

15-453 Homework # 5

1. Reverse
(20 Points)

Let $S = \{M \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Is S decidable or undecidable?

2. Going Left
(20 Points)

In this problem, you will work with two similar languages, only one of which is decidable. **Note:** Only one of these two problems will be graded. We will flip a coin to decide which.

1. Consider the problem of testing whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and determine whether it is decidable.
2. Consider the problem of testing whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w . Formulate this problem as a language and determine whether it is decidable.

3. A Powerful
Result
(40 Points)

Let P be a language of Turing machines (i.e. the strings in P are representations of Turing machines). Assume that P satisfies the following properties:

- For any TMs M_1 and M_2 , where $L(M_1) = L(M_2)$, $M_1 \in P$ if and only if $M_2 \in P$. In other words, the membership of a TM M in P depends only on the language of M .
- There exists TMs M_1 and M_2 , where $M_1 \in P$ and $M_2 \notin P$. In other words, P is nontrivial — it holds for some, but not all, TMs.

Prove that P is undecidable. Please write a clear and complete proof.

(Hint: Suppose $M_1 \in P$. Reduce deciding A_{TM} to deciding P by constructing a TM M_w that accepts on input x if and only if M accepts w and M_1 accepts x . Then run a TM for P on M_w .)