Given string w of 1s and 0s, and an integer i ($1 \le i \le |w|$) define

$TALLY_1(w,i)$ = #(1) - #(0) up until position i

w =	0	1	1	0	1	0	0	0	1	0
$TALLY_1$:	-1	0	1	0	1	0	-1	-2	-1	-2

$TALLY_1(w,1) = -1$

Give a CFG for { w | w has the same number of 1s and 0s}

1.1

Give a CFG for { w | w has more 1s than 0s }

$M \to 1S \mid 1M \mid SM \mid 1$

$S \to 1S0 \mid 0S1 \mid SS \mid \varepsilon$

If w has more 1s than 0s and does not start with a 1, then it can be split into w = yz where y has the same number of 1s and 0s and z has more 1s than 0s

2.1

Prove that $\{ 0^n1^n0^n1^n \mid n \ge 0\}$ is not context free

By the pumping lemma, $0^P1^P0^P1^P = uvxyz$, where

1. $|vy| > 0$
2. $|vxy| \le P$
3. $uv^ixy^iz \in L$ for any $i \ge 0$

Since $|vxy| \le P$, then it is either all 1s, all 0s or of the form 1^m0^n or 0^m1^n

0	1	0	1

2.2

Prove that { w#x | w is a substring of x} is not context free

Try pumping the string $0^P1^P\#0^P1^P$

By the pumping lemma, $0^P1^P\#0^P1^P$ = uvxyz, where

1. **$|vy| > 0$**
2. **$|vxy| \leq P$**
3. **$uv^ixy^iz \in L$ for any $i \geq 0$**

Three possibilities:

vxy comes before #

vxy comes after #

vxy contains #

Midterm I will cover EVERYTHING we have seen so far

It will be Closed-Book as well as Closed-Everything

I will be in my office on Monday between 2 pm and 4 pm

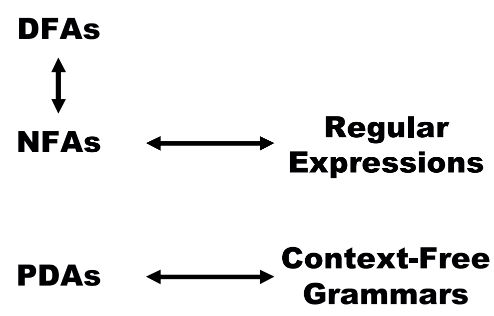
GUESS THE MIDTERM GAME

Rules: Pair up into groups of 2 people

In each pair:

Pick a scribe with LEGIBLE handwriting

Write down the names of the people in your group



THE REGULAR OPERATIONS

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

Negation: $\neg A = \{ w \mid w \notin A \}$

Concatenation: $A \cdot B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

How do we test if two regular expressions are the same?

THE PUMPING LEMMA

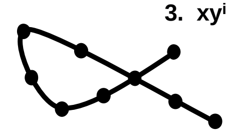
Let L be a regular language with $|L| = \infty$

Then there exists a positive integer P such that

if $w \in L$ and $|w| \geq P$

then $w = xyz$, where:

1. $|y| > 0$
2. $|xy| \leq P$
3. $xy^iz \in L$ for any $i \geq 0$



You need to be able to prove that a language is not regular

Show that if an NFA with k states accepts any string at all, then it accepts a string of length k-1 or less

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