Session-Typed Concurrent Programming[1]

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POPL 2019 TutorialFest

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Content of tutorial

Message-passing programming and session types

- types for typing such programs

Linear logic and session types

- benefits and limitations

Manifest sharing

- controlled sharing for concurrency (non-determinism)

Manifest sharing and the pi-calculus

- expressiveness of untyped asynchronous pi-calculus recovered

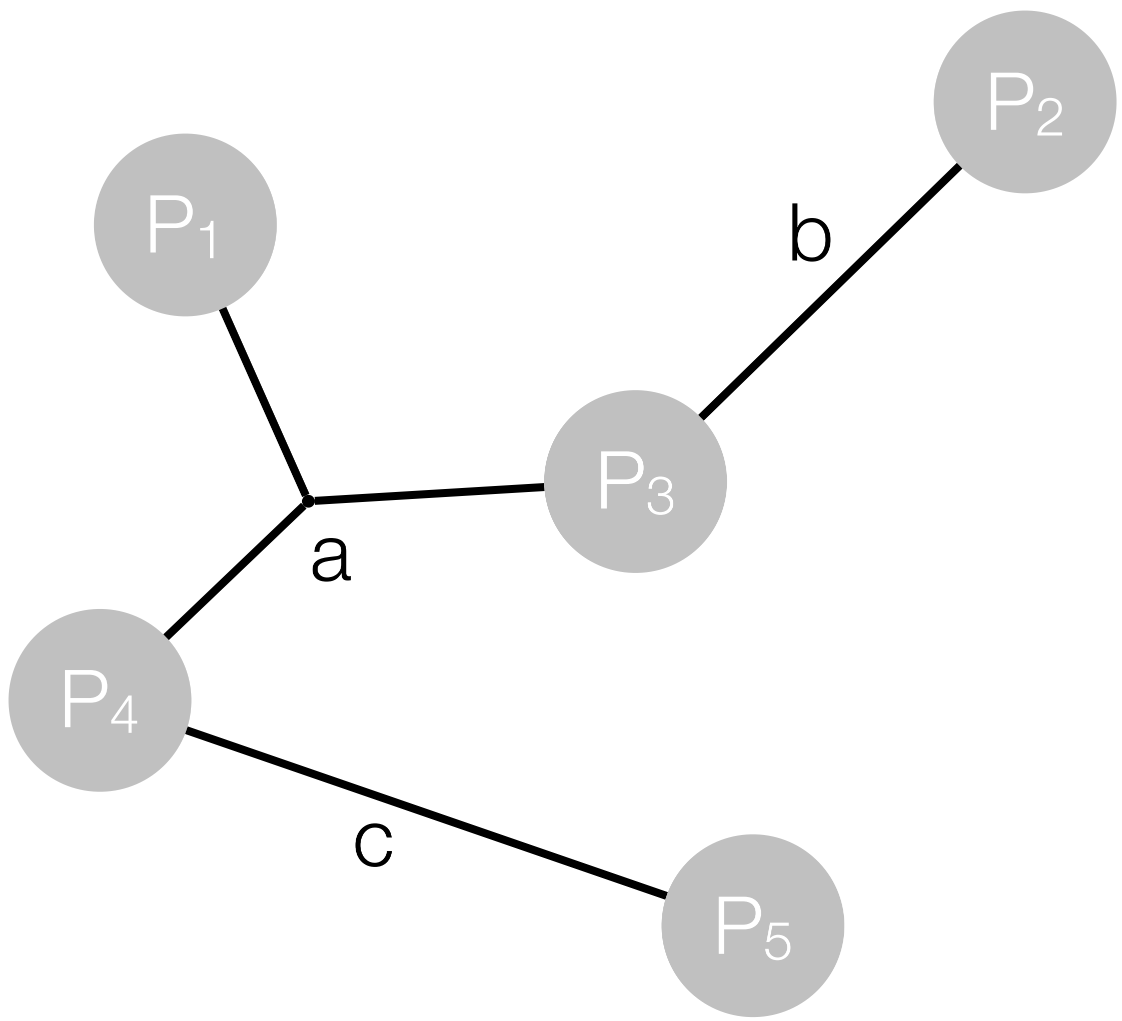
Current & future research

Learning objectives of tutorial

- How to program with session types
- What linearity is good for in programming
- How logic can guide programming language design
- Expressiveness of session-typed programming
- Hands-on experience with Concurrent C0

Session-typed message-passing programming

Message-passing concurrent programming



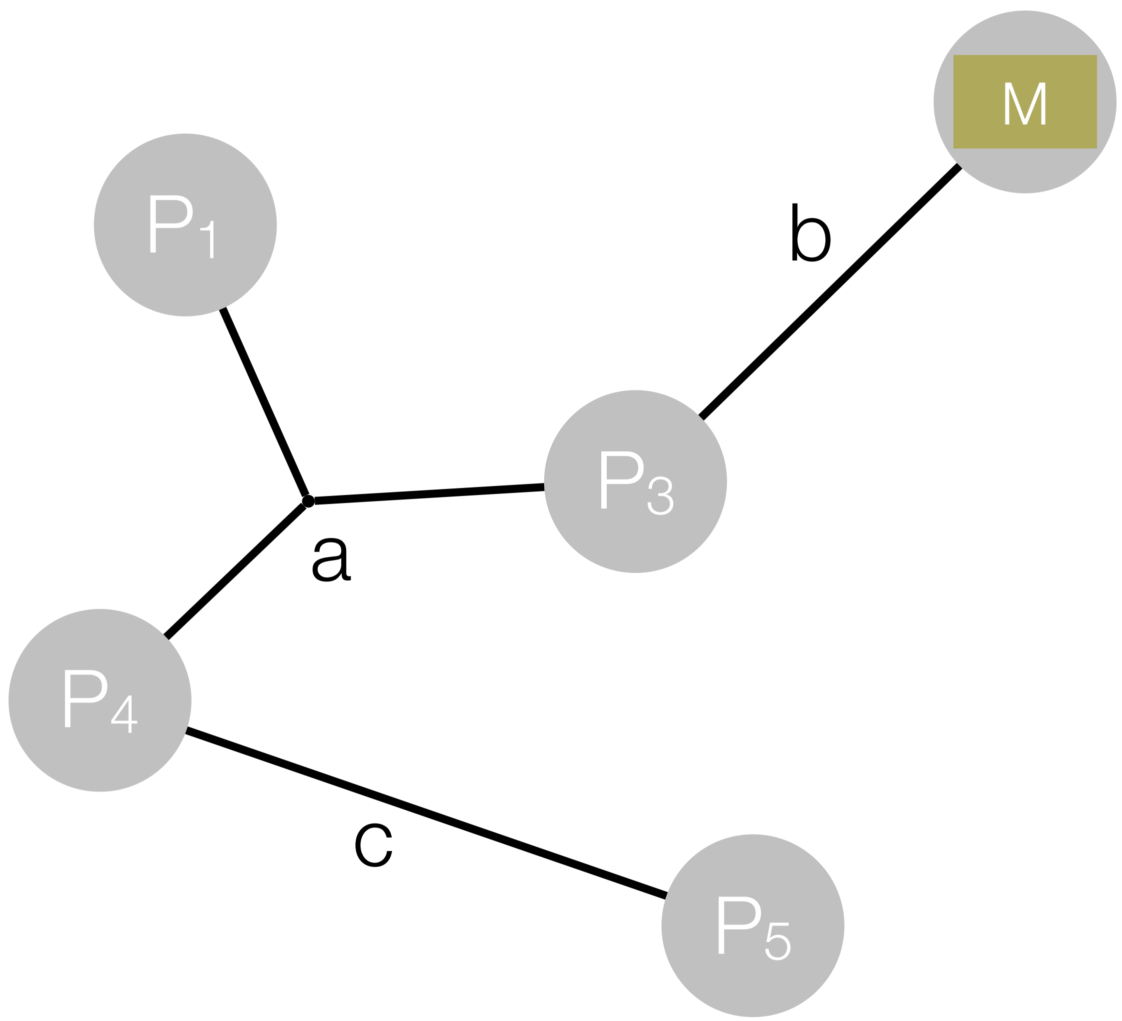
Program: network of processes connected by channels

Computation: by message exchange along channels

N-ary channels: e.g., a connects P_1, P_2, and P_3

Legend: process — channel — message

Message-passing concurrent programming



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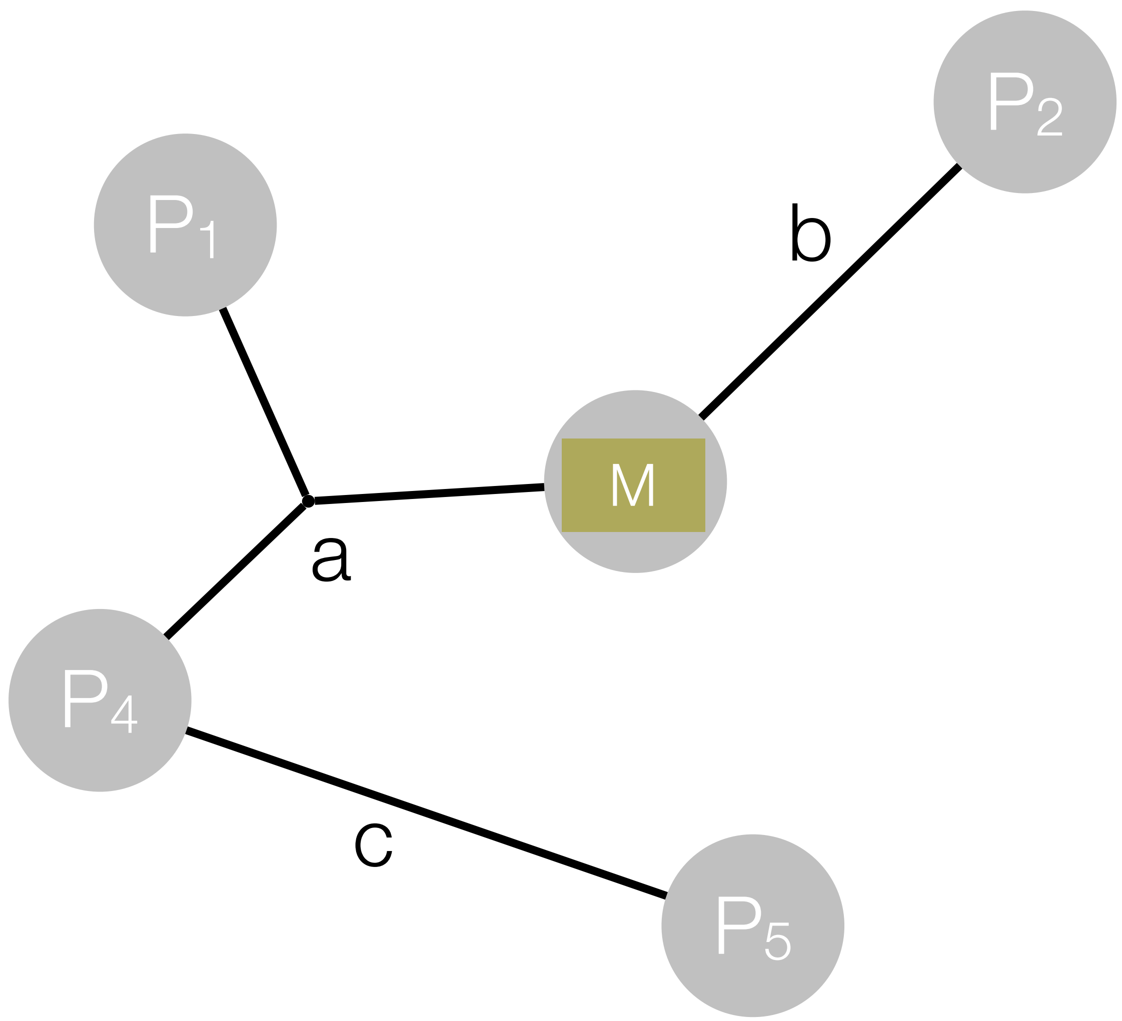

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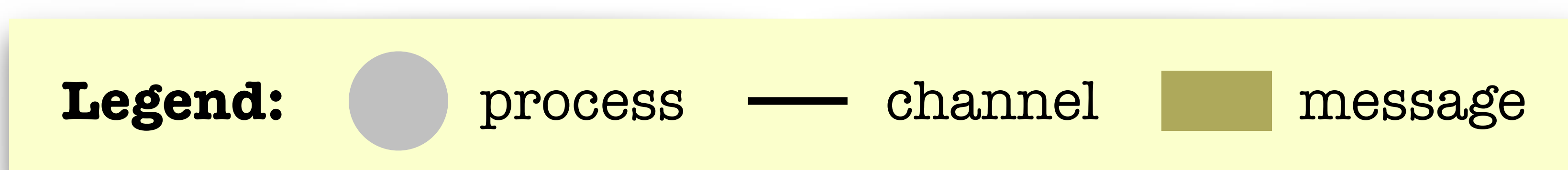

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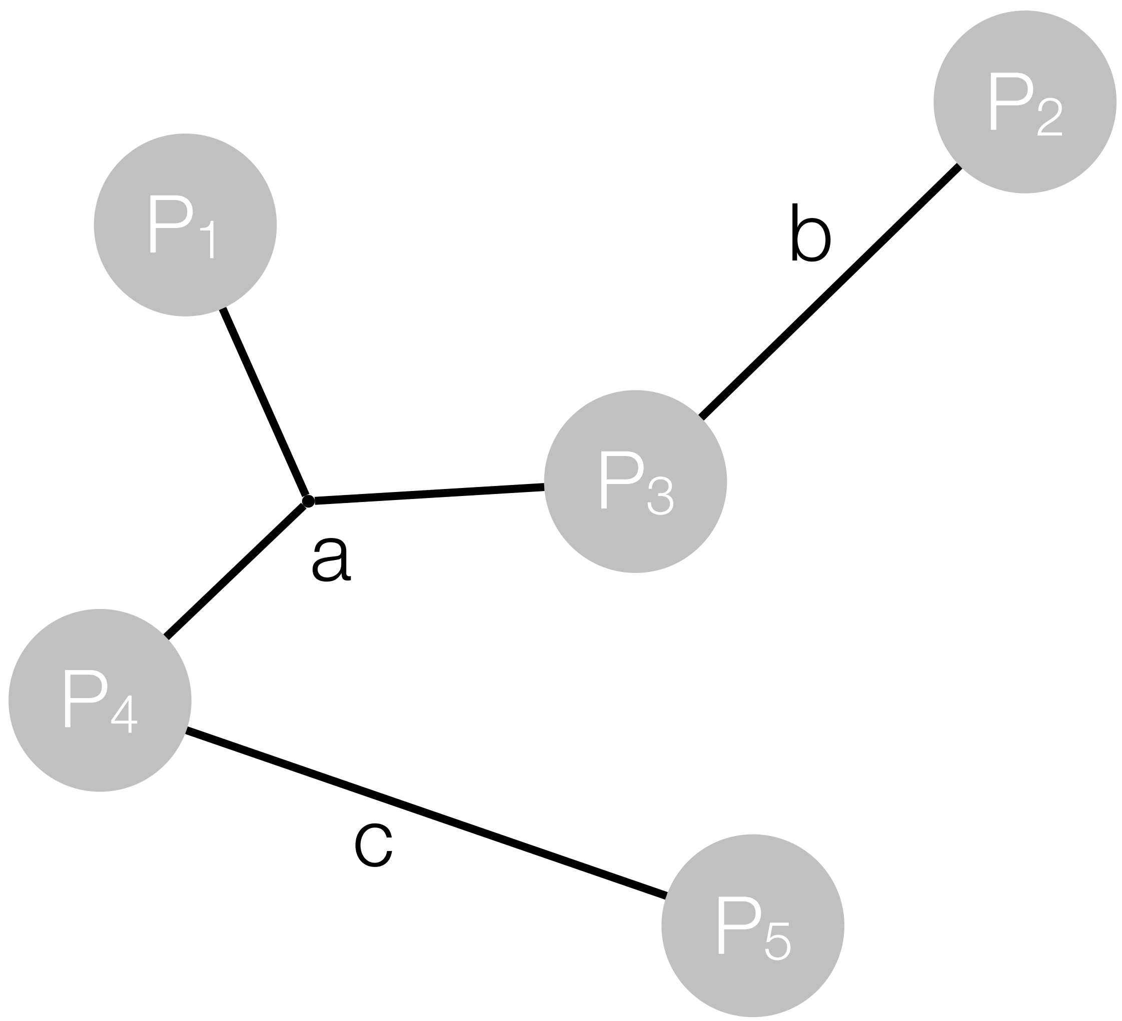

