15-453 Homework # 8

1. Double SAT (20 Points)

Let DOUBLE-SAT $= \{\phi \mid \phi$ has at least two satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.

2. HALF-CLIQUE (20 Points)

Let HALF-CLIQUE $= \{G \mid G$ is an undirected graph having a complete subgraph with at least $n/2$ nodes, where n is the number of nodes in $G\}$. Show that CLIQUE $\leq_P$ HALF-CLIQUE.

3. If P = NP... (40 Points)

Prove the following:

1. Show that if P = NP then every language $A \in$ P is NP-complete, except $A = \emptyset$ and $A = \Sigma^*$
2. Show that if P = NP, a polynomial time algorithm exists that, given a Boolean formula ϕ, actually produces a satisfying assignment for ϕ if it is satisfiable. (Note: NP is a class of languages, so saying "SAT is in NP" is not a correct answer for this problem. This problem requires you to show that if you can decide whether a formula is satisfiable in polynomial time, then you can actually find a satisfying assignment for it in polynomial time.)