

# 15-453 Homework # 3

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## 1. CFGs (10 Points)

Give context-free grammars that generate the following languages:

1.  $\{w \mid w \text{ contains more 1s than 0s}\}$ . (The alphabet here is  $\{0, 1\}$ .)
2.  $\{w\#x \mid w^R \text{ is a substring of } x \text{ for } x, w \in \{0, 1\}^*\}$ . (The alphabet here is  $\{0, 1, \#\}$ .)

## 2. Pumpin' (20 Points)

Show that the following languages are not context free:

1.  $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$ .
2.  $\{w\#x \mid w \text{ is a substring of } x, \text{ where } x \in \{a, b\}^*\}$ .

## 3. Proofs (50 Points)

As usual, this problem is designed to help you learn how to write mathematical proofs. Therefore, we will pay careful attention to how you write your proofs. Make sure you are clear, correct, and justify all your assertions.

1. A context-free language is said to be *inherently ambiguous* if it can only be generated by ambiguous grammars. Prove that the language  $A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$  is inherently ambiguous.
2. In the literature, PDAs are often called FILOs (first in, last out). Define a FIFO (first in, first out) machine  $M$  to be exactly like a FILO except that  $M$  uses a queue instead of a stack. Compare the languages recognized by FIFOs to the languages recognized by FILOs. What can you say about  $\mathcal{L} = \{L \mid L \text{ is a language recognized by a FIFO}\}$ ? Does  $\mathcal{L}$  contain the regular languages? Is  $\mathcal{L}$  equal to the set of regular languages. What about the intersection of  $\mathcal{L}$  and the context-free languages?