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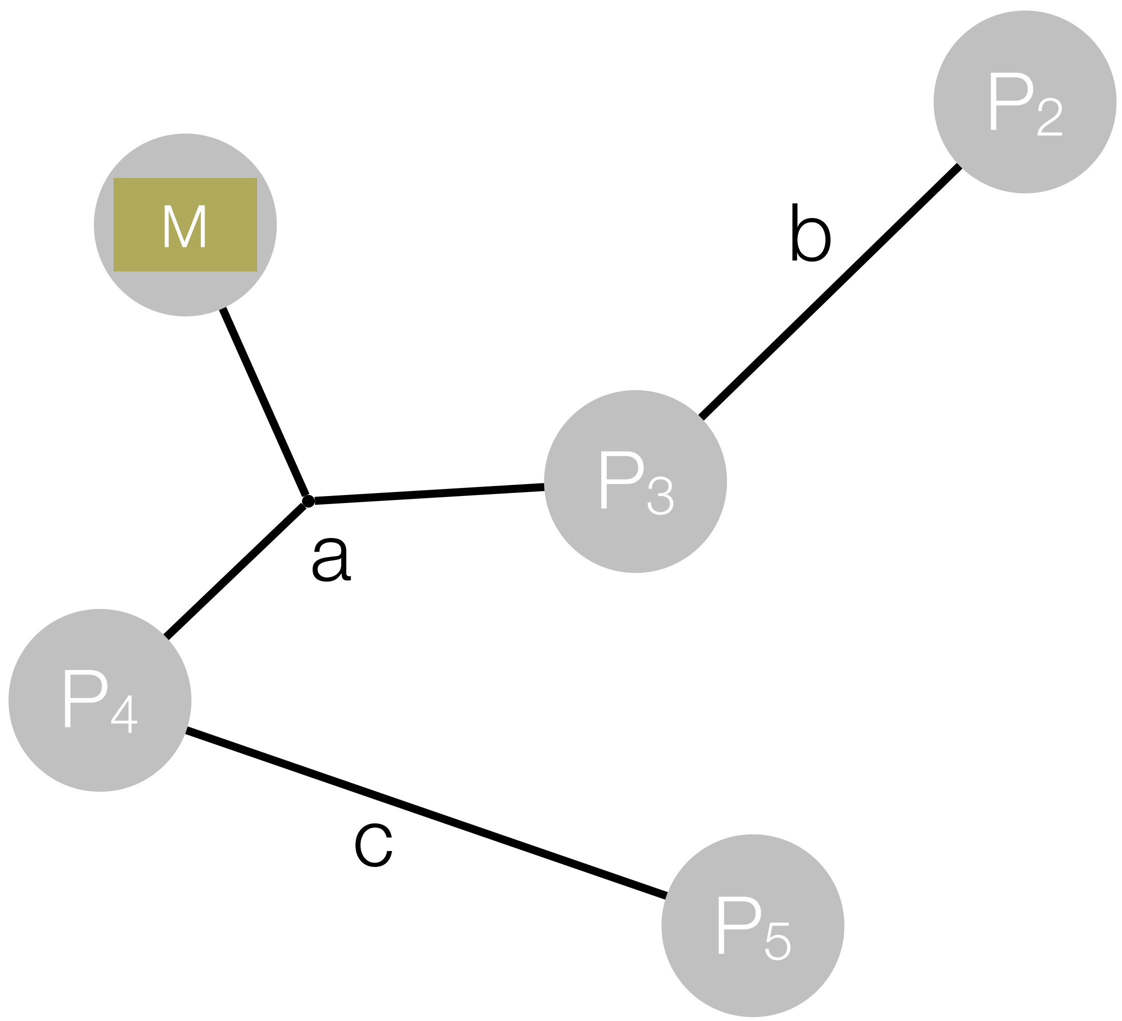

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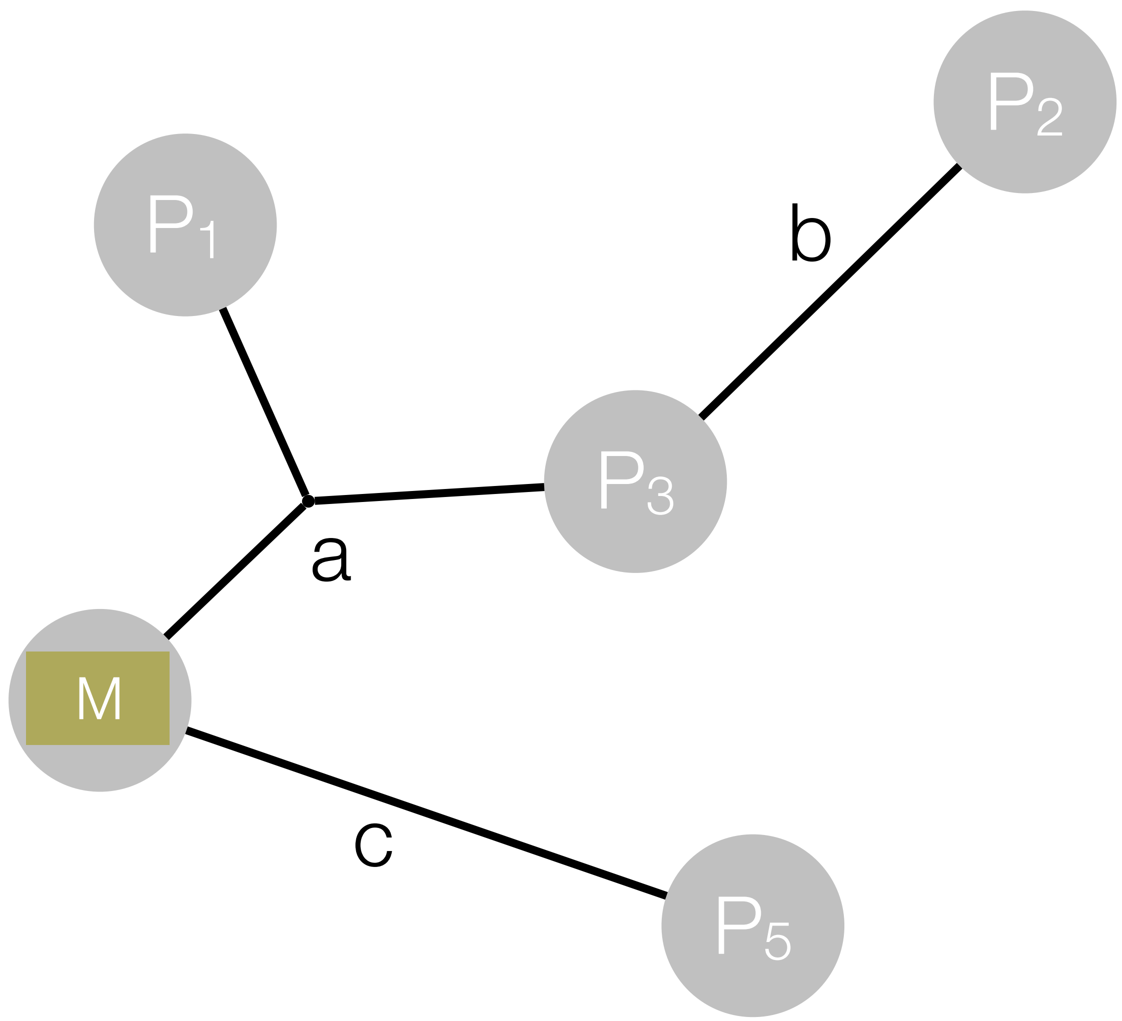

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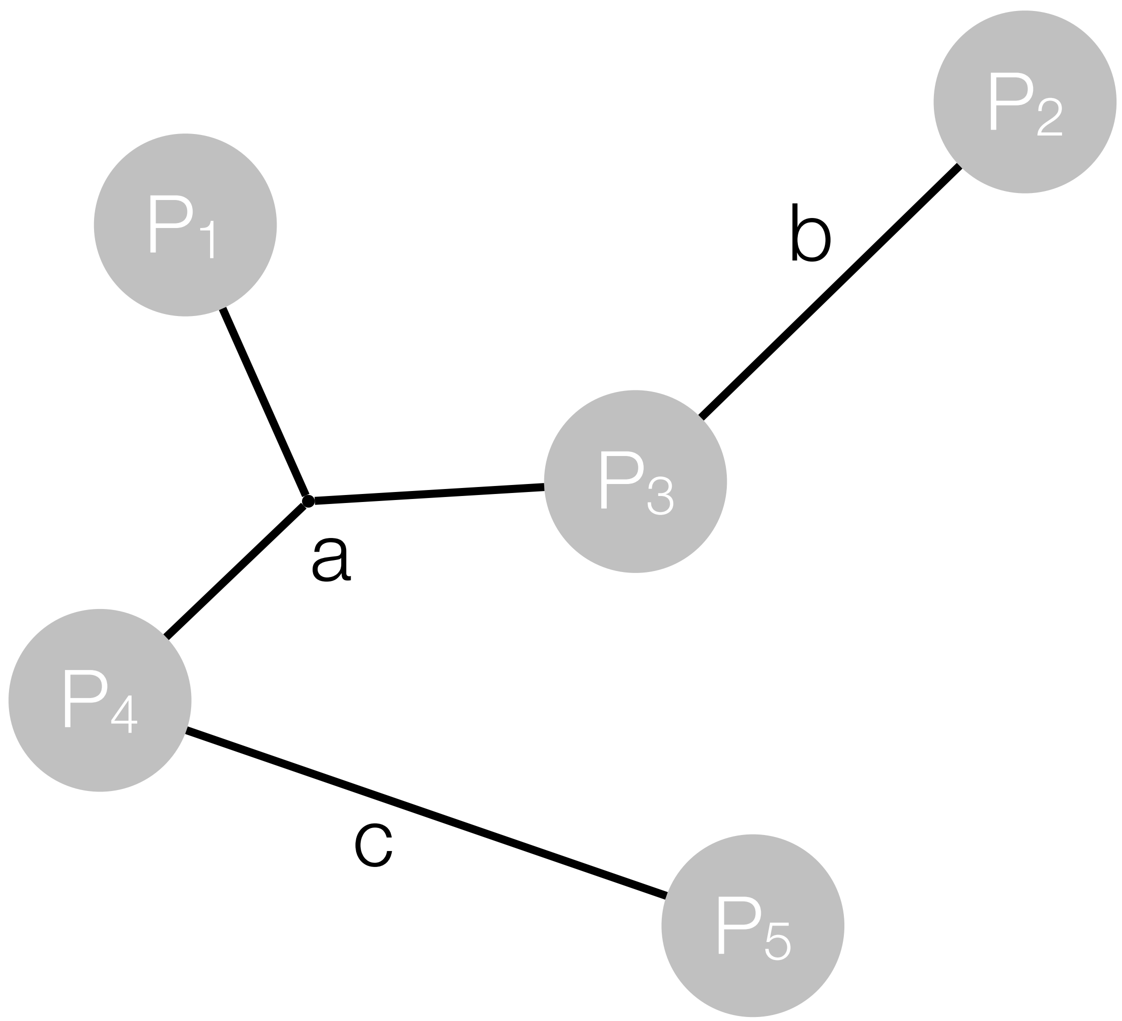
Legend: process channel message

Message-passing concurrent programming



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→ non-determinism

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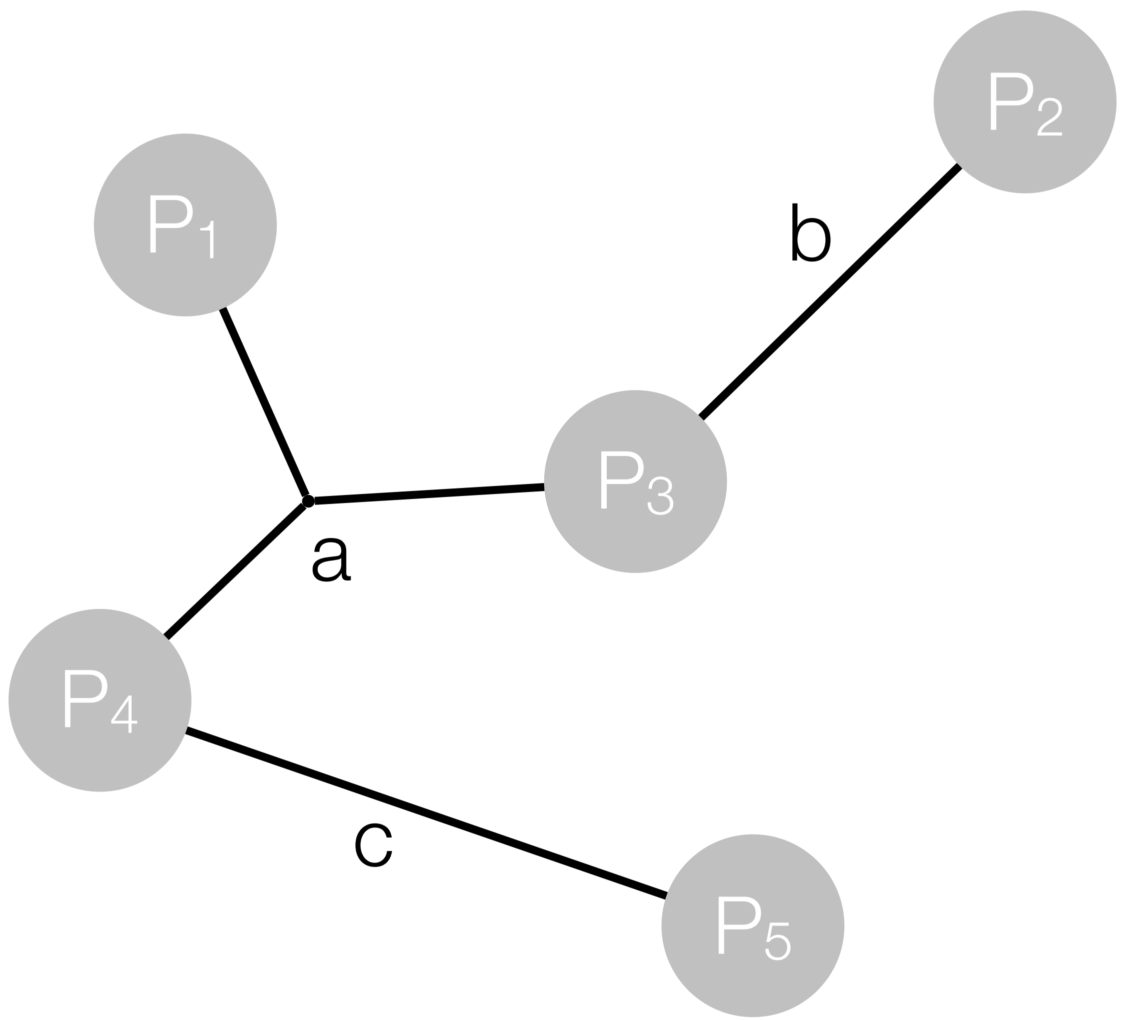

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Computation: by message exchange along channels

N-ary channels: e.g., a connects P_1, P_2, and P_3

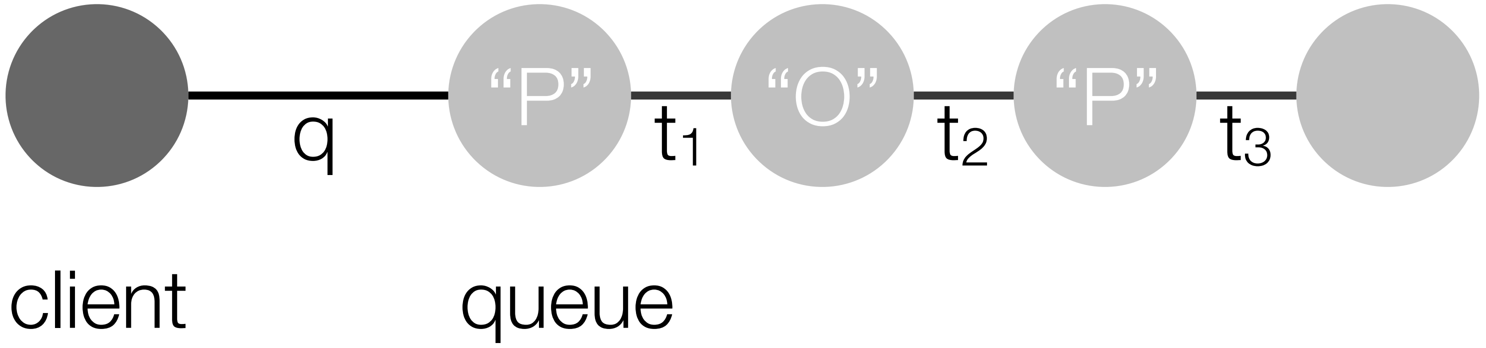
→ non-determinism

→ formal model: pi-calculus

[Milner 1992]

