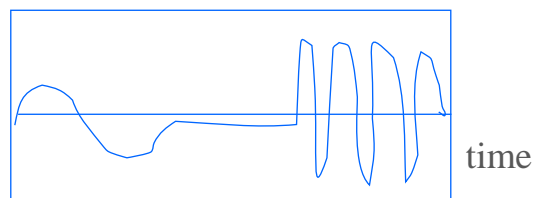


# Wavelets - DWT

- DFT is great - but, how about compressing opera? (baritone, silence, soprano?)
  - (General case: both smooth and /or locally bumpy functions)

value



Multi DB and D.M.

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# Wavelets - DWT

- Solution#1: Short window Fourier transform
- But: how short should be the window?

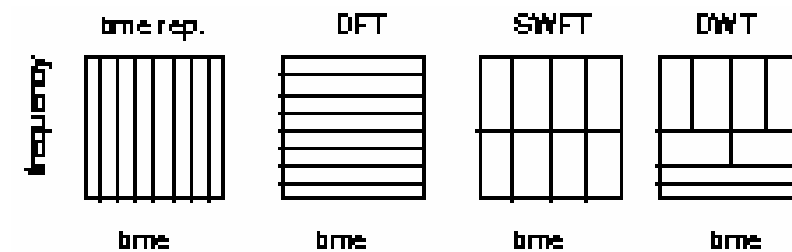
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## Wavelets - DWT

- Answer: **multiple** window sizes! -> DWT



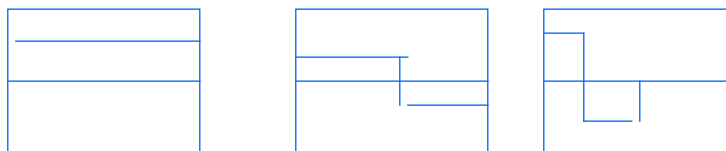
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## Haar Wavelets

- subtract sum of left half from right half
- repeat recursively for quarters, eighths ...



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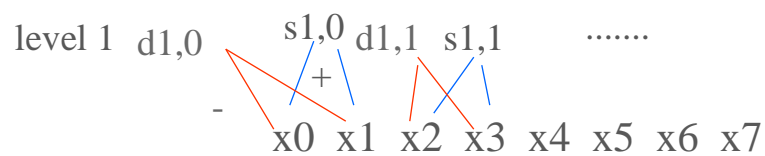
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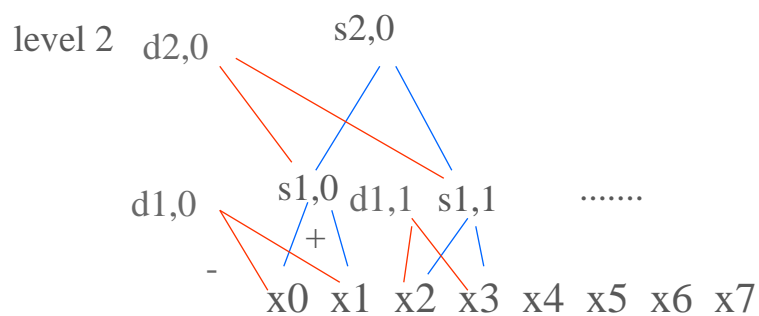
# Wavelets - construction

$x_0 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7$

# Wavelets - construction



## Wavelets - construction

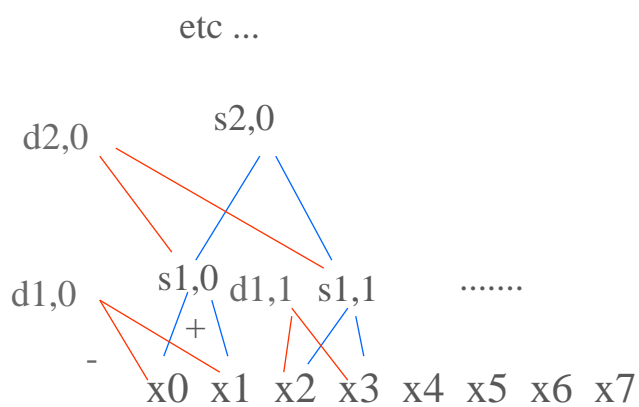


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## Wavelets - construction



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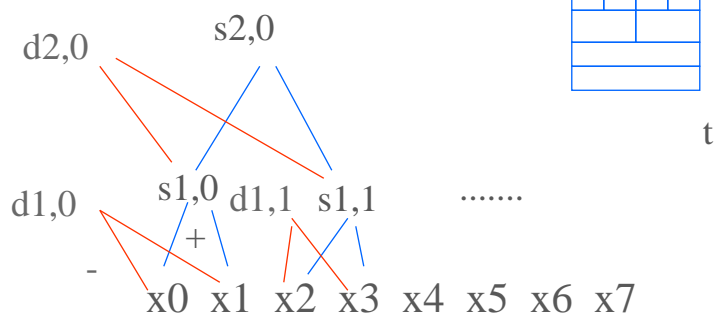
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## Wavelets - construction

