

15-453 Homework # 7

1. Pay back from
way back
(35 Points)

One way to determine if two DFAs recognize the same language is the following. Let A and B be DFAs. We construct a new DFA C that accepts all strings that are recognized by either A or B but not by both. In other words, $L(C) = (L(A) \cap \neg L(B)) \cup (\neg L(A) \cap L(B))$.

1. How can C be constructed?
2. How can C be used to determine if A and B accept the same language? Provide an algorithm that uses C to determine if $L(A) = L(B)$.
3. If A has n states and B has m states, what is the running time of this procedure?

2. Fixed Point
(10 Points)

In the fixed-point theorem let the transformation t be a function that interchanges the states q_{accept} and q_{reject} in Turing machine descriptions. Give an example of a fixed point for t . That is, give a TM M such that $L(M) = L(t(M))$.

3. Machine
Emulation
(35 Points)

1. Describe in English a TM T that can determine whether an arbitrary TM M accepts a string w in time 2^n , where $n = |w|$, and show that your TM can run in time $O(|M| \cdot 2^n)$ on every pair (M, w) .
2. Let T be any TM that can determine whether an arbitrary TM M accepts a string w in time 2^n , where $n = |w|$. Show that there exist inputs (M, w) for which T runs in time at least $\Omega(\frac{2^n}{|M|})$, where $n = |w|$.