Legend: process — channel — message

Example: queue



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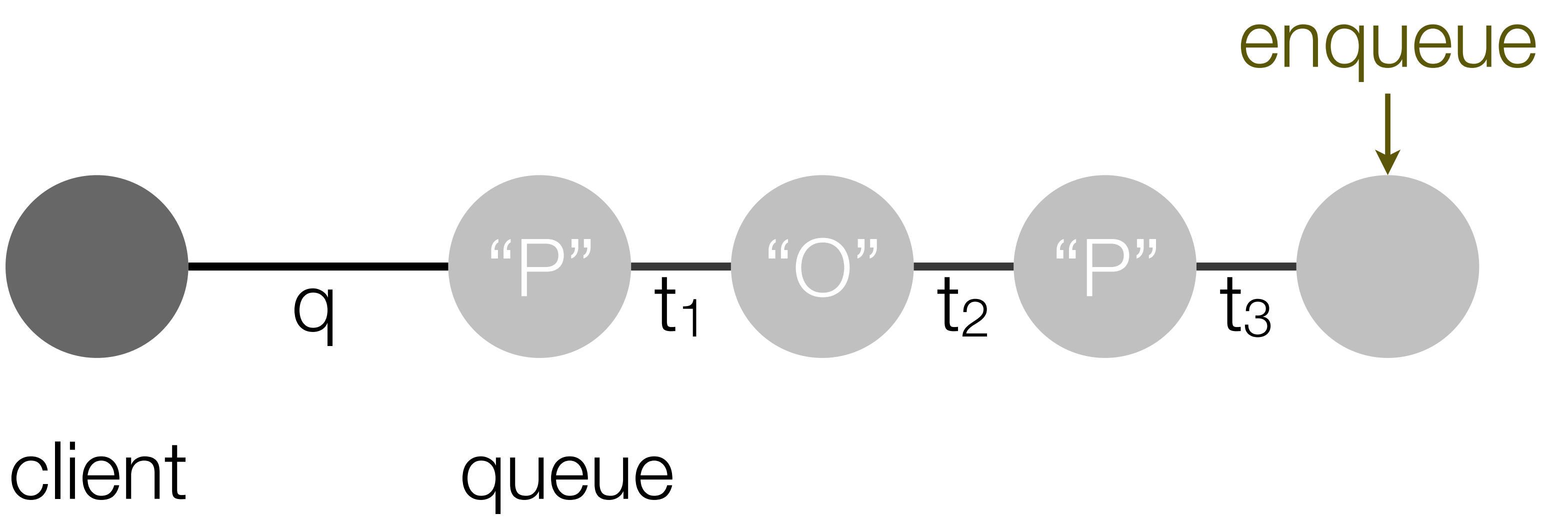

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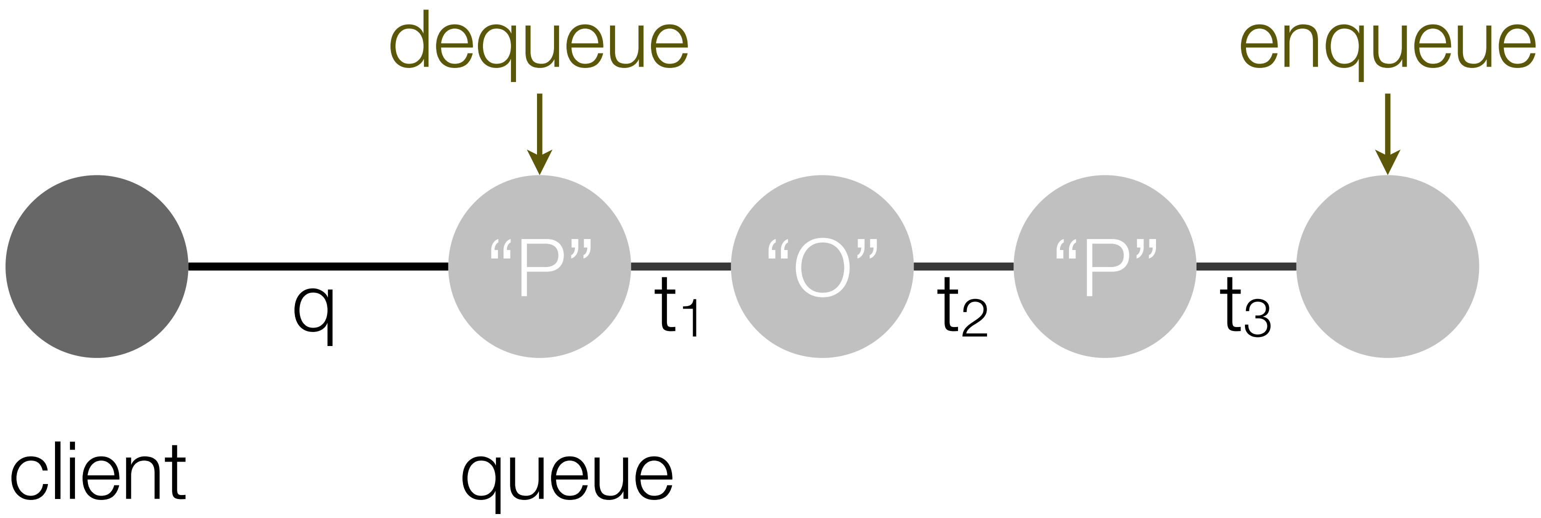

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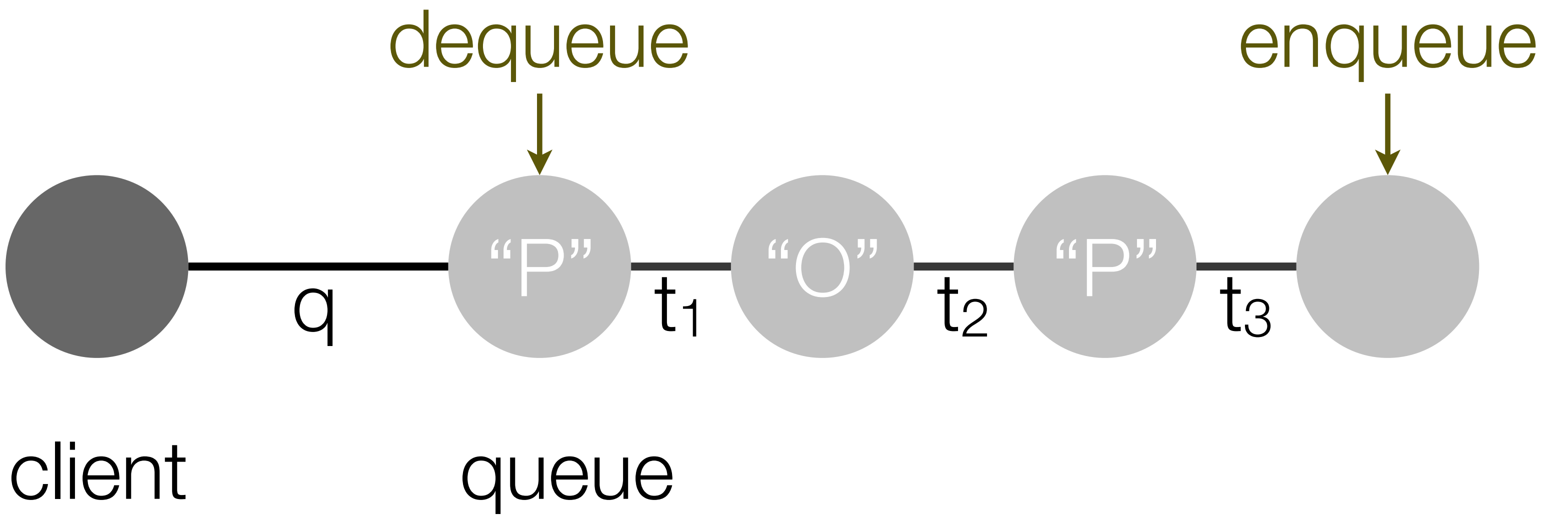

enqueue: client sends "enq" followed by "L" along q

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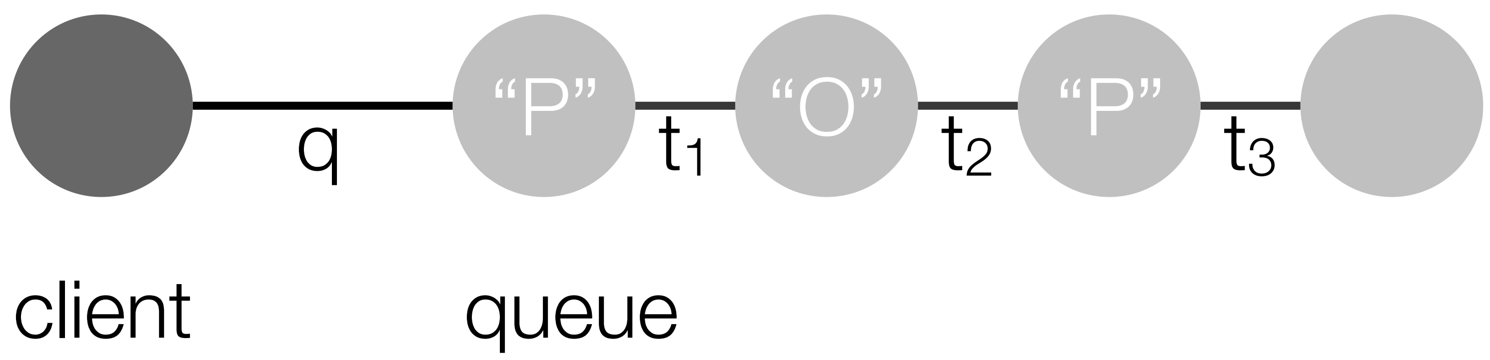

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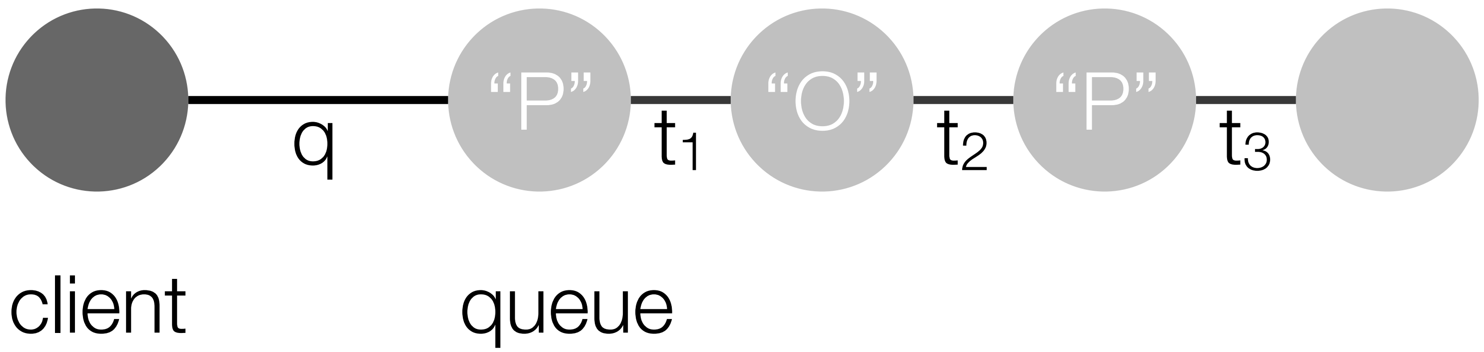

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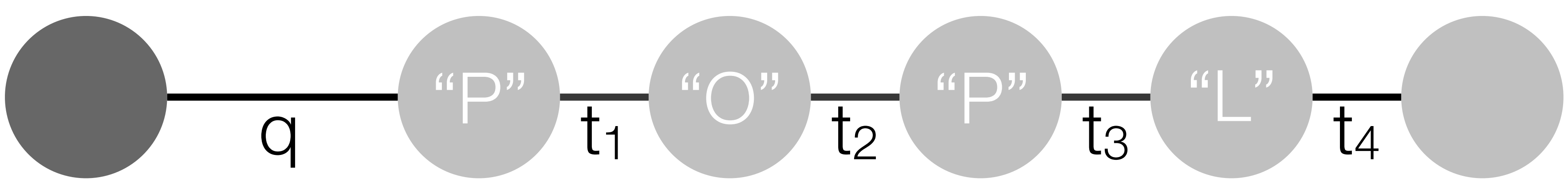

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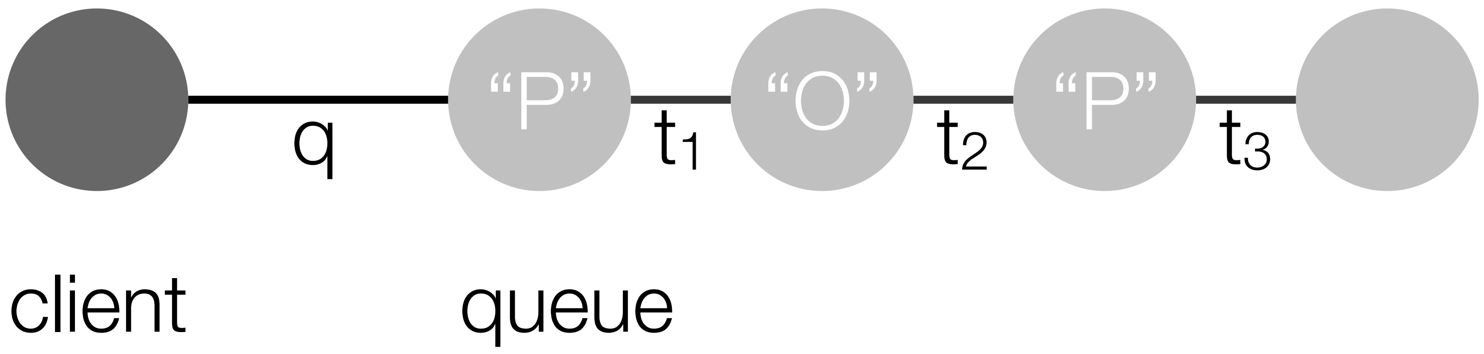

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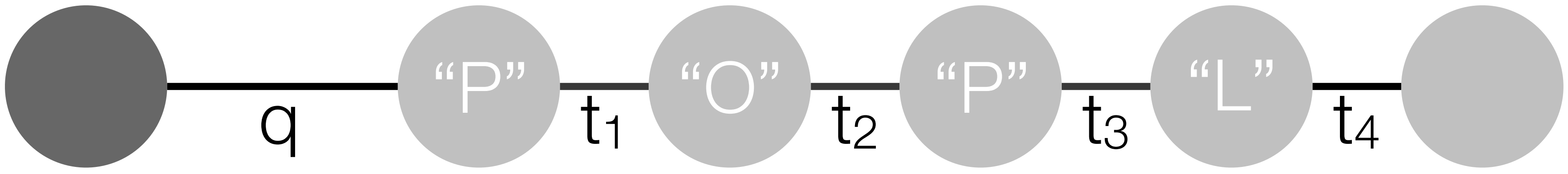

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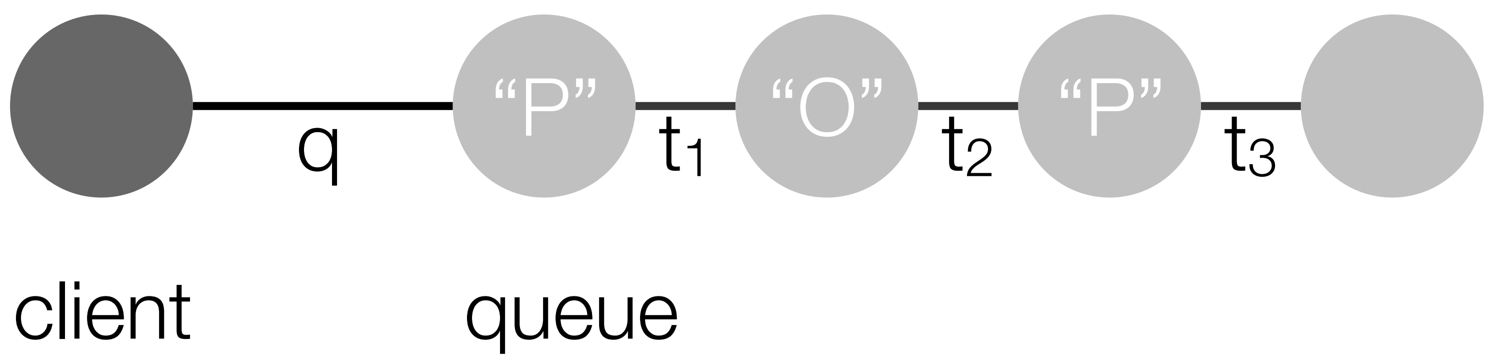

