Machine Learning for Signal Processing
Detecting faces (& other objects) in images
Class 7. 22 Sep 2015
Last Lecture: How to describe a face

• A “typical face” that captures the essence of “facehood”.
• The principal Eigen face.
A collection of least squares typical faces

- Extension: Many Eigenfaces
- Approximate every face $f$ as $f = \sum w_{f_i} V_i$ where $V_2$, $V_3$ correct errors resulting from using only $V_1$
- And so on..

$V = [V_1 \ V_2 \ V_3]$ can be computed through Eigen analysis
Detecting Faces in Images
Detecting Faces in Images

• Finding face like patterns
  – How do we find if a picture has faces in it
  – Where are the faces?

• A simple solution:
  – Define a “typical face”
  – Find the “typical face” in the image
Given an image and a ‘typical’ face how do I find the faces?

400 × 200 (RGB)

[Image of faces] + [Image of typical face] + [Diagram of process]
Finding faces in an image

• Picture is larger than the “typical face”
  – E.g. typical face is 100x100, picture is 600x800

• First convert to greyscale
  – R + G + B
  – Not very useful to work in color
Finding faces in an image

• Goal .. To find out if and where images that look like the “typical” face occur in the picture
Finding faces in an image

• Try to “match” the typical face to each location in the picture
Finding faces in an image

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Finding faces in an image

- Try to “match” the typical face to each location in the picture

- The “typical face” will explain some spots on the image much better than others
  - These are the spots at which we probably have a face!
How to “match”

• What exactly is the “match”
  – What is the match “score”
How to “match”

• What exactly is the “match”
  – What is the match “score”

• The DOT Product
  – Express the typical face as a vector
  – Express the region of the image being evaluated as a vector
  – Compute the dot product of the typical face vector and the “region” vector
What do we get

• The right panel shows the dot product at various locations
  – Redder is higher
    • The locations of peaks indicate locations of faces!
What do we get

- The right panel shows the dot product at various locations
  - Redder is higher
    - The locations of peaks indicate locations of faces!
- Correctly detects all three faces
  - Likes George’s face most
    - He looks most like the typical face
- Also finds a face where there is none!
  - A false alarm
What do we get

• The right panel shows the dot product at various locations
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Sliding windows solves only the issue of location – what about scale?

- Not all faces are the same size
- Some people have bigger faces
- The size of the face on the image changes with perspective
- Our “typical face” only represents one of these sizes
Scale-Space Pyramid

Figure 1.4: The Scale-Space Pyramid. The detector is run using the sliding windows approach over the input image at various scales. When the scale of the person matches the detector scale the classifier will (hopefully) fire yielding an accurate detection.
Speed concerns

• Sliding windows AND Scale-space pyramid may yield million’s of ‘windows’ to investigate!
• Especially for small objects in large images
Location – Scale – What about Rotation?

- The head need not always be upright!
  - Our typical face image was upright
Solution

• Create many “typical faces”
  – One for each scaling factor
  – One for each rotation
    • How will we do this?

• Match them all

• Does this work
  – Kind of .. Not well enough at all
  – We need more sophisticated models
Face Detection: A Quick Historical Perspective

1. Many more complex methods
   - Use edge detectors and search for face like patterns
   - Find “feature” detectors (noses, ears..) and employ them in complex neural networks.

2. The Viola Jones method
   - Boosted cascaded classifiers

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Figure 1: The basic algorithm used for face detection.
Face Detection: A Quick Historical Perspective

• Many more complex methods
  – Use edge detectors and search for face like patterns
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• The Viola Jones method (20K+ Citations!)
  – Boosted cascaded classifiers
And even before that – what is classification?

• Given “features” describing an entity, determine the category it belongs to
  – Walks on two legs, has no hair. Is this
    • A Chimpanzee
    • A Human
  – Has long hair, is 5’6” tall, is this
    • A man
    • A woman
  – Matches “eye” pattern with score 0.5, “mouth pattern” with score 0.25, “nose” pattern with score 0.1. Are we looking at
    • A face
    • Not a face?
Classification

• Multi-class classification
  – Many possible categories
    • E.g. Sounds “AH, IY, UW, EY..”
    • E.g. Images “Tree, dog, house, person..”

