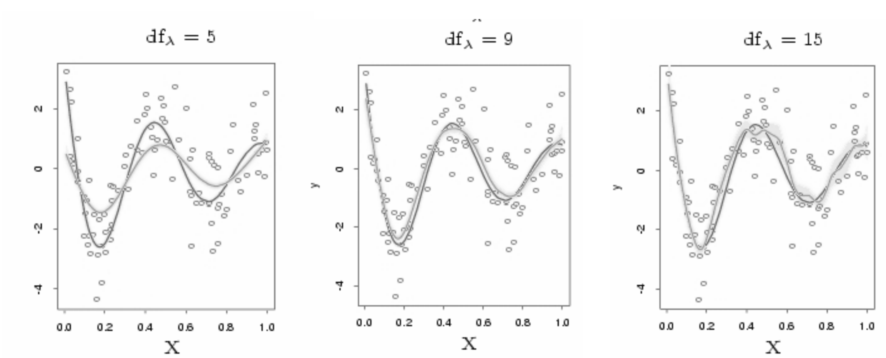
Parameters to Select

- Regression splines
  - spline degree, number and placement of knots
  - (greedy algorithm in Chapter 9)
- Smoothing splines
  - smoothing penalty
  - fix d.f. and invert this $\lambda \rightarrow S_\lambda \rightarrow trace(S_\lambda) = df_\lambda$

Effects of d.f. Choice



Sample Data Generation

- Each point (X,Y) in a batch of 100

1) $X \sim U(0,1)$
2) $f(X) = \frac{Sin(2(X+0.2))}{X+0.2}$
3) $\varepsilon \sim N(0,1)$
4) $Y = f(X) + \varepsilon$

- Standard error bands at a given point x

$$\hat{f}_\lambda(x) \pm 2se(\hat{f}_\lambda(x))$$

How to fix d.f

- If Y were known, by minimizing $EPE(\hat{f}_\lambda)$

$$EPE(\hat{f}_\lambda) = E(Y - \hat{f}_\lambda(X))^2 = Var(Y) + E[Bias^2(\hat{f}_\lambda(X)) + Var(\hat{f}_\lambda(X))]$$

$$MSE(\hat{f}_\lambda) = E[(\hat{f}_\lambda(x) - Y)^2] = Var(\hat{f}_\lambda) + (Bias(\hat{f}_\lambda))^2$$

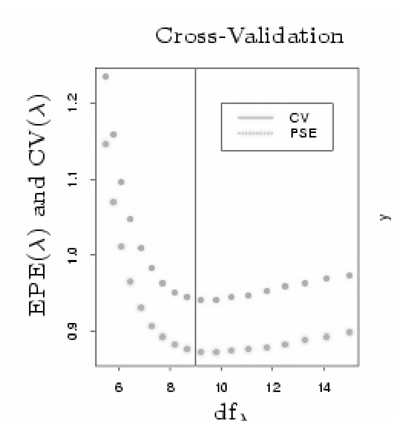
$$Bias(\hat{f}_\lambda) = E[\hat{f}_\lambda(X)] - Y$$

$$Var(\hat{f}_\lambda(X)) = E[(\hat{f}_\lambda(X) - E(\hat{f}_\lambda(X))^2]$$

Cross-Validation Estimate of EPE

- Divide data into N blocks of N-1 points (x_i, y_i)
- Compute squared error $(y_i - \hat{f}_\lambda^{(-i)}(x_i))^2$
  over all examples (x_i, y_i)
  and take its average

Cross-Validation Estimate of EPE



- Approximately unbiased!
- Washes whiter!