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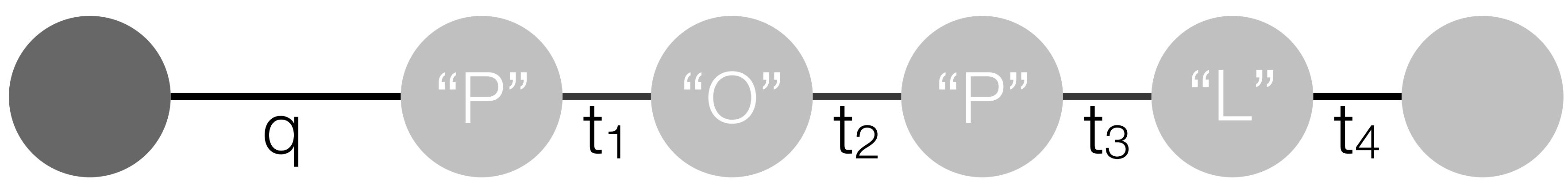

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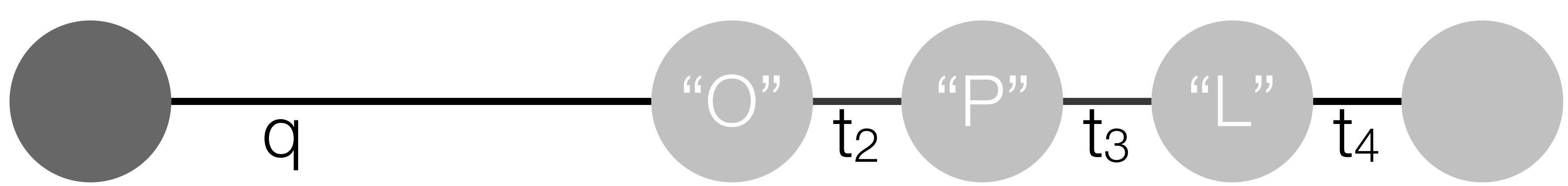

Example: queue

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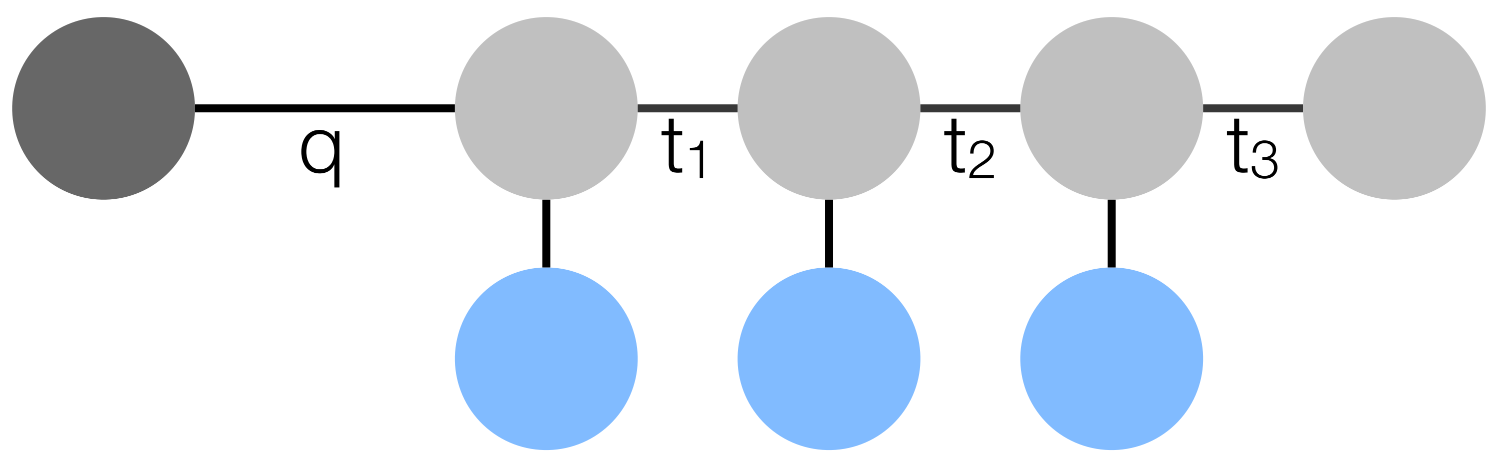

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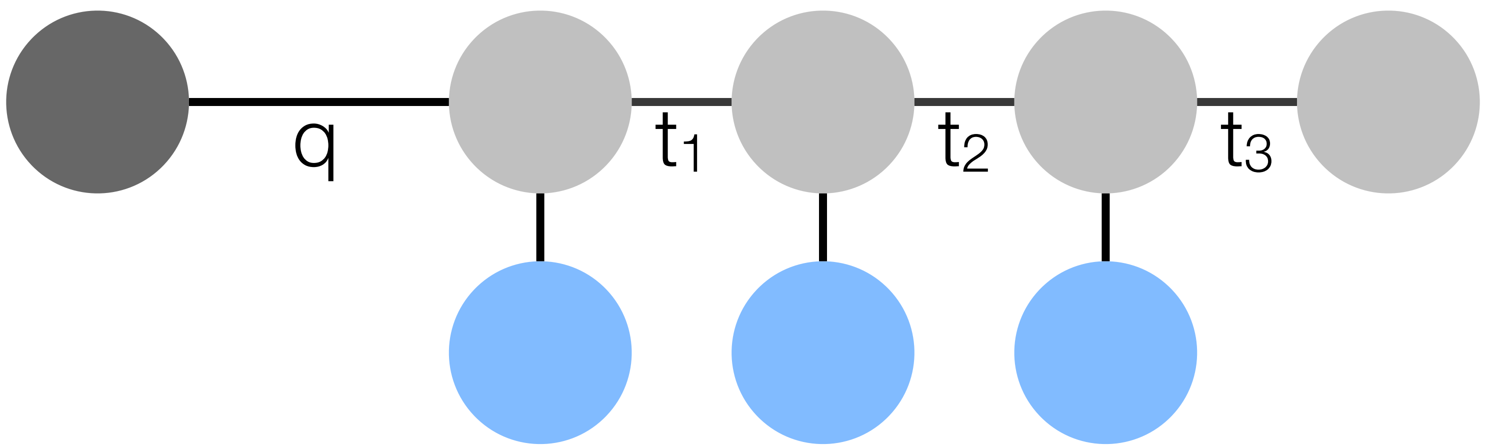

"higher-order" channels (session types)

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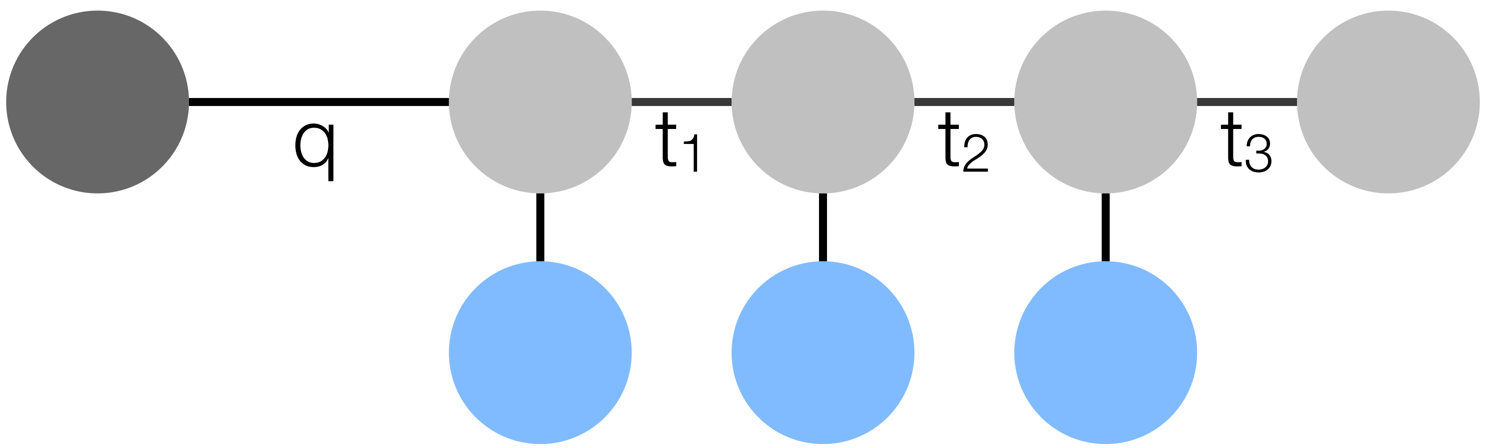

→ "higher-order" channels (session types)

→ "mobility" (pi-calculus)

Types for protocols of message exchange

Types for protocols of message exchange

Session types [Honda 1993]

$$
\begin{array}{lll}
A & \triangleq & ?[T].A' \mid ![T].A' \mid \\
& & \&\{l_1 : A_1, \ldots, l_n : A_n\} \mid \oplus\{l_1 : A_1, \ldots, l_n : A_n\} \mid \\
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input: receive message of type T, continue as type A’

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output: send message of type T, continue as type A'

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external choice: receive label l_i, continue as type A_i

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termination: close session and terminate

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recursive session types

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Example:

$$
\begin{aligned}
\mathsf{queue} = \&\{&\mathsf{enq} : ?[\mathsf{char}].\mathsf{queue}, \\
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Session types change with protocol

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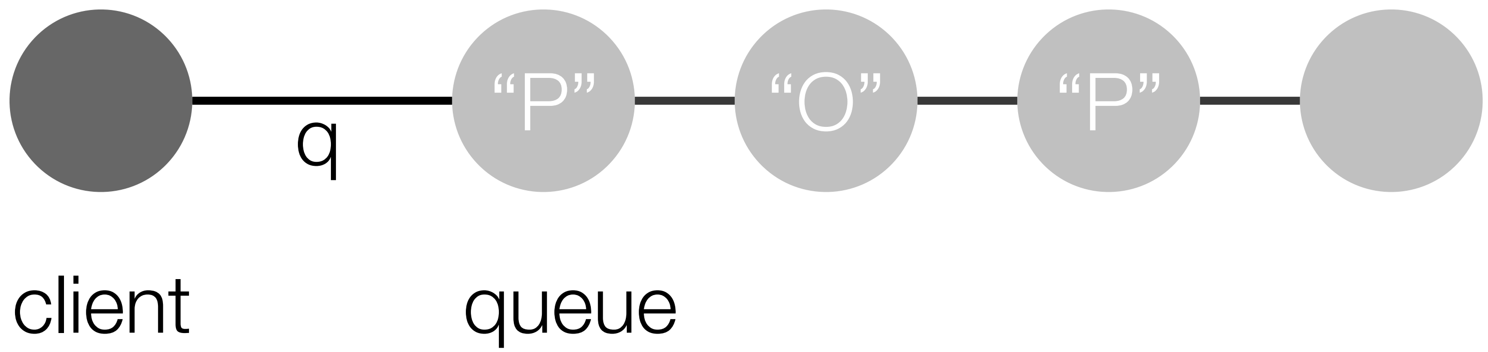

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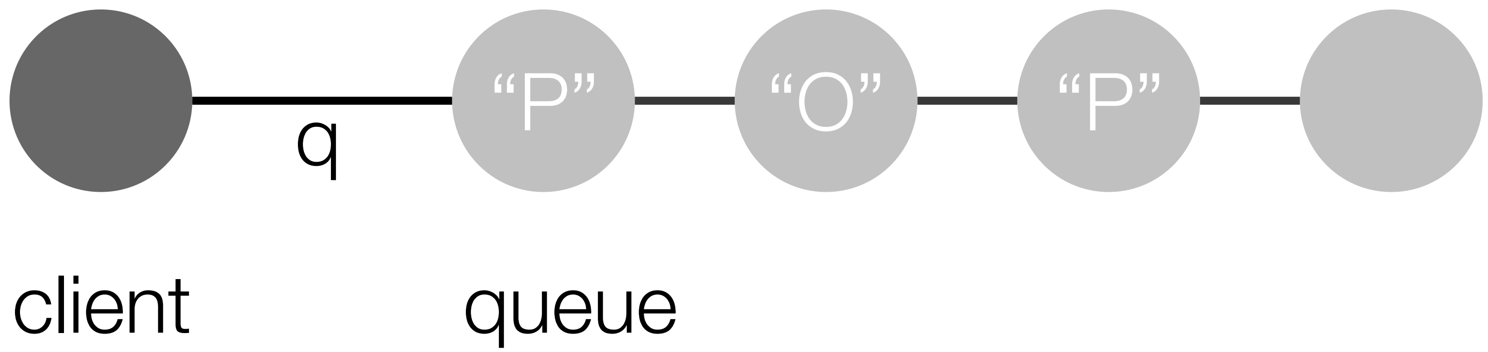

What is the type of channel q?

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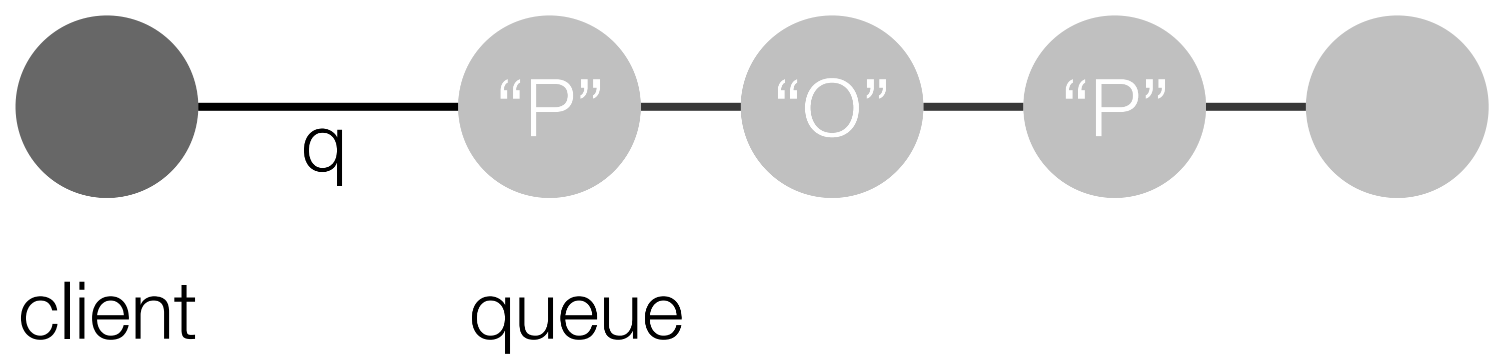

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q: queue

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Action:	Type:
	q: queue
send “enq” along q	q:

Session types change with protocol

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Action:	Type:
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send "L" along q	q:

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Session types change with protocol

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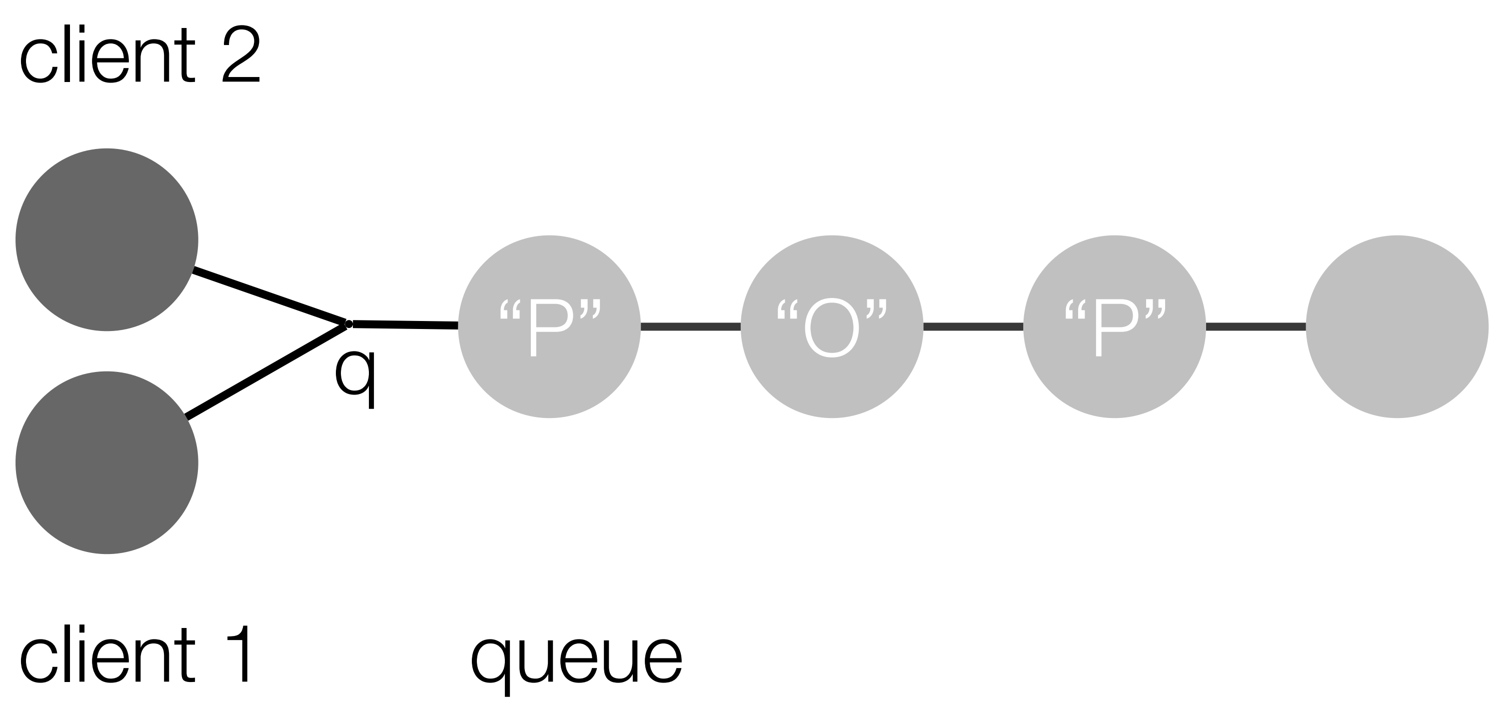
What if?



