15-453 Homework # 1

1. Designing Automata (25 Points)

Design **deterministic** finite automata for each of the following sets:

1. The set of strings in $\{2, 3, 5\}^*$ containing the substring 235.
2. The set of strings in $\{a\}^*$ whose length is divisible by either 5 or 7.
3. The set of strings $x \in \{0, 1\}^*$ such that the number of 1 characters is odd and the number of 0 characters is divisible by 3.
4. The set of strings over the alphabet $\{a, b\}$ containing at least two occurrences of three consecutive b characters, with overlaps permitted (e.g., the string $bbbb$ should be accepted).
5. The set of strings over the alphabet $\{a, b\}$ containing at least two occurrences of three consecutive b characters, with overlaps not permitted (e.g., the string $bbbb$ should not be accepted).

2. Regular Expressions (15 points)

Give regular expressions for each of the following subsets of $\{a, b\}^*$. Simplify as much as possible.

1. $\{x \mid x$ contains an even number of a's$\}$.
2. $\{x \mid x$ contains an odd number of b's$\}$.
3. $\{x \mid x$ contains an even number of a's or an odd number of b's$\}$.

3. Even Bits (40 Points)

In this problem, you will learn how to prove statements of the form *if A is a regular set, then so is A'*, where A' is formed by massaging A in some way. Most of these statements can be proved by applying the following steps:

Step 1. Assume we are given a deterministic finite automaton $M = (Q, \Sigma, \delta, q, F)$ accepting A. We want to build a **non-deterministic** automaton $M' = (Q', \Sigma, \delta', q', F')$ accepting A'. The first thing to do is to come up with an intuitive design of M' in terms of moving around on the states of M. Think about the initial configuration, how M' should react in response to each input symbol, and what the accepting configurations should be.

Step 2. Write down formal descriptions of Q', δ', q' and F' that formally capture your intuition developed in Step 1. Think about what the states Q' should be — you need to figure out how to encode formally the information that the new machine needs to remember at each step.

Step 3. Prove that $L(M') = A'$. This can usually be done by proving a relationship between δ' and δ, and then extending this relationship to account for sequences of steps.

Use the previous methodology (or any other methodology) to solve the following problem:

1. For any set of strings A, define the set $\mathsf{EvenBits}(A) = \{x = x_1x_2 \dots x_n \mid$ there exists $y \in A$ with $|y| = 2|x|$ and $y = y_1x_1y_2x_2 \dots y_nx_n\}$. For example, $\mathsf{EvenBits}(\{a, ab, bab, bbab\}) = \{b, bb\}$. If A is regular, **prove** that $\mathsf{EvenBits}(A)$ is also regular.