watershed segmentation to triangulated meshes, Mangan and Whitaker [9] extract geometrical features from a mesh. The central idea behind watershed segmentation is to partition the surface such that there is a single (degenerate or non-degenerate) minimum of a specified scalar "height" function within each region. The algorithm proceeds by the following four steps:

1) Minima computation: First, all the minima of the height function on the surface are determined. Fig. 4(a) depicts the minima of a height function defined on a line segment.
2) Minima expansion: This step essentially filters out low magnitude-high frequency features from the height function and merges adjacent minima that fall within a specified tolerance limit into a single minimum. Fig. 4(b) shows the effect of minima expansion on the same example in Fig. 4(a). Here the minima corresponding to points B, C and D are treated as the same minimum.
3) Descent to minima: Beginning at each point on the surface, the algorithm tracks a path along the steepest descent direction until a minima is reached, and all the points travelled along the descent are associated with the encountered minimum. This step essentially produces watershed segmentation, but often leads to oversegmentation of the surface owing to low magnitude-low frequency features. Fig. 4(c) shows the resultant segmentation after the descent to minima operation.
4) Region merging: In order to eliminate low magnitude-low frequency surface features, the watershed height of each region is calculated as the minimum difference in heights of points along the boundary of the region and the height at the minimum of the region. If the watershed height of a region is less than a specified height tolerance, that region is merged with the adjoining region at the boundary point with minimum watershed height. In fig. 4(d), we assume that the watershed height l_2 of region 2 is less than the tolerance value, and therefore Region 2 is merged with Region 3.

Note that the watershed segmentation can be applied to any continuous height function and guarantees that the regions of high function values are restricted to the boundaries of the resultant cells. Mangan and Whitaker [9] use the norm of the covariance matrix of the surface normal evaluated at each point on the surface as the height function, whereas Pulla [10] use the root mean square (RMS) curvature at a given point. Recall that the RMS curvature is given as

$$\kappa_{\text{rms}} = \sqrt{\frac{{\kappa_{\max}}^2 + {\kappa_{\min}}^2}{2}}, \tag{2}$$

where $\kappa_{\max}$ and $\kappa_{\min}$ are the principal curvatures of the surface.

As mentioned in Section III-A, our primary objective behind using the watershed segmentation is to ensure that the regions of high Gaussian curvature are restricted to the boundaries of the resultant cells. To achieve this goal, our algorithm uses the root mean square curvature as the height function for the watershed algorithm because 1) the maxima of root mean square curvature typically match the maxima of Gaussian curvature, thus ensuring that regions of high Gaussian curvature are confined to resulting cell boundaries, and 2) the maxima of the root mean square curvature correspond to regions where the surface normal changes its direction rapidly. Although significant changes in the surface normal do not necessarily imply regions of high Gaussian curvature (e.g. high mean curvature areas on extruded surfaces), a single robotic end-effector may not be able to completely cover cells separated by high surface normal curvatures. We discuss the results of the watershed segmentation based on RMS curvature in Section VI.

IV. Decomposition Based on Surface Topology

The cells produced by watershed segmentation are geometrically simple, however, they may not be topologically simple and may contain holes. In order to minimize the amount of paint waste and make the paint process more economical and environment-friendly, it is desirable to avoid painting over the holes. To achieve this, it is necessary to further segment the cells obtained by geometrical decomposition into sub-cells that are diffeomorphic to a disk (i.e., topologically simple) and which can be covered with simple back and forth motions of the robotic end effector (i.e., topologically monotonic with respect to a chosen orientation of the passes).

To segment a curved surface into topologically simple and monotonic cells, it is intuitive to extend the Boustrophedon decomposition procedures for planar surfaces [4] to non-planar surfaces by sweeping a plane with a fixed normal through the surface, and generating cells based on the connectivity changes of the restriction of the plane to the surface. An inherent problem with this approach is that the section plane may become tangent to the target surface during the sweeping procedure. In such cases, the intersection between the section plane and the surface becomes degenerate, and determining the connectivity of the intersection becomes ambiguous.

Our experience shows that typical automotive surfaces are designed from approximately extruded surfaces (such as doors, hood, roof and trunk of the car) joined together by regions of high curvature such as fenders and support columns. Therefore, the cells produced by applying watershed segmentation to

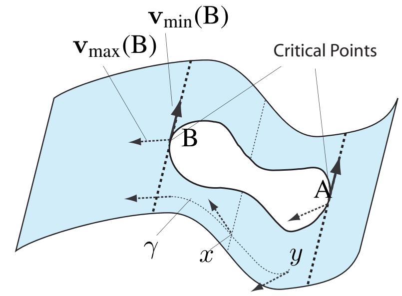

Fig. 5. Topological decomposition due to height function h_1. The boundaries of the cells are determined by level sets of h_1 at the critical points. Also, note that the ambiguity in defining $\mathbf{v}_{\max}(x)$ at an umbilic point x is resolved by parallel transport of vector $\mathbf{v}_{\max}(y)$ along a geodesic connecting x and a non-umbilic point y.

automotive surface are typically extruded surfaces which may have holes, in the form of windows. As such, in this section, we will particularly focus on topological decomposition of extruded surfaces with holes.