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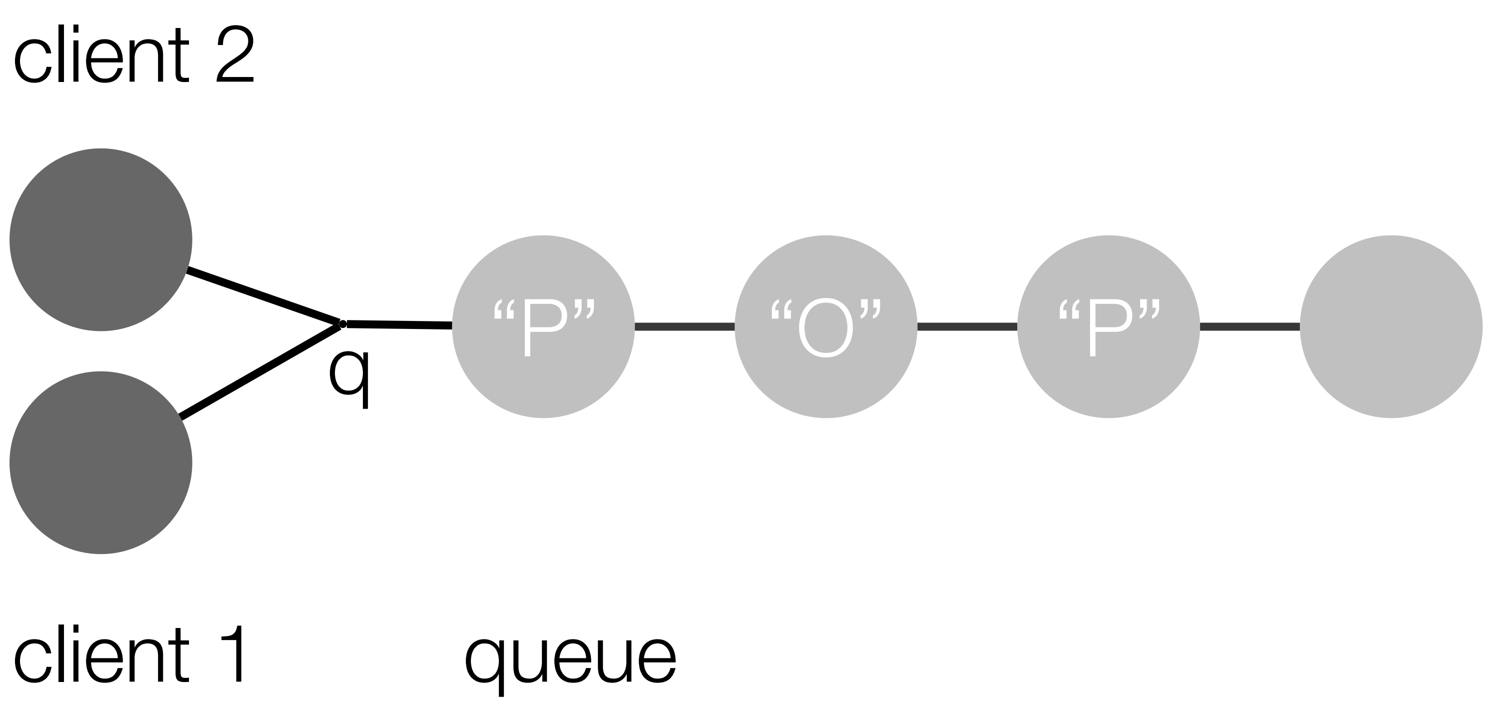

Action:

Type:

q: queue

Session types change with protocol

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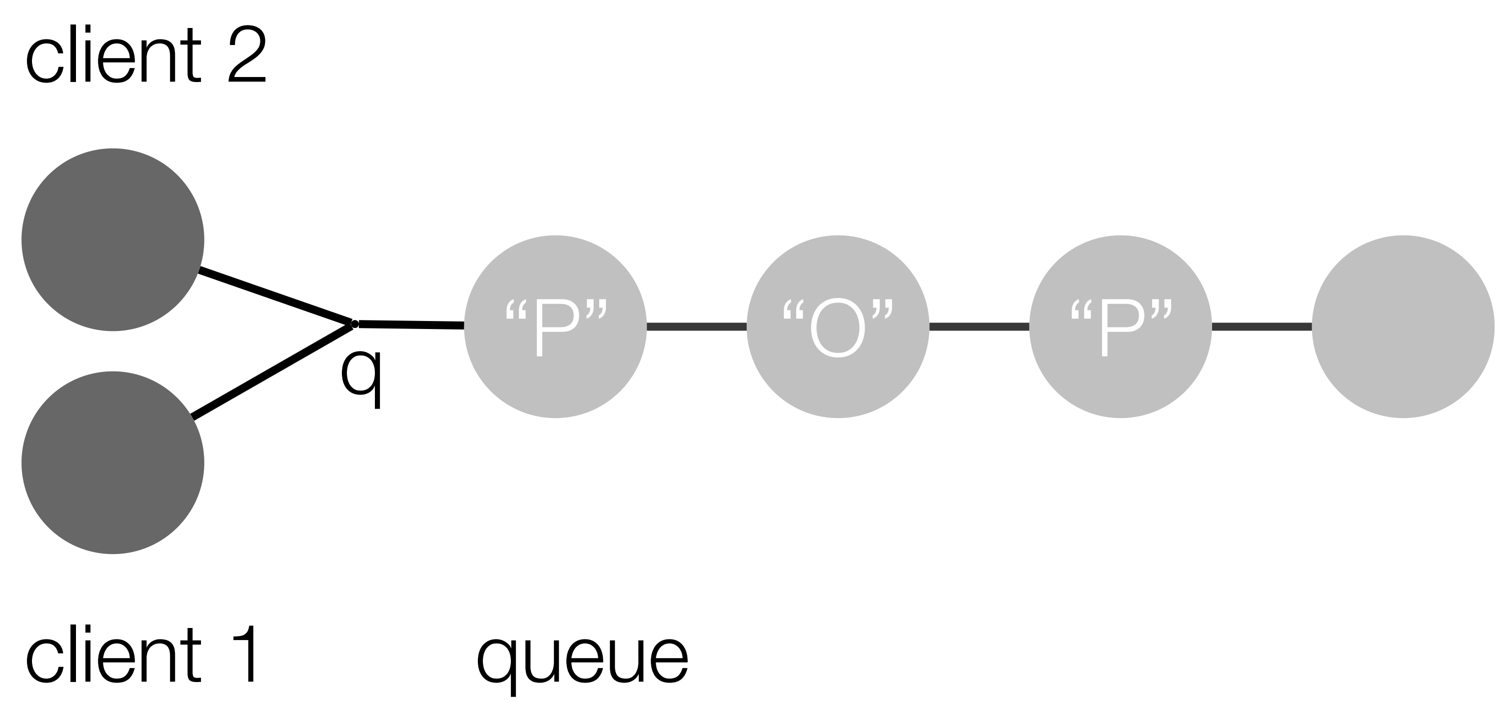


Action:

Type:

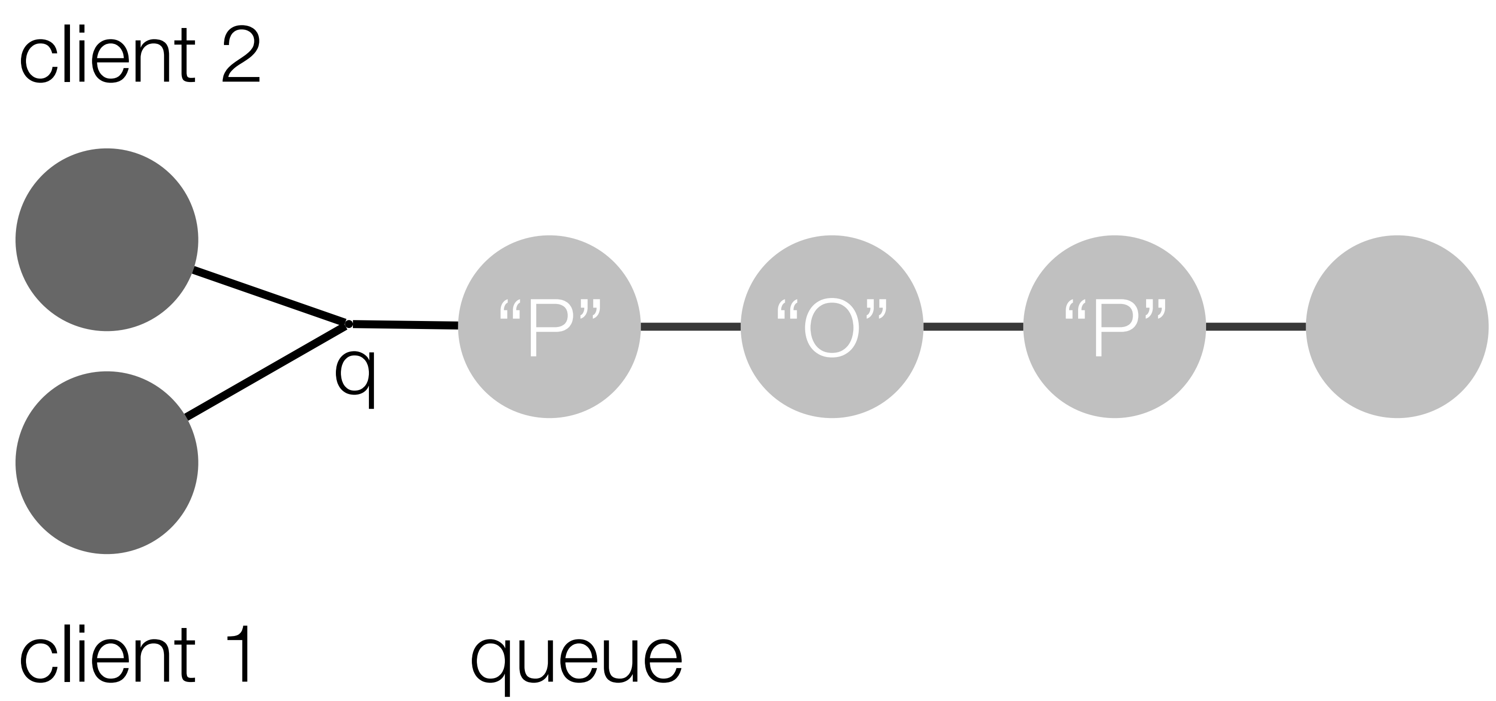
q: $\texttt{queue}$

client 1 sends “enq” along q

q:

Session types change with protocol

What if?



Action:

Type:

q: queue

client 1 sends "enq" along q

q: $?[\mathsf{char}].\mathsf{queue}$

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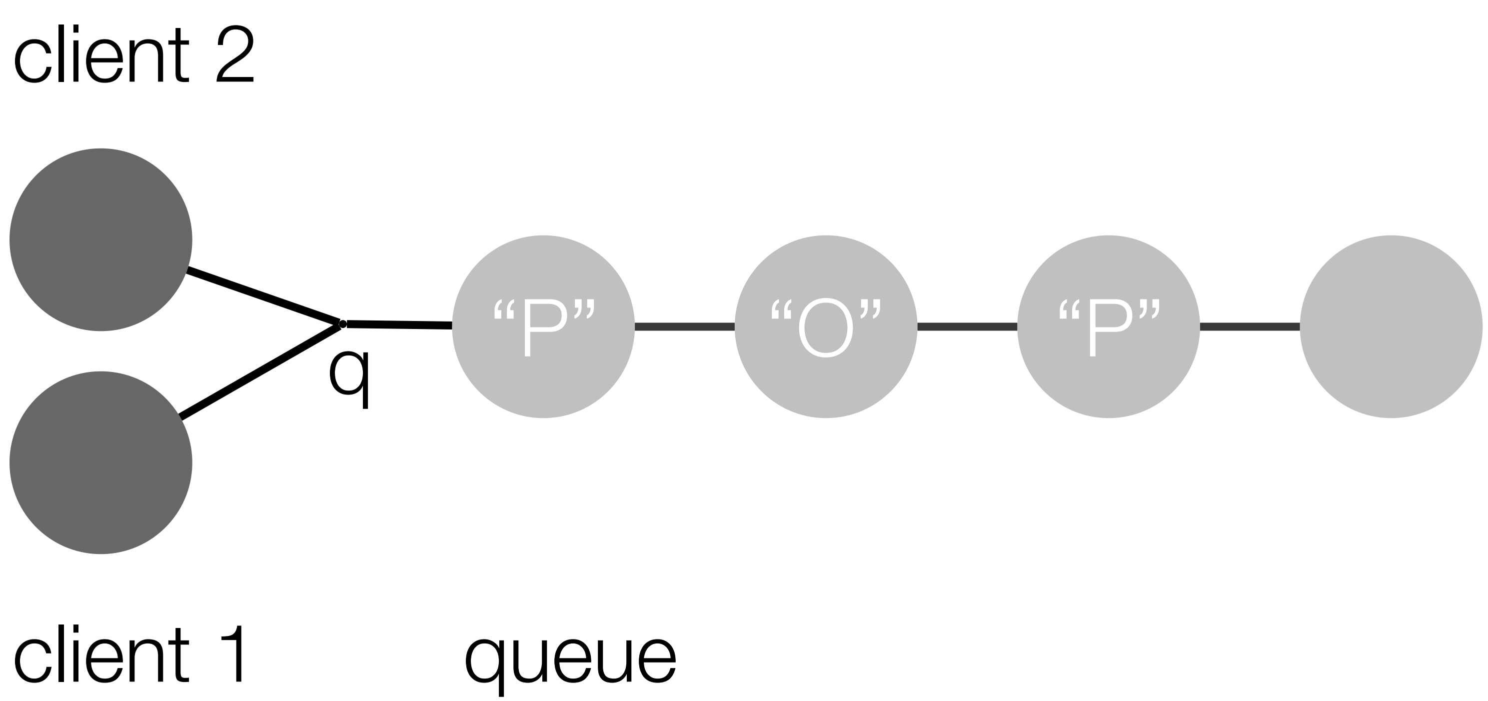

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client 2 sends “enq” along q