• Binary classification
  – Only two categories
    • Man vs. Woman
    • Face vs. not a face...
Detection vs Classification

• Detection: Find an X
• Classification: Find the correct label X,Y,Z etc.
Detection vs Classification

• Detection: Find an X
• Classification: Find the correct label X,Y,Z etc.
• Binary Classification as Detection: Find the correct label X or not-X
Face Detection as Classification

- Faces can be many sizes
- They can happen anywhere in the image
- For each face size
  - For each location
    - Classify a rectangular region of the face size, at that location, as a face or not a face
- This is a series of *binary* classification problems
Binary classification

- Classification can be abstracted as follows
- \( H: X \rightarrow (+1,-1) \)
  - A function \( H \) that takes as input some \( X \) and outputs a +1 or -1
    - \( X \) is the set of “features”
    - +1/-1 represent the two classes
- Many mechanisms (may types of “H”)
  - Any many ways of characterizing “\( X \)”
- We’ll look at a specific method based on voting with simple rules
  - A “META” method
Introduction to Boosting

• An *ensemble* method that sequentially combines many simple *BINARY* classifiers to construct a final complex classifier
  – Simple classifiers are often called “weak” learners
  – The complex classifiers are called “strong” learners

• Each weak learner focuses on instances where the previous classifier failed
  – Give greater weight to instances that have been incorrectly classified by previous learners

• Restrictions for weak learners
  – Better than 50% correct

• Final classifier is *weighted* sum of weak classifiers
Boosting: A very simple idea

• One can come up with many rules to classify
  – E.g. Chimpanzee vs. Human classifier:
    – If arms == long, entity is chimpanzee
    – If height > 5’6” entity is human
    – If lives in house == entity is human
    – If lives in zoo == entity is chimpanzee

• Each of them is a reasonable rule, but makes many mistakes
  – Each rule has an intrinsic error rate

• *Combine* the predictions of these rules
  – But not equally
  – Rules that are less accurate should be given lesser weight
Boosting and the Chimpanzee Problem

- The total confidence in all classifiers that classify the entity as a chimpanzee is
  \[ \text{Score}_{\text{chimp}} = \sum \alpha_{\text{classifier}} \]
  \( \text{classifier favors chimpanzee} \)

- The total confidence in all classifiers that classify it as a human is
  \[ \text{Score}_{\text{human}} = \sum \alpha_{\text{classifier}} \]
  \( \text{classifier favors human} \)

- If \( \text{Score}_{\text{chimp}} > \text{Score}_{\text{human}} \) then the our belief that we have a chimpanzee is greater than the belief that we have a human
Boosting

• The basic idea: Can a “weak” learning algorithm that performs just slightly better than a random guess be *boosted* into an arbitrarily accurate “strong” learner

• This is a “meta” algorithm, that poses no constraints on the form of the weak learners themselves
Boosting: A Voting Perspective

• Boosting is a form of voting
  – Let a number of different classifiers classify the data
  – Go with the majority
  – Intuition says that as the number of classifiers increases, the dependability of the majority vote increases
    • Boosting by majority

• Boosting by *weighted* majority
  – A (weighted) majority vote taken over all the classifiers
  – How do we compute weights for the classifiers?
  – How do we actually train the classifiers
ADA Boost

• Challenge: how to optimize the classifiers and their weights?
  – Trivial solution: Train all classifiers independently
  – Optimal: Each classifier focuses on what others missed
  – But joint optimization becomes impossible

• Adaptive Boosting: Greedy incremental optimization of classifiers
  – Keep adding classifiers incrementally, to fix what others missed
AdaBoost

ILLUSTRATIVE EXAMPLE
AdaBoost

First WEAK Learner
AdaBoost

The First Weak Learner makes Errors
AdaBoost

Reweighted data
AdaBoost

SECOND Weak Learner

FOCUSES ON DATA “MISSED” BY FIRST LEARNER
SECOND *STRONG* Learner Combines both Weak Learners
RETURNING TO THE SECOND WEAK LEARNER
AdaBoost

