

**TIME COMPLEXITY AND
POLYNOMIAL TIME**

TUESDAY OCTOBER 18

COMPLEXITY THEORY

Studies what can and can't be computed under limited resources such as time, space, etc

Today: Time complexity

MEASURING TIME COMPLEXITY

We measure time complexity by counting the elementary steps required for a machine to halt

Consider the language $A = \{ 0^k 1^k \mid k \geq 0 \}$

Definition: Let M be a TM that halts on all inputs. The running time or time-complexity of M is the function $f : \mathbb{N} \rightarrow \mathbb{N}$, where $f(n)$ is the maximum number of steps that M uses on any input of length n .

ASYMPTOTIC ANALYSIS

$$5n^3 + 2n^2 + 22n + 6$$

BIG-O

Let f and g be two functions $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$. We say that $f(n) = O(g(n))$ if positive integers c and n_0 exist so that for every integer $n \geq n_0$

$$f(n) \leq cg(n)$$

When $f(n) = O(g(n))$, we say that $g(n)$ is an asymptotic upper bound for $f(n)$

$$5n^3 + 2n^2 + 22n + 6 = O(n^3)$$

If $c = 6$ and $n_0 = 10$, then $5n^3 + 2n^2 + 22n + 6 \leq cn^3$

$$2n^{4.1} + 200283n^4 + 2$$

$$3n \log_2 n + 5n \log_2 \log_2 n$$

$$n \log_{10} n^{78}$$

Definition: $\text{TIME}(t(n)) = \{ L \mid L \text{ is a language decided by a } O(t(n)) \text{ time Turing Machine} \}$

$$A = \{ 0^k 1^k \mid k \geq 0 \} \in \text{TIME}(n^2)$$

$A = \{ 0^k 1^k \mid k \geq 0 \} \in \text{TIME}(n \log n)$

**We can prove that a TM cannot
decide A in less time than $O(n \log n)$**

**Can $A = \{ 0^k 1^k \mid k \geq 0 \}$ be decided in time $O(n)$
with a two-tape TM?**

**Different models of computation
yield different running times for
the same language!**

Theorem: Let $t(n)$ be a function such that $t(n) \geq n$. Then every $t(n)$ -time multi-tape TM has an equivalent $O(t(n)^2)$ single tape TM

$$P = \bigcup_{k \in \mathbb{N}} \text{TIME}(n^k)$$

OPTIMAL FACTORING ALGORITHM

On input n

- T_1 Run every other step
- T_2 Run once every 4 steps
- T_3 Run once every 8 steps
- T_4 Run once every 16 steps
- \vdots
- T_i Run once every 2^i steps
- \vdots

Whenever a machine halts with output m ,
test if m divides n

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Read Chapters 7.1 and 7.2 of the book for next time