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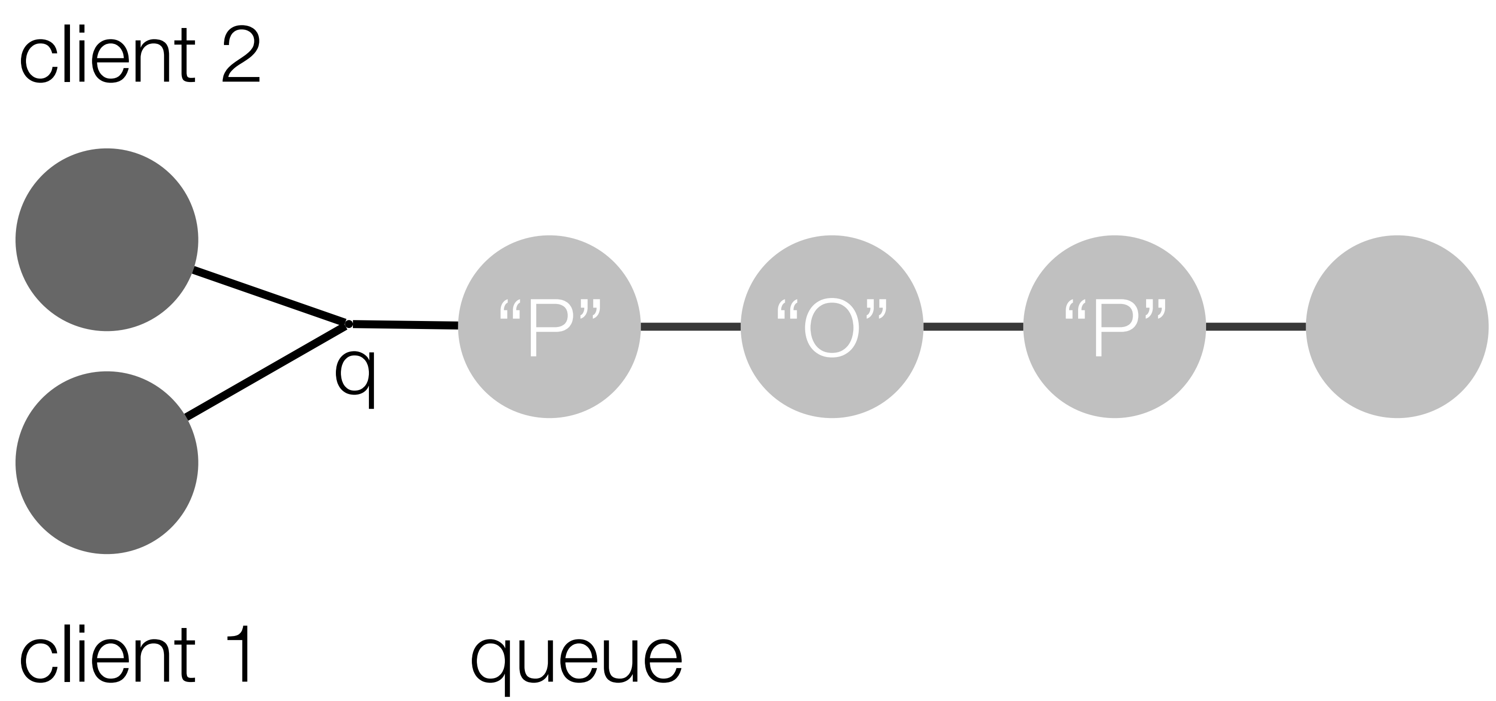

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client 1 sends “enq” along q q: $?[\mathsf{char}].\mathsf{queue}$

client 2 sends “enq” along q q is not at expected type!

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- Expectations of client and provider match, if they do initially.
- How can we recover preservation?

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→ Linear logic allows us to treat channels as “resources”

→ What does that mean?

Linear session types

Linear logic

Rejects the following two structural rules:

$$\frac{\Gamma \vdash C}{\Gamma, A \vdash C}\ \textsf{weaken} \qquad \frac{\Gamma, A, A \vdash C}{\Gamma, A \vdash C}\ \textsf{contract}$$

"drop resource" "duplicate resource"

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Presented work based on intuitionistic linear logic

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- Presented work based on intuitionistic linear logic
- Distinction of provider (right) from clients (left) of turnstile

Weakening

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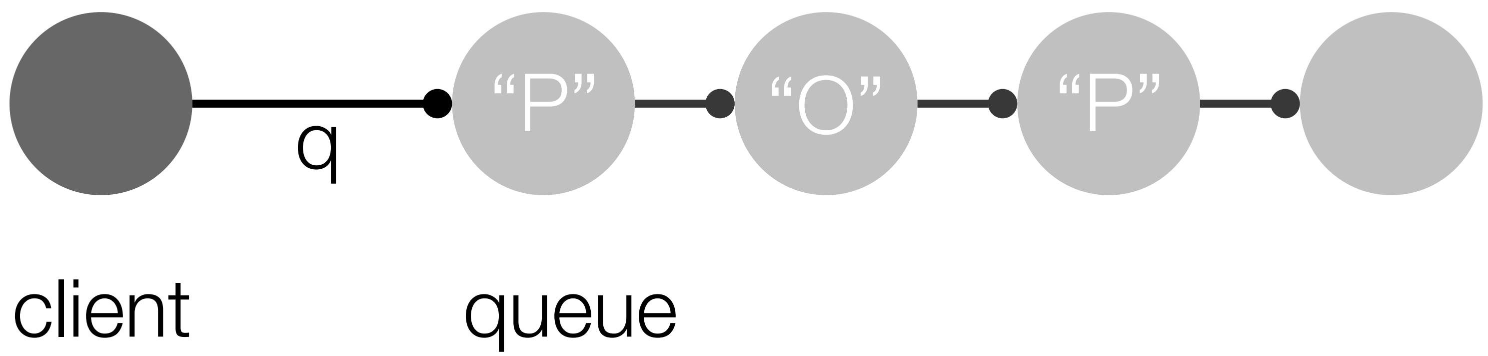

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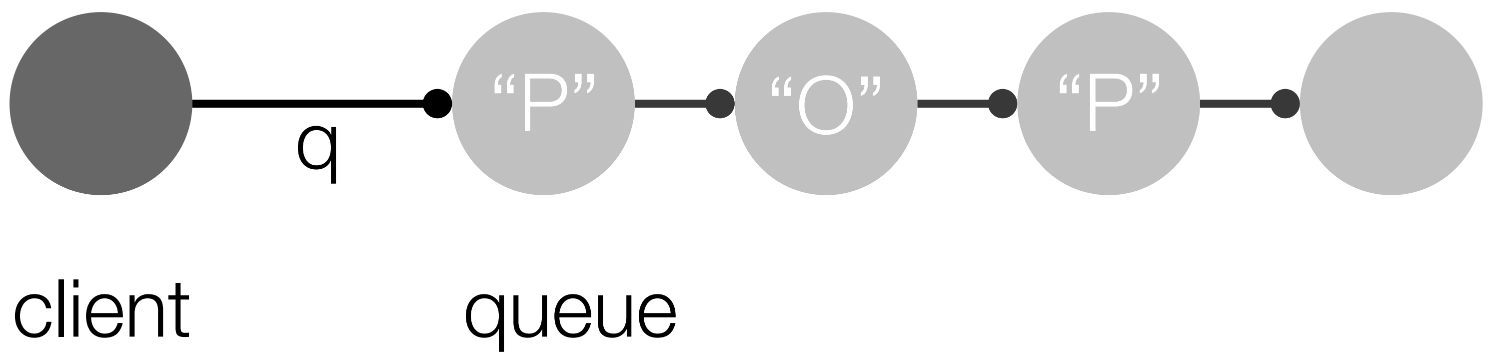

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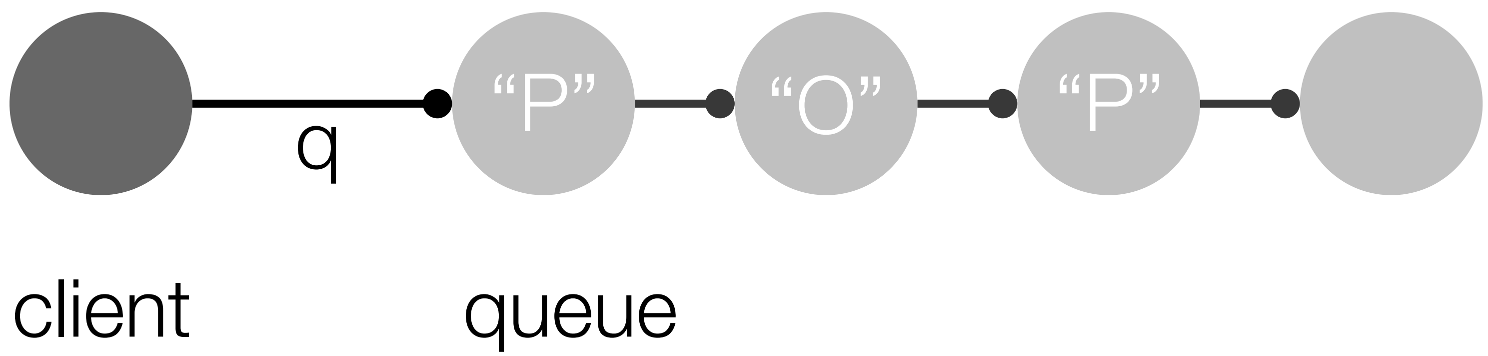

Terminating client without terminating or passing on queue

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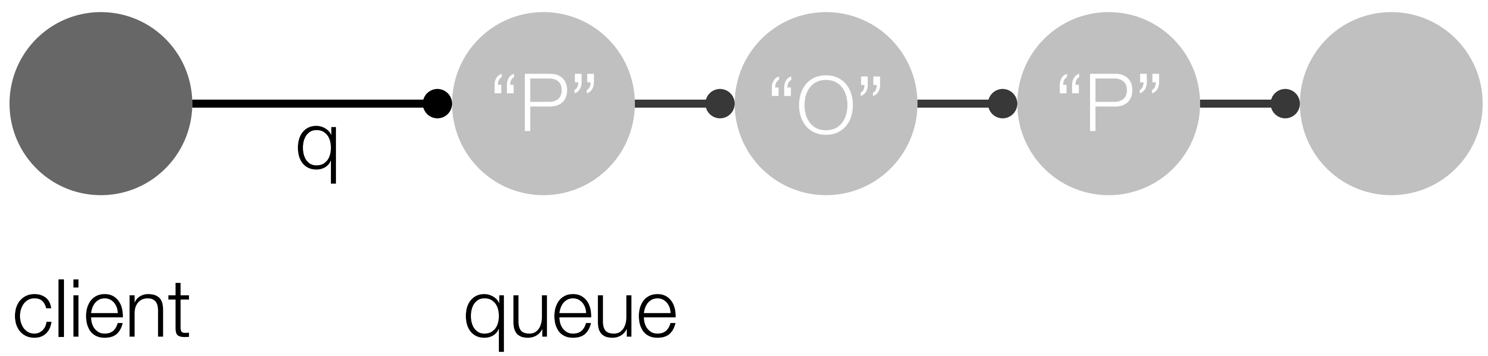

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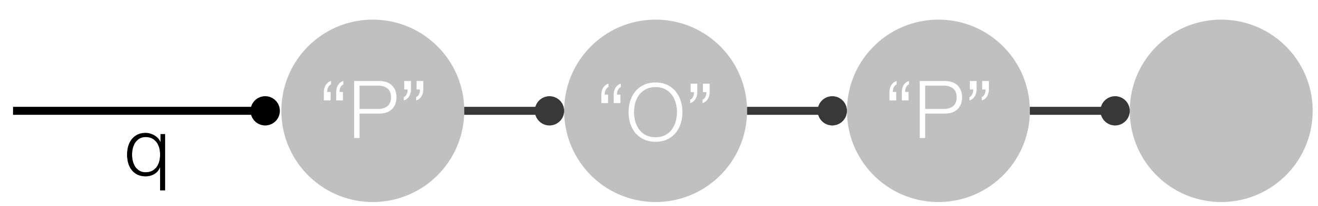

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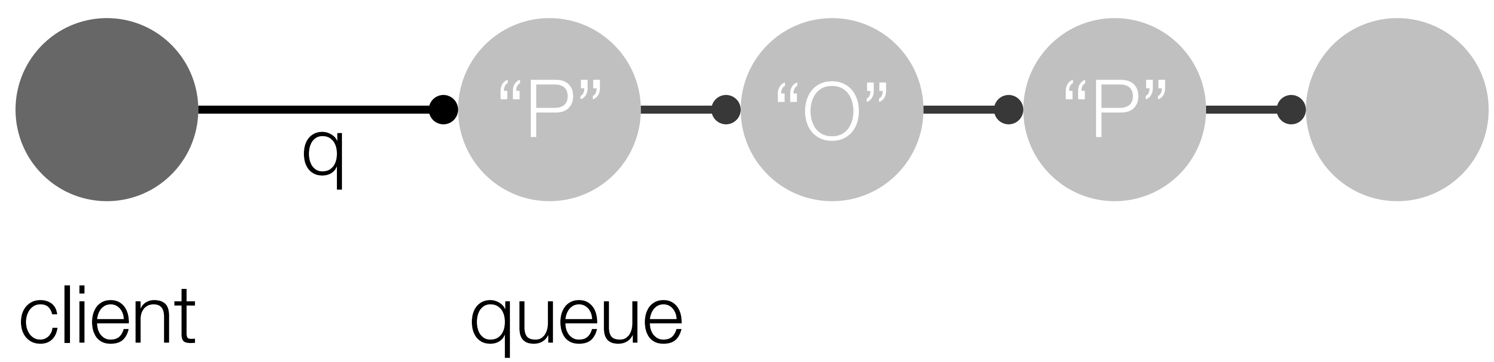

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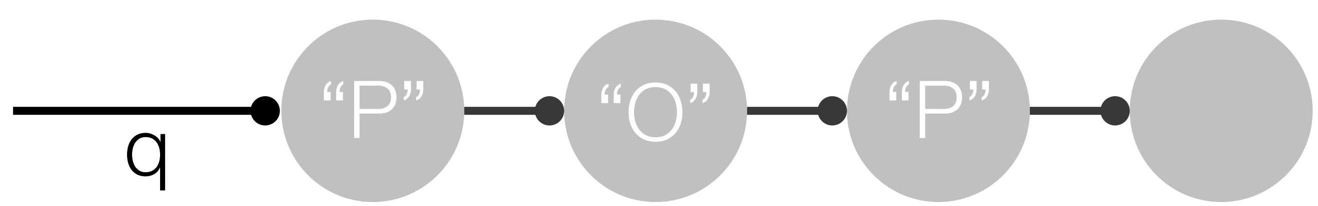

Providing process without a client

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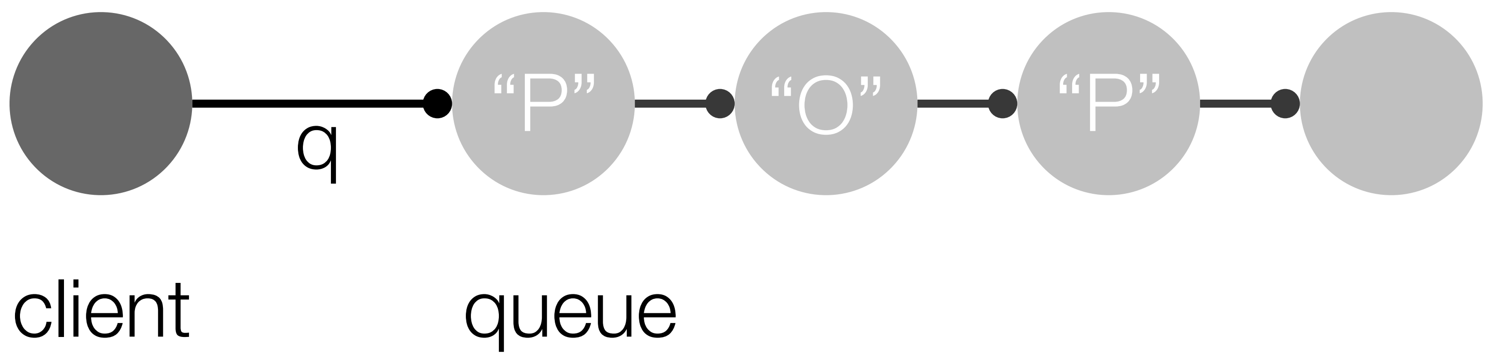