The SECOND Weak Learner makes Errors
AdaBoost

Reweighting data

$h_2$

$h_1$
AdaBoost

FOCUSES ON DATA “MISSED” BY FIRST AND SECOND LEARNERS

THIRD Weak Learner
AdaBoost

THIRD STRONG Learner
Boosting: An Example

- Red dots represent training data from Red class
- Blue dots represent training data from Blue class
The final strong learner has learnt a complicated decision boundary
• The final **strong** learner has learnt a complicated decision boundary

• Decision boundaries in areas with low density of training points assumed inconsequential
Overall Learning Pattern

- Strong learner increasingly accurate with increasing number of weak learners

- Residual errors increasingly difficult to correct
  - Additional weak learners less and less effective

![Graph showing error rate over number of weak learners]

Error of n\textsuperscript{th} weak learner
**Overfitting**

- Note: Can continue to add weak learners EVEN after strong learner error goes to 0!
  - Shown to IMPROVE generalization!
AdaBoost: Summary

- No relation to Ada Lovelace
- Adaptive Boosting
- Adaptively Selects Weak Learners
- ~8K citations for just one paper for Freund and Schapire
The ADABoost Algorithm

- Initialize $D_1(x_i) = 1/N$
- For $t = 1, \ldots, T$
  - Train a weak classifier $h_t$ using distribution $D_t$
  - Compute total error on training data
    - $\varepsilon_t = \text{Sum} \{D_t(x_i) \frac{1}{2}(1 - y_i h_t(x_i))\}$
  - Set $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \varepsilon_t}{\varepsilon_t} \right)$
  - For $i = 1 \ldots N$
    - set $D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t y_i h_t(x_i))$
  - Normalize $D_{t+1}$ to make it a distribution
- The final classifier is
  - $H(x) = \text{sign}(\sum_t \alpha_t h_t(x))$
First, some example data

- Face detection with multiple Eigen faces
- Step 0: Derived top 2 Eigen faces from Eigen face training data
- Step 1: On a (different) set of examples, express each image as a linear combination of Eigen faces
  - Examples include both faces and non faces
  - Even the non-face images are explained in terms of the Eigen faces

\[
\begin{align*}
\text{Image} &= a \cdot E_1 + b \cdot E_2 \\
E_1 &= 0.3 \cdot E_1 - 0.6 \cdot E_2 \\
E_2 &= 0.5 \cdot E_1 - 0.5 \cdot E_2 \\
E_3 &= 0.7 \cdot E_1 - 0.1 \cdot E_2 \\
E_4 &= 0.6 \cdot E_1 - 0.4 \cdot E_2 \\
E_5 &= 0.2 \cdot E_1 + 0.4 \cdot E_2 \\
E_6 &= -0.8 \cdot E_1 - 0.1 \cdot E_2 \\
E_7 &= 0.4 \cdot E_1 - 0.9 \cdot E_2 \\
E_8 &= 0.2 \cdot E_1 + 0.5 \cdot E_2
\end{align*}
\]
# Training Data

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- **Face** = +1
- **Non-face** = -1
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The E1 “Stump”

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1

Sign = +1, error = 3/8
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<th>E1</th>
<th>E2.</th>
<th>Class</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.3</td>
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<td>+1</td>
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<td>C</td>
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</tr>
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<td>0.2</td>
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<td>-1</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Sign = +1, error = 3/8
Sign = -1, error = 5/8
The E1 “Stump”

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1

Sign = +1, error = 2/8
Sign = -1, error = 6/8

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<tr>
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</table>
The E1 “Stump”

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0) face = true

sign = +1 or -1

ID | E1  | E2. | Class | Weight |
---|-----|-----|-------|--------|
A  | 0.3 | -0.6| +1    | 1/8    |
B  | 0.5 | -0.5| +1    | 1/8    |
C  | 0.7 | -0.1| +1    | 1/8    |
D  | 0.6 | -0.4| +1    | 1/8    |
E  | 0.2 | 0.4 | -1    | 1/8    |
F  | -0.8| -0.1| -1    | 1/8    |
G  | 0.4 | -0.9| -1    | 1/8    |
H  | 0.2 | 0.5 | -1    | 1/8    |
The E1 “Stump”

