# Language Learnability

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#### Outline

- Gold's "Identification in the Limit" (1967)
  - Learnability models
  - Results and implications
  - Proof sketches
  - Learning time
- Empirical Studies: Bohannon & Stanowicz's Experiment on Adult Feedback, and Gordon's Attack

# Gold's "Identification in the Limit" (1967)

- Q: How to model natural languages in artificial systems?
  - Lower bound: rich enough to simulate the linguistic phenomena
  - Upper bound: a training procedure exists
- Q: What are the implications to cognitive systems brought by the artificial models?

- Toward a formal model of natural languages
  - Step 0: Power of different classes of formal languages (naïve models)
  - Step 1: Learnability/identifiability of the formal languages
  - Step 2: Complex models for real-life linguistic phenomena
  - Step 3: Learnability of the complex models

properly inclusive

Language	Grammar	Machine	Chomsky Hierarchy
Non-computable		??	
Recursively enumerable (RE)	Unrestricted	Turing machines	0
Recursive		Turing machines that always halt	
Context-sensitive	Context-sensitive grammar	Linear-bounded automata	1
Context-free	Context-free grammar	Non-deterministic pushdown automata	2
Regular	Regular expressions	Finite state automata	3

- Q: Is human mind a Turing machine?
  - If yes
    - what machines can't learn, human can't learn either, and vice versa
    - Natural languages are recursive (unless humans have time-out or probabilistic reasoning capability?)
  - If no: all bets are off

- Q: Why study the inductive inference "in the limit"
  - "in the limit" = at time
  - It's unlikely to get answers to the question "given the information and a set of possible conclusions, at *specific time t* what are the *correct* conclusion?
  - The 'power' question: the behavior of a learner in the limit

- A learnability model consists of
  - A definition of learnability: what do you mean by saying that a language is learned?
  - A method of information presentation: how does an instructor teach the learner?
  - A naming relation which assigns names to languages: what is the result of the learning?

- Basic concepts
  - Alphabet A is a non-empty finite set of symbols;
     A\* is the (inifinite) set of all finite strings over
  - Language  $\square$  is a subset of  $A^*$ ; a language class  $\square$  is the set of languages of the same underlying machine (some language is non-computable because  $2^{A*}$  is uncountable but the set of all possible TMs is countable)
  - Time t is discrete (t=1, 2, ...)

- Basic concepts
  - A primitive recursive function is
    - a recursive function (but not vice versa, e.g., Ackermann's function)
    - composed by a finite number of applications of composition and primitive recursion over null(0), successor and projection functions.
    - a total function, i.e., defined on all natural numbers
  - A string can be encoded into a single integer i.e., we can always have a  $A*\square\square$  function

- Learnability models
  - -Definitions of learnability
  - -A method of information presentation
  - –A naming relation

• Learnability:

$$g_t = G(i_1, i_2, ..., i_t)$$
, where

- $-g_t$  is the guess of the name of unknown language at time t
- -G is the guessing/learning algorithm
- $-i_1, i_2, ..., i_t$  is the information sequence received up to time t, where  $i_j$  is an information taken from the set of all possible units I at time j

- Three learnability definitions

  - Finite identification  $h(i_1, i_2, ..., i_t)$  is a decision function returning 0/1;  $g = G(i_1, i_2, ..., i_t)$  iff  $h(i_1, i_2, ..., i_t)=1$
  - Fixed-time identification  $g = G(i_1, i_2, ..., i_{\square})$  where  $\square$  is a constant

- Learnability models
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- Method of information presentation
  - For a language  $\square$ , I ( $\square$ ) is its set of allowable information sequences (each one has infinite length)
  - For a language  $\square$ :  $i_1, i_2, ..., i_t$  is a prefix of some sequence in I ( $\square$ )

- Two information presentation methods
  - Text: each  $i_t$  is a string of [] and [] [] [] []  $i_t = []$ . In fact  $i_t$  is a function [] [] and three classes of texts based on the type of functions are
    - Arbitrary: arbitrary functions
    - Recursive: recursive functions
    - Primitive recursive: primitive recursive functions This class of texts are effectively enumerable

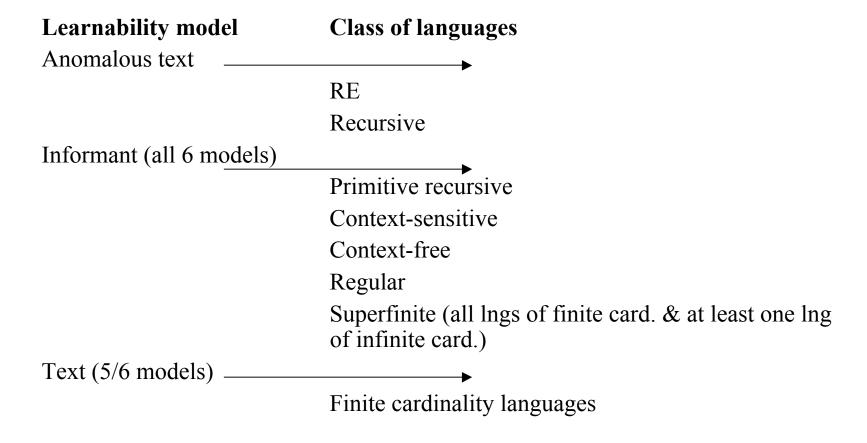
- Informant: each  $i_t$  is a string together with a binary signal indicating if the string is in  $\square$ . Again  $i_t$  is a function  $\square \square \square$  and three classes of informants based on the type of functions are
  - Arbitrary: arbitrary functions
  - Methodical:  $i_t$  is the *i*-th string in  $A^*$
  - Request:  $i_t$  is requested by the learner (or equivalently  $i_t$  is *defined* by the learner)

- Learnability models
  - -Definitions of learnability
  - -A method of information presentation
  - –A naming relation

- A naming relation consists of a set of names N and a function f:N
- So the problem of language identification is to find a procedure by which given  $\square\square\square$  and  $I(\square)$ , find the name n such that  $f(n)=\square$

- Two naming relations
  - Tester (comprehensive/analysis): a binary decision procedure (TM) 1(0) if the input string is (not) in  $\square$ .
  - Generator (productive/generation): a TM generating a string in 
     ☐ given an input integer.
  - A tester exists to test a recursive language, a generator exists to generate RE language.
  - It is possible to translate from testers to generators, but not vice versa [] if testers are identifiable than so are the corresponding generators, but not vice versa, e.g., anomalous text.