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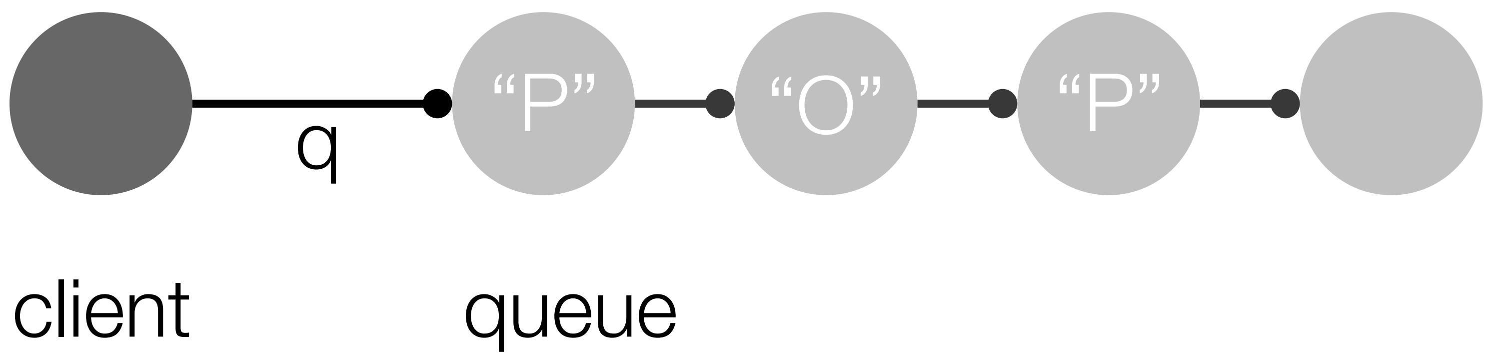

Passing on channel reference and keeping it

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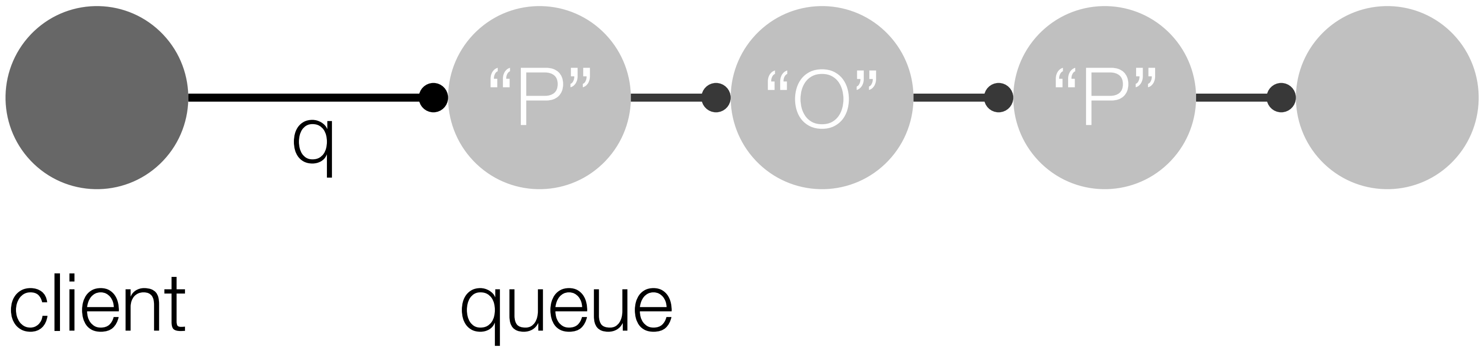

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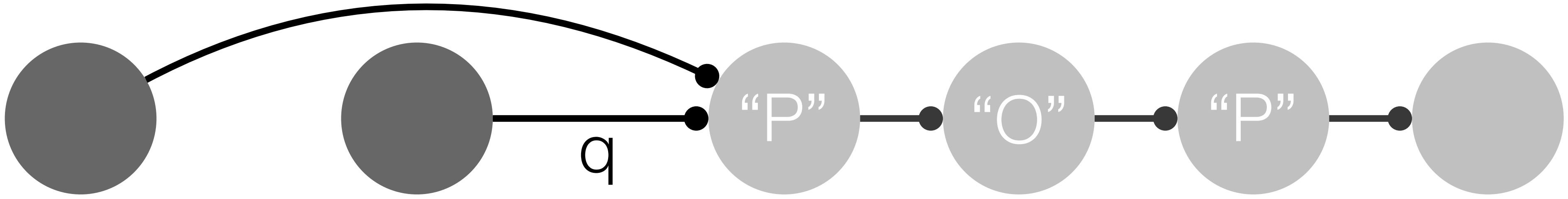

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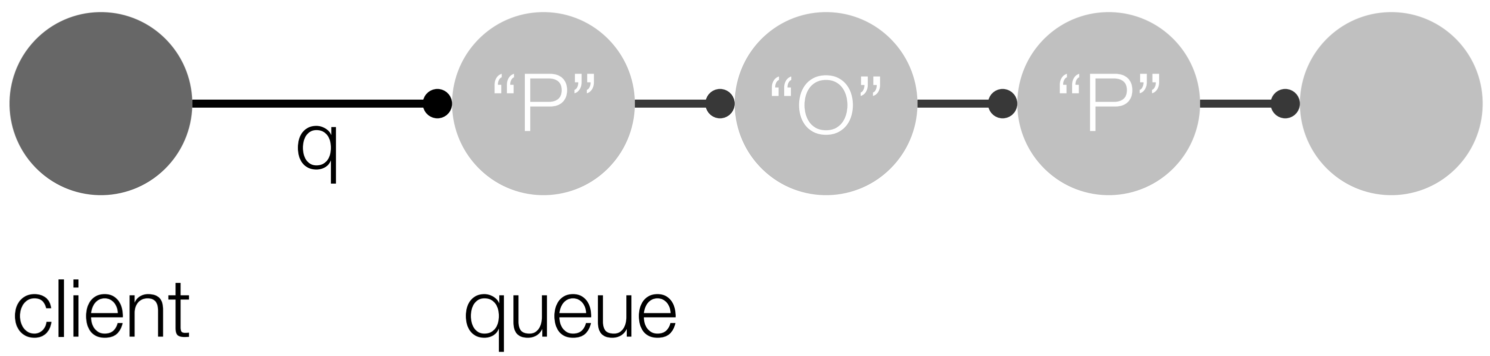

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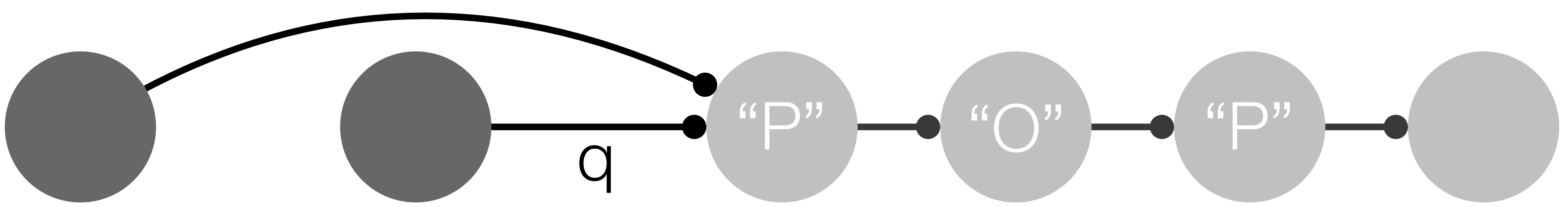
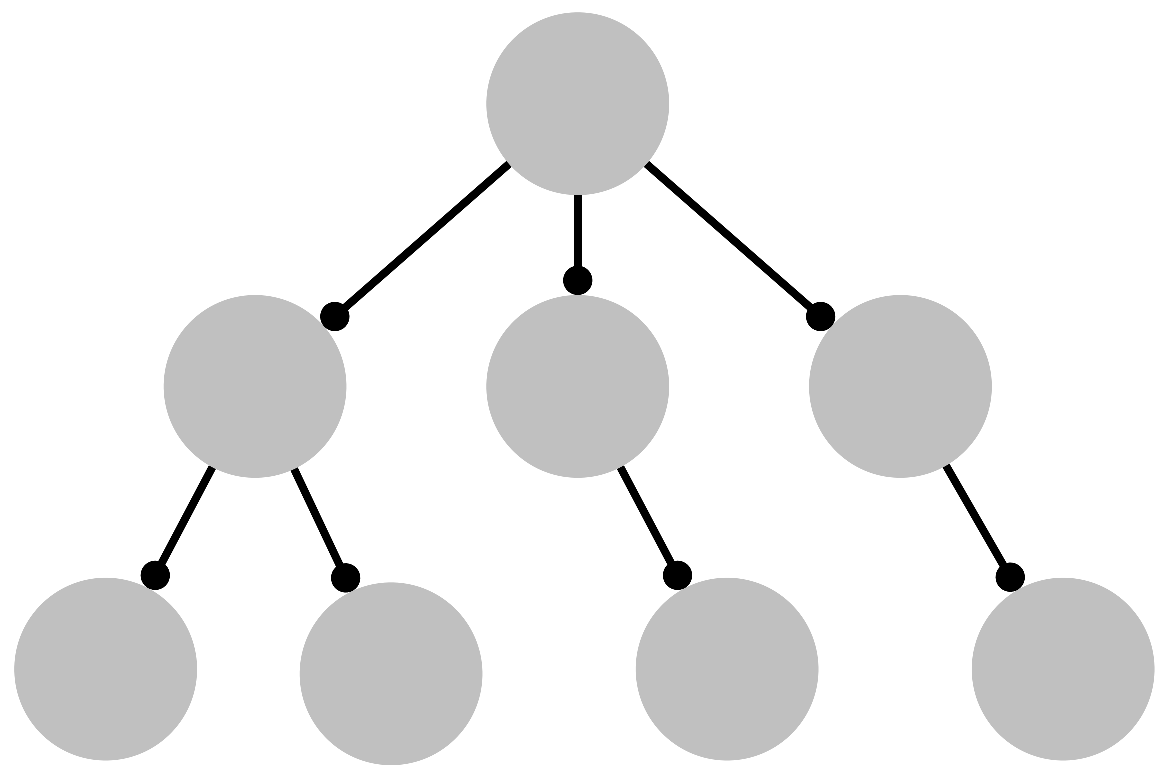


Providing process has multiple clients

Tree structure

Without weakening and contraction, process graph forms a tree.



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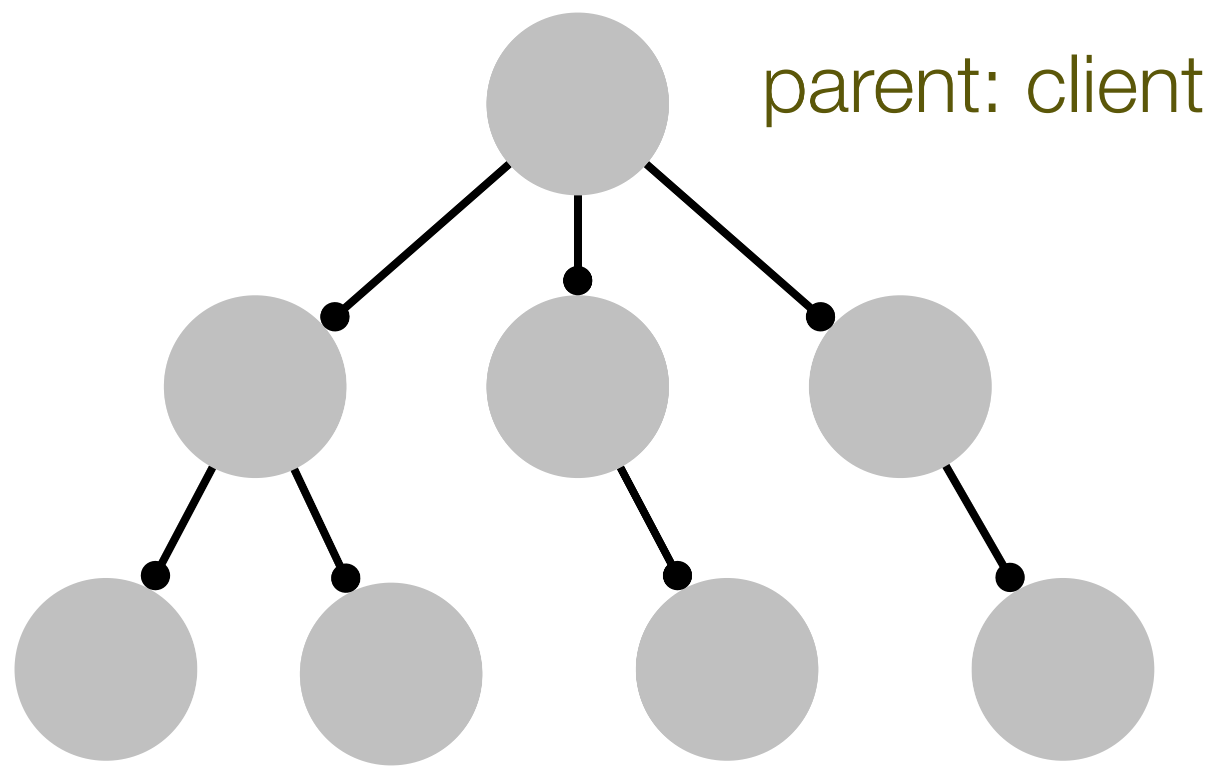

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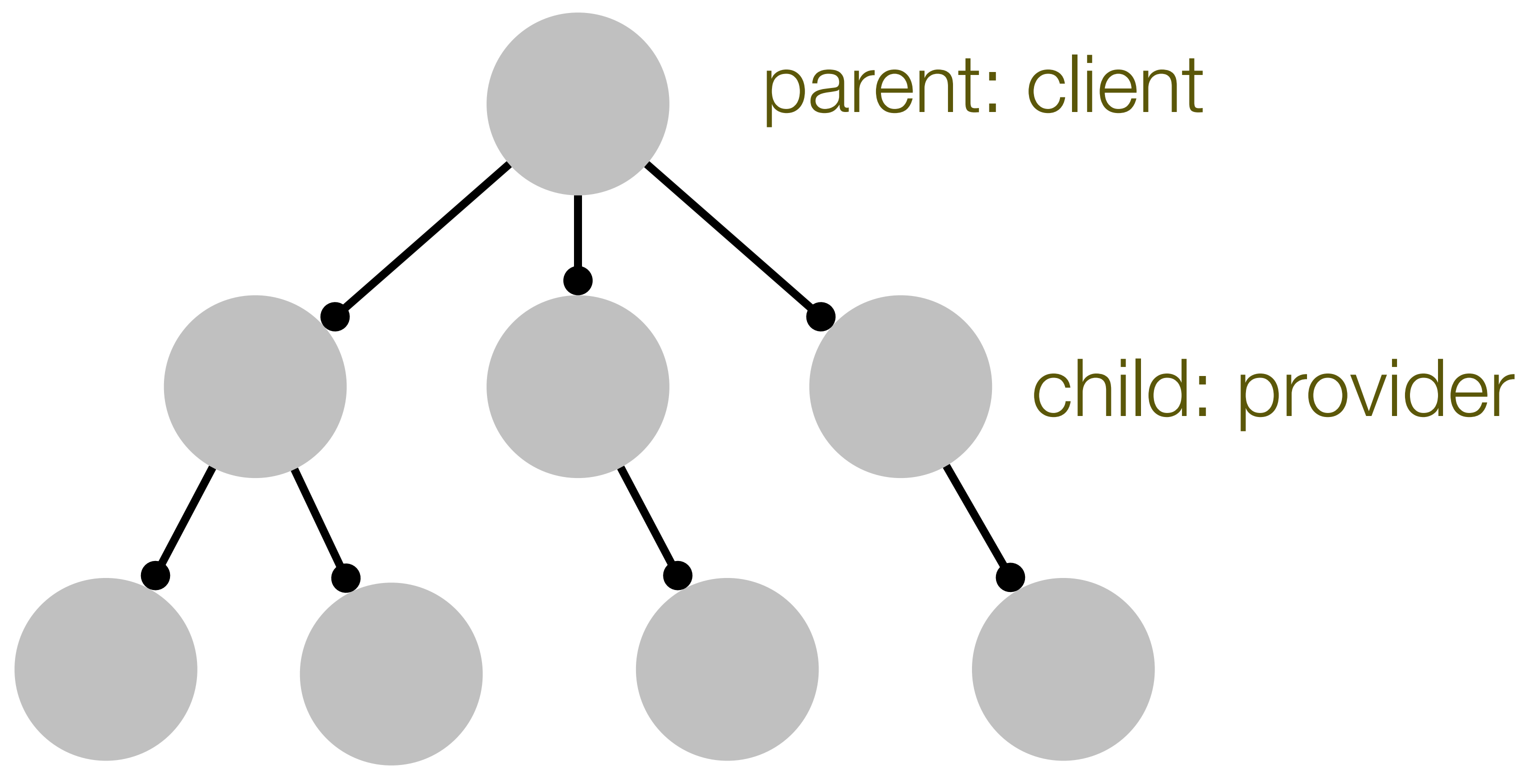