To guarantee that the section plane never becomes tangent to an extruded surface, we vary the orientation of the section plane as we sweep it through the surface. We characterize this behavior of the section planes by defining a *height function* such that each level set of the height function is a section plane that is orthogonal to the surface. Ideally, we wish to design a height function such that the intersection of its level sets (i.e., planes) with the surface are geodesic curves. In general, for curved surfaces, such a height function may not exist. However, for extruded surfaces, there are two choices of such height functions that yield passes either parallel or orthogonal to the direction of extrusion. These height functions are:

$$h_1(x) = \langle \mathbf{v}_{\min}(x) \times \mathbf{n}_s(x), x \rangle \quad \text{or} \tag{3}$$

$$h_2(x) = \langle \mathbf{v}_{\max}(x) \times \mathbf{n}_s(x), x \rangle \quad , \tag{4}$$

where $\mathbf{v}_{\min(x)}$ and $\mathbf{v}_{\max}(x)$ are respectively the principal minimum (identically zero at all points) and maximum (absolute values) curvature directions of surface S at point $x \in S$, and $\mathbf{n}_s(x)$ is the normal to the surface at point x. Note that the choice of $h_1(x)$ and $h_2(x)$ becomes ambiguous if x is an umbilic point (i.e., point at which principal curvatures are the same). This ambiguity can easily be resolved by determining a non-umbilic point y such that x lies on the geodesic γ starting from point y, and doing a parallel transport of $\mathbf{v}_{\max}(y)$ along the geodesic to get corresponding vector $\mathbf{v}_{\max}(x)$ at x (see Fig. 5). Similarly, $\mathbf{v}_{\min}(x)$ can be determined by parallel transport of $\mathbf{v}_{\min}(y)$ along γ.

Next, we determine the critical points of function h_1, where the connectivity of the level sets of h_1 changes. These critical points appear at the surface boundary (including hole boundaries). At each critical point p, the tangent $\mathbf{t}_p$ to the surface boundary is parallel to $\mathbf{v}_{\min}$ (see Fig. 5). Once all the critical points of h_1 are determined from this tangency test, we form topologically monotonic cells using the level sets of h_1 passing through the critical points as the cell boundaries. Similarly, for height function h_2, topologically monotonic cells are obtained after determining all critical points by checking if the tangent $\mathbf{t}_q$ to the surface boundary is collinear with $\mathbf{v}_{\max}$ at each point q on the surface boundary.

Among the two choices of height functions for non-planar extruded surfaces, we will choose the one for segmentation that yields minimal sum of the heights (measured along geodesics orthogonal to the level sets of the selected height function) of the cells to reduce the number of turns in the path, and equivalently the cycle time. Note that since we consider only two choices of spatial orientation of passes, neither of the two may be cycle time optimal. However, here our objective is to exploit the special structure of the resultant passes (that is, zero geodesic curvature and the symmetry of the surface in the direction of the passes or the direction orthogonal to it) resulting from these two height functions to ensure good uniformity of resultant paint deposition. Our topological segmentation framework is justified because in automobile industry, the uniformity constraints have a higher precedence than the cycle time constraints; and fortunately for most of the automotive surfaces h_1 or h_2 will minimize the cycle-time.

For planar surfaces, for any given spatial orientation of the passes, the offsets of geodesic passes (i.e., straight lines) are also geodesics. Therefore, uniformity of resultant paint deposition is independent of the spatial orientation of passes, and thus, we choose a cycle-time minimizing height function and obtain the corresponding topological segmentation. On the other hand, for surfaces with high Gaussian curvature, in order to minimize the possibility of section plane becoming tangent to the surface, we consider only one spatial orientation of the section plane that is maximally orthogonal to the surface [12]. We discuss the results of topological decomposition on a few test surfaces in Section VI.

V. Decomposition Based on Path Geometry

The successive application of segmentation procedures based on the surface geometry and topology yields geometrically and topologically simple cells. Each of these individual cells can be completely covered with trajectories generated using our coverage procedures for simple surfaces [3]. For each simple cell, our algorithm selects a geodesic start curve that splits the surface into two parts with equal integrals of Gaussian curvature. Then, additional passes are obtained by offsetting the start curve sideways within the cell (see Fig. 1(a)). The geometrical segmentation ensures that the regions of high Gaussian curvature are confined near the cell boundaries; therefore, for most of the surfaces, the geodesic curvature of the offset passes remains low as we offset the start curve in each cell. However, it is possible that the surface has only one minimum of Gaussian curvature on the surface, and yet the integral of Gaussian curvature over the surface is high, for example, consider a spherical surface. In such cases, the geodesic curvature of the offset passes may increase significantly and the subsequent offset passes may self-intersect. Therefore, it is necessary that the hierarchical segmentation procedures not only produce geometrically and topologically simple cells, but also ensure low geodesic curvature of resultant offset passes.