## Results and Implications



## Results and Implications

- With pure positive data the model can only learn language with finite cardinality!
- If human mind is a computation device we have a problem explaining why children can pick up their mother tongues without negative evidence.
- Information presentation provides negative information in this framework.

#### Results and Implications

- Possible ways out
  - Human languages is none of the classes we studied (possibly a subset of some class such as context-sensitive)
  - Babies receive negative evidence anyway we just don't know what that is (B&S's attempt)
  - Innateness claim: hardwired constraints may infer indirect negative evidence from the data

- Definitions
  - Effective vs. ineffective identification
    - There exists an algorithm for the former but the latter
    - The former implies the latter

- Collapsing uncertainty condition Let  $\Box_t$  be the set of languages agreeing the information received so far, i.e.,  $\Box_t = \{\Box_j: i_1, i_2, ..., i_t \text{ is a prefix of some sequence in } I (\Box_j) \}$ , then  $\Box_t = \{\Box\}$  ( $\Box$  is the correct language), or equivalently,  $\Box \Box' \neq \Box$ ,  $\Box' \Box \Box_t$ . Or intuitively, the size of  $\Box_t$  is keeps decreasing.

- Identification by enumeration
  An enumeration ☐ is an onto function ☐ ☐☐
  (this assumes ☐ is countable). At time t we find the first ☐ in ☐ that is in ☐ t.
  In the limit we return the name of the only element left in ☐ t, if collapsing uncertainty condition holds.
  - To make this *effective* we need
    - an effective procedure to test [(n)]
    - an effective procedure to find a name of  $\square(n)$

#### • Theorem

- Ineffective identifiability ☐ distinguishability
- Collapsing uncertainty [] identification by enumeration gives ineffective identification in the limit for any enumeration
- $\square \square I (\square)$  is countable plus distinguishability  $\square$  ineffective identification in the limit

- Informant method satisfies the collapsing uncertainty condition so all 6 models can learn almost all classes of languages
  - Q: why not recursive and RE? Are they countable? (there're countable sets which are not RE, but how about the opposite direction?)

- Text method satisfies the distinguishability condition, which alone doesn't guarantee anything
  - but if *I* (□) is countable then ineffective identification in the limit is guaranteed this is part of the reasons why anomalous text (generated by primitive recursive functions, using the generator naming relation) is identifiable.

- Why text is weak?
  - For a super-finite language we can always fool the learner by provide a successive larger finite subsets of the infinite language, so the learner makes mistakes for infinite number of times.
  - How to prevent repetitions? Probabilistic assumptions?

## Learning Time

• Seemingly counterintuitive result: identification-by-enumeration is the most efficient method for the identification in the limit, and none of them (each using different enumerations) performs uniformly better than the other!

## Learning Time

- Let [G, [], []] denotes the time step when the guessing algo. G correctly identifies [], given the information sequence []
- Prove  $[\![G, [\!], [\!] < [\![G_0, [\!], [\!]] ]\!] [\![G_0, [\!]', [\!]] < [\![G, [\!]', [\!]]).$

Empirical Studies: Bohannon & Stanowicz's Experiment on Adult Feedback, and Gordon's Attack

## Discrepancy

- Gold showed with pure positive data (text) only languages of finite cardinality can be identified.
- It's a common belief that parents do not give negative evidence, or, do not perform the informant role as defined by Gold.
- How to account for the discrepancy?
- Hidden assumptions: mind is a TM

## Discrepancy

- Solution 1: positing innate knowledge (Chomsky, Pinker, Wexler & Cullicover, etc.)
- Solution 2: there *is* negative evidence (B&S, etc.)
- The definition of "negative evidence" is somewhat relaxed in B&S, as it consists of various types of repetitions and questions)

## B&S on Negative Evidence

- Adults including parents & non-parents, male and female.
- Adult responses are categorized into three types of repetitions (exact, contracted, recasts and expansions) and two types of questions (repetitious/non-repetitious) followed by three types of children's language errors (semantic, syntactic & phonological).

## B&S on Negative Evidence

#### Claims

- The experiment showed adults did respond children differentially based on the linguistic errors they made.
- The responses give more information to language learners than the negative evidence defined in the strictest sense.
- The result undermines the belief of innate knowledge (Occam's Razor)

#### Gordon's Attack

- Negative evidence and innateness are two orthogonal issues: even with reliable negative evidence we might need innate knowledge to learn a "human" language (which is different from the formal languages)
- B&S's results showed substantial proportion of the ill-formed utterances were not responded, and substantial proportion of the well-formed utterances received feedback pertinent to the illformed ones. It is not clear without innate knowledge how children knew which to ignore.

#### Gordon's Attack

- There are strong evidence supporting innate knowledge: the children whose parental input was unstructured Pidgin languages still acquired structured Creole language. In this case the parents cannot provide meaningful feedback.
- (My criticism) In B&S the use of MacWhinney's claim that low frequency events do not necessarily imply they aren't important (1982) has contradicting implications: this implies the children actually have the innate ability to avoid being misled by low frequencies.