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)  
face = true

sign = +1 or -1

Sign = +1, error = 2/8
Sign = -1, error = 6/8

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</tbody>
</table>
The E1 “Stump”

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
   face = true

sign = +1 or -1

Sign = +1, error = 1/8
Sign = -1, error = 7/8

<table>
<thead>
<tr>
<th>ID</th>
<th>E1</th>
<th>E2</th>
<th>Class</th>
<th>Weight</th>
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<tbody>
<tr>
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<td>0.5</td>
<td>-1</td>
<td>1/8</td>
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</tbody>
</table>
The E1 “Stump”

Classifier based on E1:
if ( sign\*wt(E1) > thresh) > 0) 
face = true

sign = +1 or -1

Sign = +1, error = 2/8
Sign = -1, error = 6/8

<table>
<thead>
<tr>
<th>ID</th>
<th>E1</th>
<th>E2</th>
<th>Class</th>
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<tbody>
<tr>
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</tbody>
</table>
The Best E1 “Stump”

Classifier based on E1:
if (sign*wt(E1) > thresh) > 0)
face = true

Sign = +1
Threshold = 0.45

<table>
<thead>
<tr>
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</tbody>
</table>
The E2 “Stump”

Classifier based on E2:
if ( sign*wt(E2) > thresh) > 0)  
  face = true

sign = +1 or -1

Note order

<table>
<thead>
<tr>
<th>ID</th>
<th>E1</th>
<th>E2.</th>
<th>Class</th>
<th>Weight</th>
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<tr>
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<td>0.2</td>
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<td>-1</td>
<td>1/8</td>
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</tbody>
</table>

Sign = +1, error = 3/8
Sign = -1, error = 5/8
The Best E2 “Stump”

Classifier based on E2:
if ( sign*wt(E2) > thresh) > 0)
   face = true

sign = -1
Threshold = 0.15

Sign = -1, error = 2/8

<table>
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<tr>
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The Best “Stump”

The Best overall classifier based on a single feature is based on E1

If \( wt(E1) > 0.45 \) \( \rightarrow \) Face

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The Best “Stump”
The ADABOost Algorithm

• Initialize $D_1(x_i) = 1/N$
• For $t = 1, ..., T$
  – Train a weak classifier $h_t$ using distribution $D_t$
  – Compute total error on training data
    • $\varepsilon_t = \text{Sum } \{D_t(x_i) \frac{1}{2}(1 - y_i h_t(x_i))\}$
  – Set $\alpha_t = \frac{1}{2} \ln \left( \frac{\varepsilon_t}{1 - \varepsilon_t} \right)$
  – For $i = 1...N$
    • set $D_{t+1}(x_i) = D_t(x_i) \exp(- \alpha_t y_i h_t(x_i))$
  – Normalize $D_{t+1}$ to make it a distribution
• The final classifier is
  – $H(x) = \text{sign}\left(\sum_t \alpha_t h_t(x)\right)$
The Best “Stump”
The Best Error

The Error of the classifier is the sum of the weights of the misclassified instances.

Sign = +1, error = 1/8

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NOTE: THE ERROR IS THE SUM OF THE WEIGHTS OF MISCLASSIFIED Instances
The ADABoost Algorithm

• Initialize \( D_1(x_i) = 1/N \)

• For \( t = 1, \ldots, T \)
  – Train a weak classifier \( h_t \) using distribution \( D_t \)
  – Compute total error on training data
    \[ \varepsilon_t = \text{Sum} \{ D_t(x_i) \frac{1}{2}(1 - y_i h_t(x_i)) \} \]
    
  – Set \( \alpha_t = \frac{1}{2} \ln \left( \frac{1 - \varepsilon_t}{\varepsilon_t} \right) \)
  – For \( i = 1 \ldots N \)
    • set \( D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t y_i h_t(x_i)) \)
  – Normalize \( D_{t+1} \) to make it a distribution