To ensure that the geodesic curvature of the offset passes does not exceed a specified limit in each cell, we propose a procedure that segments a geometrically and topologically simple cell on-the-fly during the path generation procedure. In each cell, our algorithm first generates the optimum geodesic start curve that splits the surface into two parts with equal integrals of Gaussian curvature [2]. We offset this start curve sideways within the surface and compute the average geodesic curvature κ_{avg} (integral of the geodesic curvature of the curve divided by the length of the curve) of each newly generated pass. The process of generating new offset passes by subsequent offsetting continues until the average geodesic curvature of the last generated offset curve exceeds a specified limit, or the cell is covered completely. In the first case, the cell is split into two cells along the newly generated offset curve.

A new start curve is determined for each newly generated cell that is not covered yet, and the process continues until the original cell is covered completely. In Section VI, we examine the results of such path geometric decomposition on a spherical surface.

Thus, the successive application of segmentation procedures based on surface geometry, surface topology and path geometry ensures that the resultant cells are geometrically and topological simple and satisfy the constraints on the quality of path (in terms of geodesic curvature) in each cell. Note that we make no explicit effort to merge multiple cells together and form larger geometrically and topologically simple cells or minimize the cross-deposition effects near the cell boundaries; these issues will be addressed in future work.

VI. Simulation Results

We have tested our hierarchical segmentation procedure on a variety of test surfaces in simulation. The results of our geometrical segmentation techniques on surfaces similar to real automotive surfaces can be seen in Fig. 6. The surface depicted in this figure is loosely based on the profile of a Ford Focus. Here, the hood, the roof, the trunk, the front and back windows are segmented as different regions from each other, whereas each side surface (typically comprising of the doors, and the front and back side fenders on a real automotive surface – cells A and J in Fig. 6) is clustered as a single cell.

Fig. 7 illustrates the result of only topological decomposition of an extruded surface. The height function chosen in this case is h_1 as defined in Section IV. This height function ensures that the section planes used for the decomposition are always orthogonal to the surface. Note that for the conventional planar sweep techniques, any orientation of section planes which yields passes along minimum curvature directions will have degenerate intersections with this surface.

Finally, for a spherical surface as shown in Fig. 8 the application of geometrical segmentation followed by topological segmentation yields a single cell (i.e., the original surface). Here, during the trajectory generation phase, as we offset the geodesic start curve (which also splits the Gaussian curvature

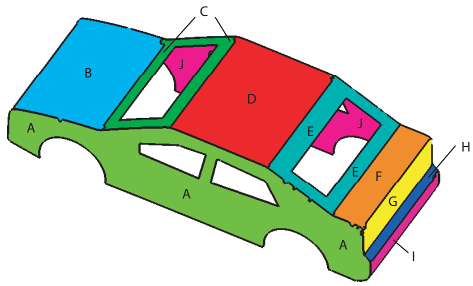

Fig. 6. Resultant cells by applying watershed algorithm to RMS curvature on a surface similar to a car shape. Each letter indicates a separate cell.

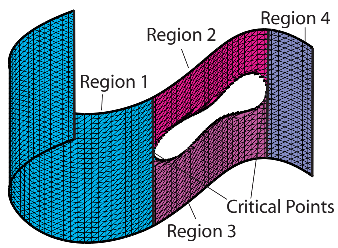

Fig. 7. Topological decomposition of an extruded surface with a hole.

of the surface), the geodesic curvature of the offset curves goes on increasing and accordingly the surface is segmented into three regions as shown in Fig. 8. This segmentation based on the path geometry ensures that the geodesic curvature of passes in each cell remains acceptable. Thus, these simulation results illustrate the principles behind our hierarchical segmentation procedure.

VII. Conclusion and Future Work

By examining the relationships between the desired properties of the coverage paths and the geometry as well as the topology of the surface, we develop a novel hierarchical procedure that segments a complex surface into smaller subsets. These subsets are geometrically as well as topologically simple and can be covered using simple coverage procedures. These surface segmentation techniques take a step towards the ultimate goal of developing turn-key automated trajectory planning systems that drastically reduce the programming time required for spray painting applications, with significant improvements in the economical and environmental quality of the paint-job.

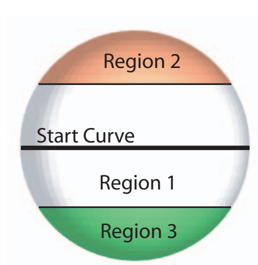

Fig. 8. Segmentation of a sphere based on geodesic curvature of offset passes.

In this work, we assume each cell that results from the hierarchical segmentation can be painted independently from the others. In other words, we assume that the motion of spray gun along the coverage trajectory in any particular cell does not deposit paint in any of its neighboring cells. This assumption is not realistic, and care must be taken to "stitch" the coverage trajectories in neighboring cells appropriately near the cell boundaries to ensure uniform paint deposition in each cell. Our future work will focus on merging multiple cells together and optimizing the ordering in which resultant cells are painted, the orientations of coverage paths in each cell and the end-effector velocity in order to improve the uniformity of overall paint deposition and the process cycle time.

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