2. Smoothing Splines (roadmap)

locally weighted regression

(2.4) df

Smoothing Spline (2.1) ← (2.3) → Projection spline (basis expansion) (2.2)

Wavelet Smoothing

(2.5) (2.5) (5)

Ridge Regression

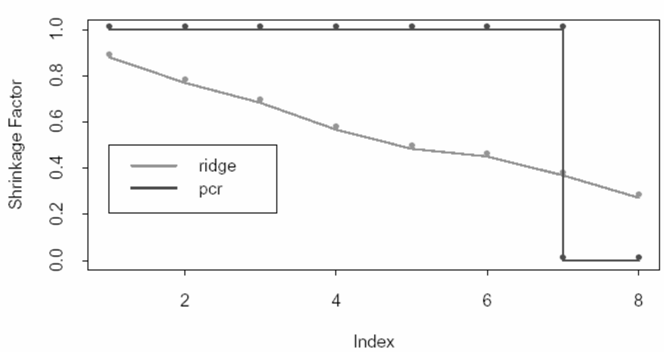
Principal Component Regression

The Lasso

global fitting

2.5 Smoothing spline – Ridge Regression, Projection spline – PCR

- PCR and Ridge regression (Figure 3.10, P67)



2.5 Smoothing spline – Ridge Regression, Projection spline – PCR

- Smoothing splines
  - f''(x): $\lambda\int\{f''(t)\}^2 dt$
  - Solution

    $\hat{\theta}=(N^T N+\lambda\Omega_N)^{-1}N^T y$
  - Then

    $N\hat{\theta}=\sum_{k=1}^{N}\frac{1}{1+\lambda d_k}u_k u_k^T y$
  - d_k is eigenvalue of K
  - df

$df(\lambda)=tr[N(N^T N+\lambda\Omega_N)^{-1}N^T]$

- Ridge Regression
  - Parameter $\lambda\sum_{j=1}^{p}\beta_j^2$
  - Solution:

    $\hat{\beta}=(X^T X+\lambda I)^{-1}X^T y$
  - Then

    $X\hat{\beta}=\sum_{j=1}^{p}\frac{d_j^2}{d_j^2+\lambda}u_j u_j^T y$
  - d_j is eigenvalue of X
  - df

$df(\lambda)=tr[X(X^T X+\lambda I)^{-1}X^T]$

Questions

- Fan Li
  - P127. Formular 5.9 why we want to f''(x) instead of f'(x)
    - f''(x) preserves the lines' curvature
  - Compare ridge regression and s_lamda
    - Both are smoothing the coefficiences
    - Ridge regression is a global fitting, Smoothing spline is local fitting method, much like the locally weighted regression procedure. (P132)
  - Formula 5.19: two N confused
    - One is the number of the basis functions
    - The other is the basis matrix: N matrix which is N*N

Questions

- Jie Lu
  - 5. P129, second bullet, what does "right-hand side exceeds the left-hand side by a positive semidefinite matrix" mean?
    - S_lamda is a positive semidefinite matrix

Questions

- Ashish
  - p127 - In what scenarios are smoothing splines more effective than the cubic spline technique given the cubic spline can use the "natural" number of knots.
    - Avoid the selection of knots?
    - Shrink more on the small eigenvector direction
- Kenvy:
  - p128 In chapter 3 we learned about ridge vs lasso shrinkage. In ch 5 we derive a nice general ridge regression solution when the constraint is a quadratic form (using the omega matrix of integrals). What does the "lasso" version of this look like for splines?
    - Wavelet splines

Questions

- Jian Zhang
  - Page 129, para 3: The author called the smoothing spline a "linear smoother" in the sense that estimated parameters are linear in y_i. I also remembered that for laso, they also mentioned similar ideas saying that "parameter estimation for aso isn't linear in y_i". Why does this bother?
    - ???
    - Ridge regression /Smoothing splines: proportional smoothing
    - Lasso / Wavelet SURE smoothing: soft thresholding
  - Which kind of good properties do we have if the estimation of parameters is a linear combination of a series of functions "y=f(x)"?
    - Computational tractable ???