• The final classifier is
  – \( H(x) = \text{sign}(\sum_t \alpha_t h_t(x)) \)
Computing Alpha

\[
\alpha = 0.5 \ln \left( \frac{1 - 1/8}{1/8} \right)
\]

\[
= 0.5 \ln(7) = 0.97
\]

Sign = +1, error = 1/8
The Boosted Classifier Thus Far

\[
\begin{align*}
\text{F} & \quad \text{E} & \quad \text{H} & \quad \text{A} & \quad \text{G} & \quad \text{B} & \quad \text{C} & \quad \text{D} \\
-0.8 & \quad 0.2 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5 & \quad 0.6 & \quad 0.7 \\
1/8 & \quad 1/8 & \quad 1/8 & \quad 1/8 & \quad 1/8 & \quad 1/8 & \quad 1/8 & \quad 1/8
\end{align*}
\]

\[
\text{Alpha} = 0.5 \ln((1 - 1/8) / (1/8)) = 0.5 \ln(7) = 0.97
\]

Sign = +1, error = 1/8

\[
h_1(X) = \text{wt}(E_1) > 0.45 \ ? \ +1 : -1
\]

\[
H(X) = \text{sign}(0.97 \times h_1(X))
\]

It's the same as \(h_1(x)\)
The ADABoost Algorithm

• Initialize \( D_1(x_i) = 1/N \)
• For \( t = 1, ..., T \)
  – Train a weak classifier \( h_t \) using distribution \( D_t \)
  – Compute total error on training data
    • \( \varepsilon_t = \text{Average} \{ \frac{1}{2} (1 - y_i h_t(x_i)) \} \)
    – Set \( \alpha_t = \frac{1}{2} \ln \left( \frac{(1 - \varepsilon_t)}{\varepsilon_t} \right) \)
  – For \( i = 1...N \)
    • set \( D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t y_i h_t(x_i)) \)
    – Normalize \( D_{t+1} \) to make it a distribution
• The final classifier is
  – \( H(x) = \text{sign}(\sum_t \alpha_t h_t(x)) \)
The Best Error

\[ D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_i y_i h_t(x_i)) \]

\[ \exp(\alpha_i) = \exp(0.97) = 2.63 \]

\[ \exp(-\alpha_i) = \exp(-0.97) = 0.38 \]

Multiply the correctly classified instances by 0.38
Multiply incorrectly classified instances by 2.63

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<th>E2</th>
<th>Class</th>
<th>Weight</th>
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<tbody>
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<td>0.5</td>
<td>-0.5</td>
<td>+1</td>
<td>1/8 * 0.38</td>
<td>0.05</td>
</tr>
<tr>
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AdaBoost
AdaBoost
The ADABoost Algorithm

1. Initialize $D_1(x_i) = 1/N$
2. For $t = 1, ..., T$
   - Train a weak classifier $h_t$ using distribution $D_t$
   - Compute total error on training data
     - $\varepsilon_t = \text{Average}\{\frac{1}{2} (1 - y_i h_t(x_i))\}$
   - Set $\alpha_t = \frac{1}{2} \ln ((1 - \varepsilon_t) / \varepsilon_t)$
   - For $i = 1... N$
     - set $D_{t+1}(x_i) = D_t(x_i) \exp(-\alpha_t y_i h_t(x_i))$
   - Normalize $D_{t+1}$ to make it a distribution
3. The final classifier is
   - $H(x) = \text{sign}(\sum_t \alpha_t h_t(x))$
# The Best Error

\[
D' = \frac{D}{\text{sum}(D)}
\]

<table>
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<tr>
<th>ID</th>
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<tbody>
<tr>
<td>A</td>
<td>0.3</td>
<td>-0.6</td>
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<td>1/8 \times 2.63</td>
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Multiply the correctly classified instances by 0.38  
Multiply incorrectly classified instances by 2.63  
Normalize to sum to 1.0
The Best Error

\[ D' = \frac{D}{\text{sum}(D)} \]