Q: map each coefficient

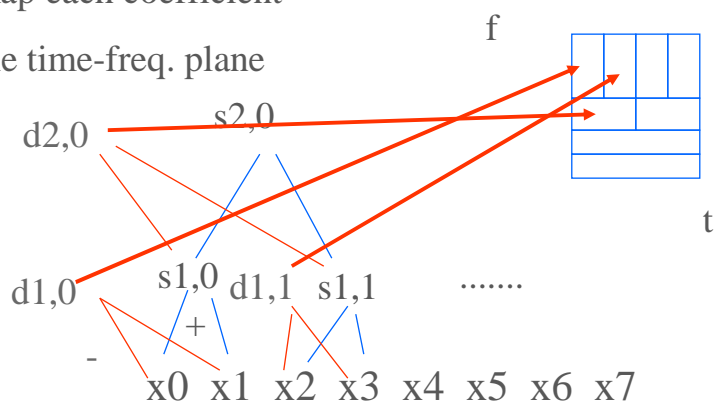
on the time-freq. plane



## Wavelets - construction

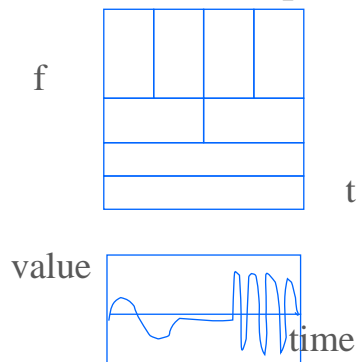
Q: map each coefficient

on the time-freq. plane



## Wavelets - Drill:

- Q: baritone/silence/soprano - DWT?



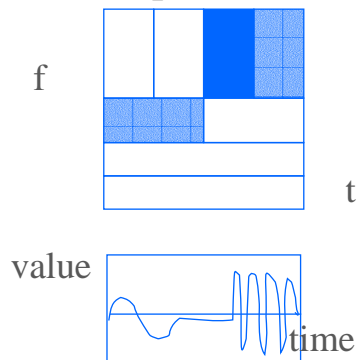
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## Wavelets - Drill:

- Q: baritone/soprano - DWT?



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## Wavelets - construction

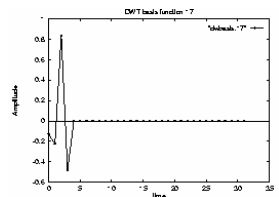
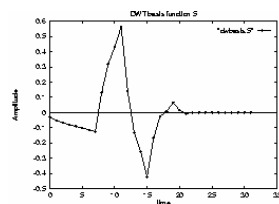
### Observation1:

- ‘+’ can be some weighted addition
- ‘-’ is the corresponding weighted difference  
(‘Quadrature mirror filters’)

### Observation2: unlike DFT/DCT,

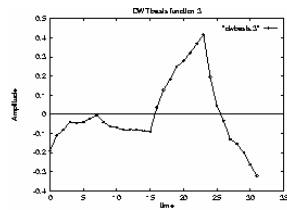
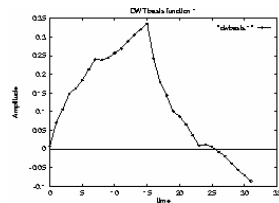
- there are *\*many\** wavelet bases: Haar, Daubechies-4, Daubechies-6, ...

## Wavelets - how do they look like?



- E.g., Daubechies-4

## Wavelets - how do they look like?



- Daubechies-4
- Weighted sum and difference, plus ...

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## Advantages of Wavelets

- Better compression (better RMSE with same number of coefficients)
- closely related to the processing of the mammalian eye and ear
- Good for progressive transmission
- handle spikes well
- usually, fast to compute ( $O(n)!$ )

Multi DB and D.M.

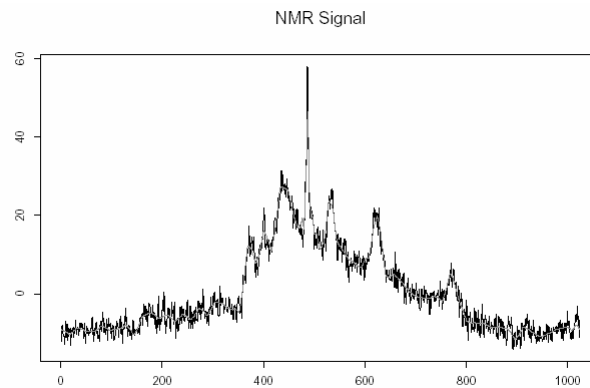
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# NMR Signal

- NMR



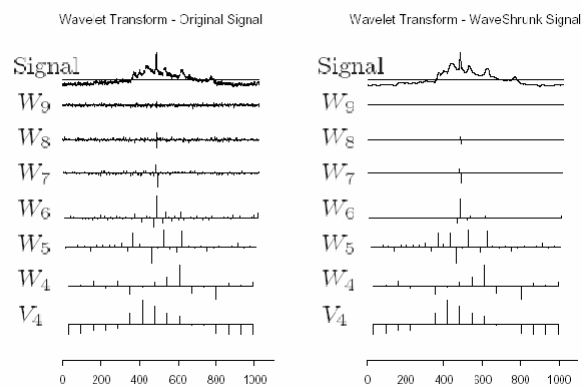
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# Wavelet Smoothing

- N



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