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• The final classifier is
  – $H(x) = \text{sign}(\sum_t \alpha_t h_t(x))$
E1 classifier

Classifier based on E1:
if (sign*wt(E1) > thresh) > 0) face = true

sign = +1 or -1

Sign = +1, error = 0.222
Sign = -1, error = 0.778

<table>
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</table>
E1 classifier

Classifier based on E1:
if (sign*wt(E1) > thresh) > 0)
    face = true

sign = +1 or -1

Sign = +1, error = 0.148
Sign = -1, error = 0.852

<table>
<thead>
<tr>
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</tr>
</tbody>
</table>
The Best E1 classifier

Classifier based on E1:
if ( sign*wt(E1) > thresh) > 0)
   face = true

sign = +1 or -1

Sign = +1, error = 0.074

<table>
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</tbody>
</table>
The Best E2 classifier

Classifier based on E2:
if ( sign\*wt(E2) > thresh) > 0)
face = true

sign = +1 or -1

Sign = -1, error = 0.148

<table>
<thead>
<tr>
<th>ID</th>
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<td>0.074</td>
</tr>
</tbody>
</table>
The Best Classifier

Classifier based on E1:
if \((\text{wt}(E1) > 0.45)\) face = true

Alpha = \(0.5\ln((1-0.074) / 0.074)\)
= 1.26

<table>
<thead>
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Sign = +1, error = 0.074
### The Boosted Classifier Thus Far

<table>
<thead>
<tr>
<th>F</th>
<th>E</th>
<th>H</th>
<th>A</th>
<th>G</th>
<th>B</th>
<th>C</th>
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</tr>
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<tbody>
<tr>
<td>-0.8</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\[ H(X) = \text{sign}(0.97 \times h_1(X) + 1.26 \times h_2(X)) \]

\[ h_1(X) = \text{wt}(E1) > 0.45 \ ? \ +1 : -1 \]

\[ h_2(X) = \text{wt}(E1) > 0.25 \ ? \ +1 : -1 \]
Reweighting the Data

<table>
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<tr>
<td>A</td>
<td>0.3</td>
<td>-0.6</td>
<td>+1</td>
<td>0.48*0.28</td>
</tr>
<tr>
<td>B</td>
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<td>-0.5</td>
<td>+1</td>
<td>0.074*0.28</td>
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<td>-0.9</td>
<td>-1</td>
<td>0.074*3.5</td>
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<td>H</td>
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<td>-1</td>
<td>0.074*0.28</td>
</tr>
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</table>

Sign = +1, error = 0.074

Exp(alpha) = exp(1.26) = 3.5
Exp(-alpha) = exp(-1.26) = 0.28

RENORMALIZE
Reweighting the Data

NOTE: THE WEIGHT OF “G” WHICH WAS MISCLASSIFIED BY THE SECOND CLASSIFIER IS NOW SUDDENLY HIGH

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AdaBoost

• In this example both of our first two classifiers were based on E1
  – Additional classifiers may switch to E2
• In general, the reweighting of the data will result in a different feature being picked for each classifier

• This also automatically gives us a feature selection strategy
  – In this data the wt(E1) is the most important feature
AdaBoost

- NOT required to go with the best classifier so far
- For instance, for our second classifier, we might use the best E2 classifier, even though it's worse than the E1 classifier
  - So long as it's right more than 50% of the time

- We can *continue* to add classifiers even after we get 100% classification of the training data
  - Because the weights of the data keep changing
  - Adding new classifiers beyond this point is often a good thing to do
The final classifier is

\[ H(x) = \text{sign}(\sum_t \alpha_t h_t(x)) \]

The output is 1 if the total weight of all weak learners that classify \(x\) as 1 is greater than the total weight of all weak learners that classify it as -1.
Boosting and Face Detection

- Boosting is the basis of one of the most popular methods for face detection: The Viola-Jones algorithm
  - Current methods use other classifiers like SVMs, but adaboost classifiers remain easy to implement and popular
  - OpenCV implements Viola Jones..
The problem of face detection

• 1. Defining Features
  – Should we be searching for noses, eyes, eyebrows etc.?  
    • Nice, but expensive 
  – Or something simpler

• 2. Selecting Features
  – Of all the possible features we can think of, which ones make sense

• 3. Classification: Combining evidence
  – How does one combine the evidence from the different features?
Features: The Viola Jones Method

- Integral Features!!
  - Like the Checkerboard
- The same principle as we used to decompose images in terms of checkerboards:
  - The image of any object has changes at various scales
  - These can be represented coarsely by a checkerboard pattern
- The checkerboard patterns must however now be localized
  - Stay within the region of the face

\[ \text{Image} \approx w_1 B_1 + w_2 B_2 + w_3 B_3 + \ldots \]
Features

• Checkerboard Patterns to represent facial features
  – The white areas are subtracted from the black ones.
  – Each checkerboard explains a *localized* portion of the image
• Four types of checkerboard patterns (only)

A

B

C

D

11755/18979
Explaining a portion of the face with a checker..

- How much is the difference in average intensity of the image in the black and white regions
  - \( \text{Sum(white region)} - \text{Sum(black region)} \)

- This is actually the dot product of the region of the face covered by the rectangle and the checkered pattern itself
  - White = 1, Black = -1
“Integral” features

- Each checkerboard has the following characteristics
  - Length
  - Width
  - Type
    - Specifies the number and arrangement of bands

- The four checkerboards above are the four used by Viola and Jones
Integral images

- Summed area tables

For each pixel store the sum of ALL pixels to the left of and above it.
Fast Computation of Pixel Sums

To compute the sum of the pixels within “D”:
- \( \text{Pixelsum}(1) = \text{Area}(A) \)
- \( \text{Pixelsum}(2) = \text{Area}(A) + \text{Area}(B) \)
- \( \text{Pixelsum}(3) = \text{Area}(A) + \text{Area}(C) \)
- \( \text{Pixelsum}(4) = \text{Area}(A) + \text{Area}(B) + \text{Area}(C) + \text{Area}(D) \)

Area(D) = Pixelsum(4) – Pixelsum(2) – Pixelsum(3) + Pixelsum(1)
A Fast Way to Compute the Feature

- Store pixel table for every pixel in the image
  - The sum of all pixel values to the left of and above the pixel
- Let A, B, C, D, E, F be the pixel table values at the locations shown
  - Total pixel value of black area = D + A − B − C
  - Total pixel value of white area = F + C − D − E
  - Feature value = (F + C − D − E) − (D + A − B − C)
How many features?

- Each checker board of width P and height H can start at any of \((N-P)(M-H)\) pixels

- \((M-H)*(N-P)\) possible starting locations
  - Each is a unique checker feature
    - E.g. at one location it may measure the forehead, at another the chin
How many features

• Each feature can have many sizes
  – Width from (min) to (max) pixels
  – Height from (min ht) to (max ht) pixels

• At each size, there can be many starting locations
  – Total number of possible checkerboards of one type:
    No. of possible sizes x No. of possible locations

• There are four types of checkerboards
  – Total no. of possible checkerboards: VERY VERY LARGE!
Learning: No. of features

- Analysis performed on images of 24x24 pixels only
  - Reduces the no. of possible features to about 180000

- Restrict checkerboard size
  - Minimum of 8 pixels wide
  - Minimum of 8 pixels high
    - Other limits, e.g. 4 pixels may be used too
  - Reduces no. of checkerboards to about 50000
No. of features

- Each possible checkerboard gives us one feature
- A total of up to 180000 features derived from a 24x24 image!
- Every 24x24 image is now represented by a set of 180000 numbers
  - This is the set of features we will use for classifying if it is a face or not!

<table>
<thead>
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<th></th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>.....</th>
<th>F180000</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>7</td>
<td>9</td>
<td>2</td>
<td>-1</td>
<td>.....</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>-11</td>
<td>3</td>
<td>19</td>
<td>17</td>
<td>.....</td>
<td>2</td>
</tr>
</tbody>
</table>
The Classifier

• The Viola-Jones algorithm uses AdaBoost with “stumps”

• At each stage find the best feature to classify the data with
  – I.e the feature that gives us the best classification of all the training data
    • Training data includes many examples of faces and non-face images
  – The classification rule is of the kind
    • If feature > threshold, face (or if feature < threshold, face)
    • The optimal value of “threshold” must also be determined.
To Train

• Collect a large number of facial images
  – Resize all of them to 24x24
  – These are our “face” training set

• Collect a much much much much larger set of 24x24 non-face images of all kinds
  – Each of them is
  – These are our “non-face” training set

• Train a boosted classifier
The Viola Jones Classifier

• During tests:
  – Given any new 24x24 image
    • \( R = \sum_f \alpha_f (f > p_f \theta(f)) \)
    • Only a small number of features (f < 100) typically used

• Problems:
  – Only classifies 24 x 24 images entirely as faces or non-faces
    • Pictures are typically much larger
    • They may contain many faces
    • Faces in pictures can be much larger or smaller
  – Not accurate enough
Multiple faces in the picture

- Scan the image
  - Classify each 24x24 rectangle from the photo
  - All rectangles that get classified as having a face indicate the location of a face
- For an NxM picture, we will perform \((N-24)\)*(M-24) classifications
- If overlapping 24x24 rectangles are found to have faces, merge them
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Picture size solution

• We already have a classifier
  – That uses weak learners

• *Scale the Picture*
  – Scale the picture down by a factor $a$
  – Keep decrementing down to a minimum reasonable size
False Rejection vs. False Detection

- False Rejection: There’s a face in the image, but the classifier misses it
  - Rejects the hypothesis that there’s a face
- False detection: Recognizes a face when there is none.

- Classifier:
  - Standard boosted classifier: \( H(x) = \text{sign}(\sum_t \alpha_t h_t(x)) \)
  - Modified classifier \( H(x) = \text{sign}(\sum_t \alpha_t h_t(x) + Y) \)
    - \( \sum_t \alpha_t h_t(x) \) is a measure of certainty
      - The higher it is, the more certain we are that we found a face
    - If \( Y \) is large, then we assume the presence of a face even when we are not sure
      - By increasing \( Y \), we can reduce false rejection, while increasing false detection
• Ideally false rejection will be 0%, false detection will also be 0%

• As Y increases, we reject faces less and less
  – But accept increasing amounts of garbage as faces

• Can set Y so that we rarely miss a face
Problem: Not accurate enough, too slow

• If we set Y high enough, we will never miss a face
  – But will classify a lot of junk as faces
• Solution: Classify the output of the first classifier with a second classifier
  – And so on.
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Useful Features Learned by Boosting
A Cascade of Classifiers

All Sub-windows

Reject Sub-window

Further Processing
Detection in Real Images

• Basic classifier operates on 24 x 24 subwindows

• Scaling:
  – Scale the detector (rather than the images)
  – Features can easily be evaluated at any scale
  – Scale by factors of 1.25

• Location:
  – Move detector around the image (e.g., 1 pixel increments)

• Final Detections
  – A real face may result in multiple nearby detections
  – Postprocess detected subwindows to combine overlapping detections into a single detection
Training

- In paper, 24x24 images of faces and non faces (positive and negative examples).
Sample results using the Viola-Jones Detector

• Notice detection at multiple scales
More Detection Examples
Practical implementation

• Details discussed in Viola-Jones paper

• Training time = weeks (with 5k faces and 9.5k non-faces)

• Final detector has 38 layers in the cascade, 6060 features

• 700 Mhz processor:
  – Can process a 384 x 288 image in 0.067 seconds (in 2003 when paper was written)
Best Window/Background Issues
Best Window/Background Issues
Best Window/Background Issues

- Patch Height
- Person Height

Images with different window sizes:
- 95/100
- 100/100
- 110/100
- 120/100
- 140/100
Key Ideas

• EigenFace feature
• Sliding windows & scale-space pyramid
• Boosting an ensemble of weak classifiers
• Integral Image / Haar Features
• Cascaded Strong Classifiers