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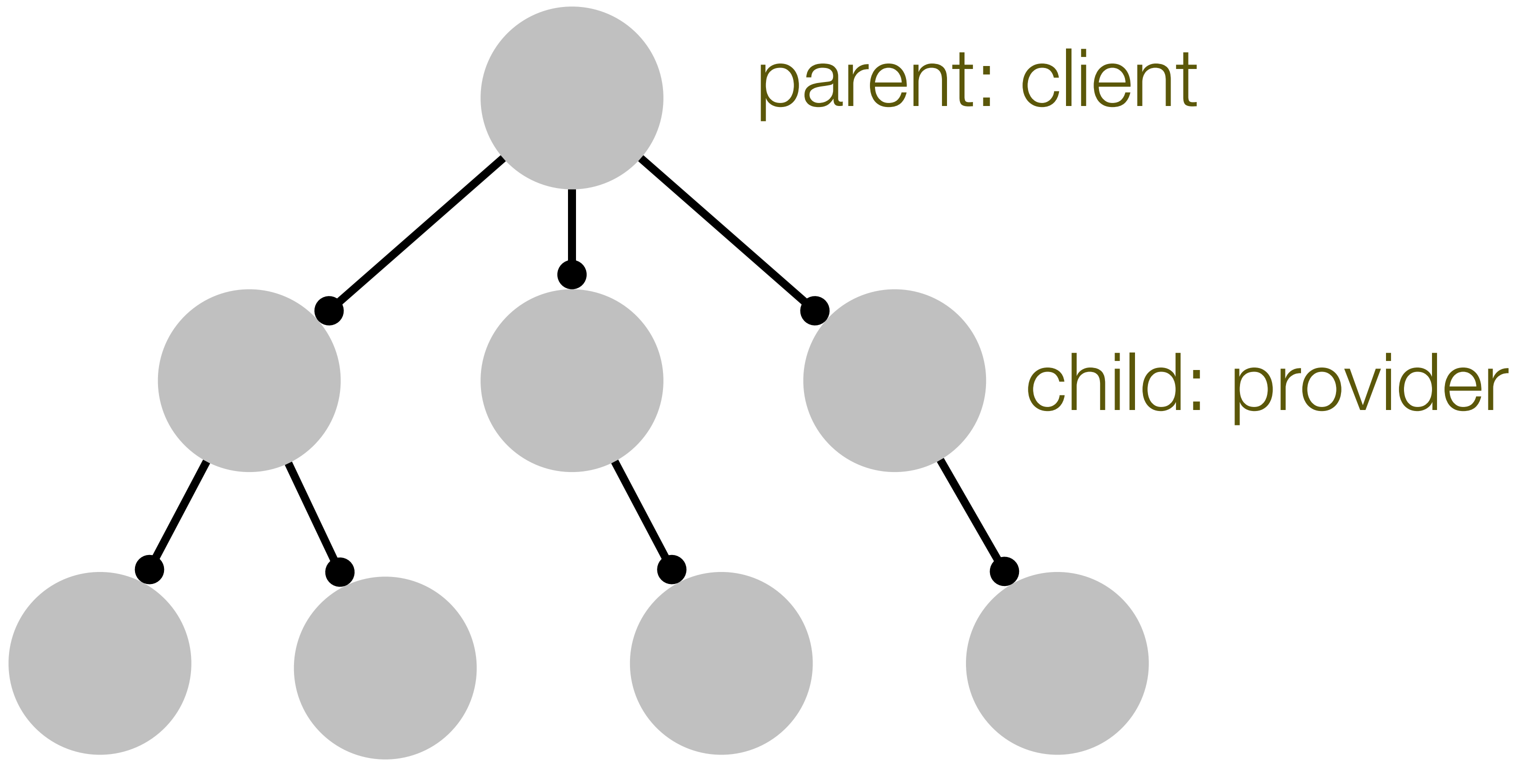

Every providing process has exactly one client.

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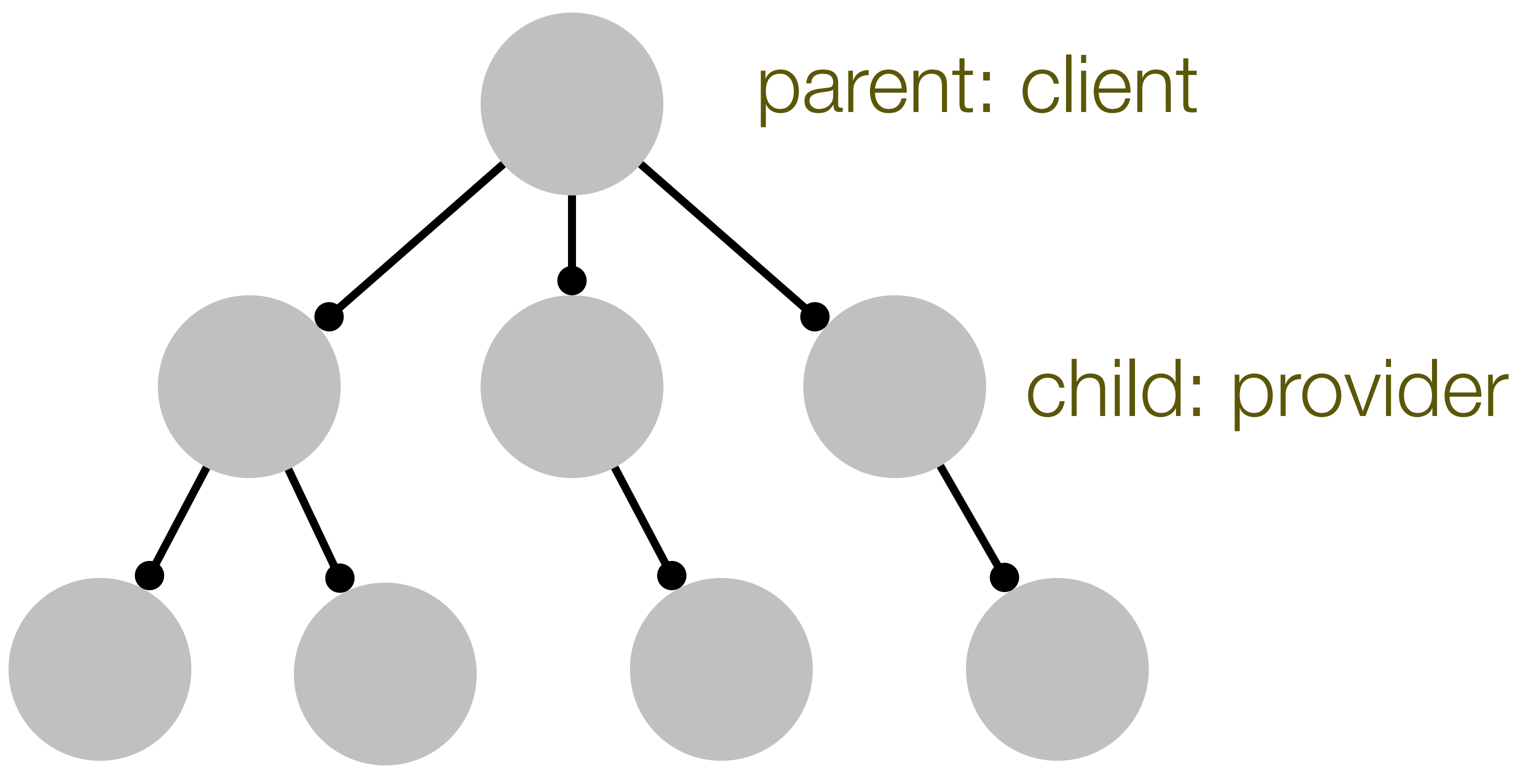

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Session fidelity restored.

Reconstructing session types from linear propositions [Caires & Pfenning 2010, Wadler 2012]

Curry-Howard correspondence: intuitionistic linear logic - session-typed pi-calculus [Caires & Pfenning 2010]

Logic:	Type theory:
Linear propositions	Session types
Proofs	Programs
Cut reduction	Communication

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Let’s discover this correspondence together!

Intuitionistic linear logic

Connectives:

$$
\begin{array}{lcll}
A, B & \triangleq & A \otimes B & \text{multiplicative conjunction} \\
 & & A \multimap B & \text{multiplicative implication} \\
 & & A \& B & \text{additive conjunction} \\
 & & A \oplus B & \text{additive disjunction} \\
 & & !A & \text{"of course", persistent truth}
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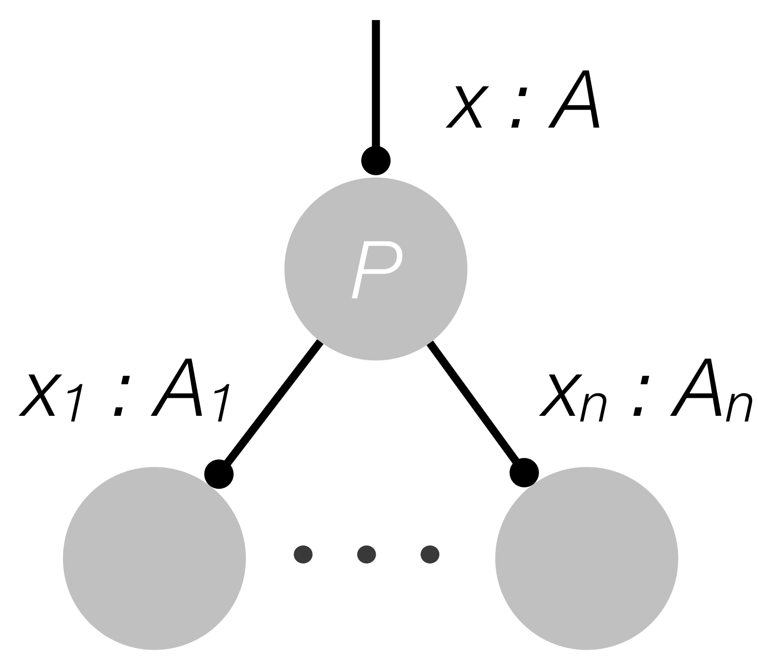

Additive conjunction

$$\frac{\Delta \vdash \quad :: (\quad : A) \qquad \Delta \vdash \quad :: (\quad : B)}{\Delta \vdash \qquad\qquad :: (\quad : A \& B)} \;(\text{T-}\&_{\text{R}})$$

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External choice: client chooses. Generalizes to $\&\{\overline{l : A}\}$.

Additive disjunction

$$\frac{\Delta \vdash P :: (x : A)}{\Delta \vdash x.\mathsf{inl}; P :: (x : A \oplus B)} \ (\text{T-}\oplus_{\text{R}_1})$$

$$\frac{\Delta \vdash P :: (x : B)}{\Delta \vdash x.\mathsf{inr}; P :: (x : A \oplus B)} \ (\text{T-}\oplus_{\text{R}_2})$$

$$\frac{\Delta, x : A \vdash Q_1 :: (z : C) \qquad \Delta, x : B \vdash Q_2 :: (z : C)}{\Delta, x : A \oplus B \vdash \mathsf{case}\, x\, \mathsf{of}(Q_1, Q_2) :: (z : C)} \ (\text{T-}\oplus_{\text{L}})$$

Internal choice: provider chooses. Generalizes to $\oplus\{\overline{l : A}\}$.

Multiplicative implication

$$\frac{\Delta, \qquad A \vdash \qquad :: (\quad : B)}{\Delta \vdash \qquad\qquad\qquad :: (\quad : A \multimap B)} \ (\text{T-}\multimap_{\text{R}})$$

Multiplicative implication

$$\frac{\Delta, \quad\quad A \vdash \quad\quad :: (\quad : B)}{\Delta \vdash \quad\quad\quad\quad\quad\quad\quad\quad :: (x : A \multimap B)} \; (\mathrm{T}\text{-}\multimap_{\mathrm{R}})$$

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Multiplicative implication

$$\frac{\Delta, y : A \vdash \quad :: (x : B)}{\Delta \vdash y \leftarrow \mathsf{recv}\ x; P :: (x : A \multimap B)}\ (\mathrm{T}\text{-}{\multimap}_{\mathrm{R}})$$

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Channel input: provider receives a channel.

Multiplicative implication

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Channel input: provider receives a channel.

Multiplicative conjunction

$$\frac{\Delta \vdash Q :: (y : A) \qquad \Delta' \vdash P :: (x : B)}{\Delta, \Delta' \vdash \mathsf{send}\ x\ (y \leftarrow Q); P :: (x : A \otimes B)}\ (\text{T-}\otimes_{\text{R}})$$

$$\frac{\Delta, x : B, y : A \vdash Q' :: (z : C)}{\Delta, x : A \otimes B \vdash y \leftarrow \mathsf{recv}\ x; Q' :: (z : C)}\ (\text{T-}\otimes_{\text{L}})$$

Channel output: provider sends a channel.

Unit of multiplicative conjunction

$$\frac{}{\cdot \vdash \qquad\qquad\qquad :: (\quad : \mathbf{1})}\ (\text{T-}\mathbf{1}_{\mathrm{R}})$$

Unit of multiplicative conjunction

$$\frac{}{\cdot \vdash \qquad\qquad :: (x : \mathbf{1})} \text{(T-}\mathbf{1}_{\mathrm{R}}\text{)}$$

Unit of multiplicative conjunction

$$\frac{}{\cdot \vdash \mathsf{close}\ x :: (x : \mathbf{1})}\ (\text{T-}\mathbf{1}_{\text{R}})$$

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→ Termination

Unit of multiplicative conjunction

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- Termination
- $\&$ and $\oplus$ must consist of at least one label.

Recap

Recap

Connectives:

A, B	$\triangleq$	$A \otimes B$	channel output
		$A \multimap B$	channel input
		$\oplus\{\overline{l : A}\}$	internal choice
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		$\mathbf{1}$	termination

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Example:

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Judgmental rules

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$$\frac{\Delta \vdash P(x : A) \qquad \Delta', x : A \vdash Q :: (z : C)}{\Delta, \Delta' \vdash x \leftarrow P; Q :: (z : C)} \; (\textsc{T-cut})$$

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Parallel composition (spawning new process)

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Forward: process offering along x terminates, client henceforth interacts with process offering along y.

Let's program in Concurrent C0!

C0 [Pfenning 2010, Arnold 2010]

- safe subset of C supporting contracts
- teaching language developed at CMU
- http://c0.typesafety.net

Concurrent C0 [Willsey & Prabhu & Pfenning 2016]

- extends C0 with session types

Installing Concurrent C0

- see http://www.cs.cmu.edu/~balzers/popl_tutorial_2019

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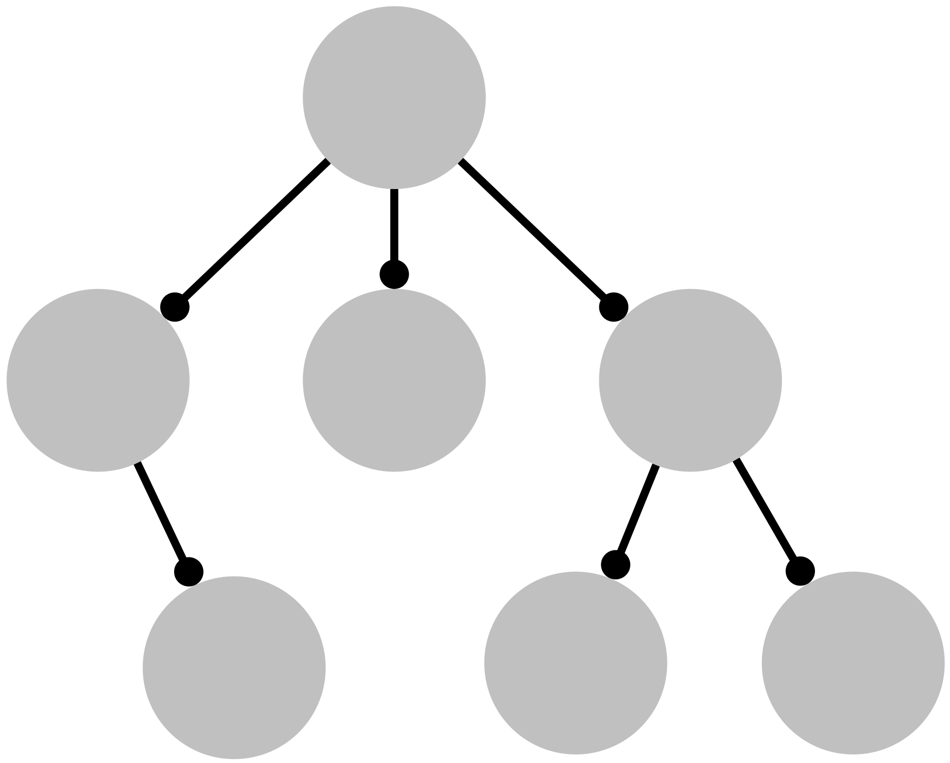

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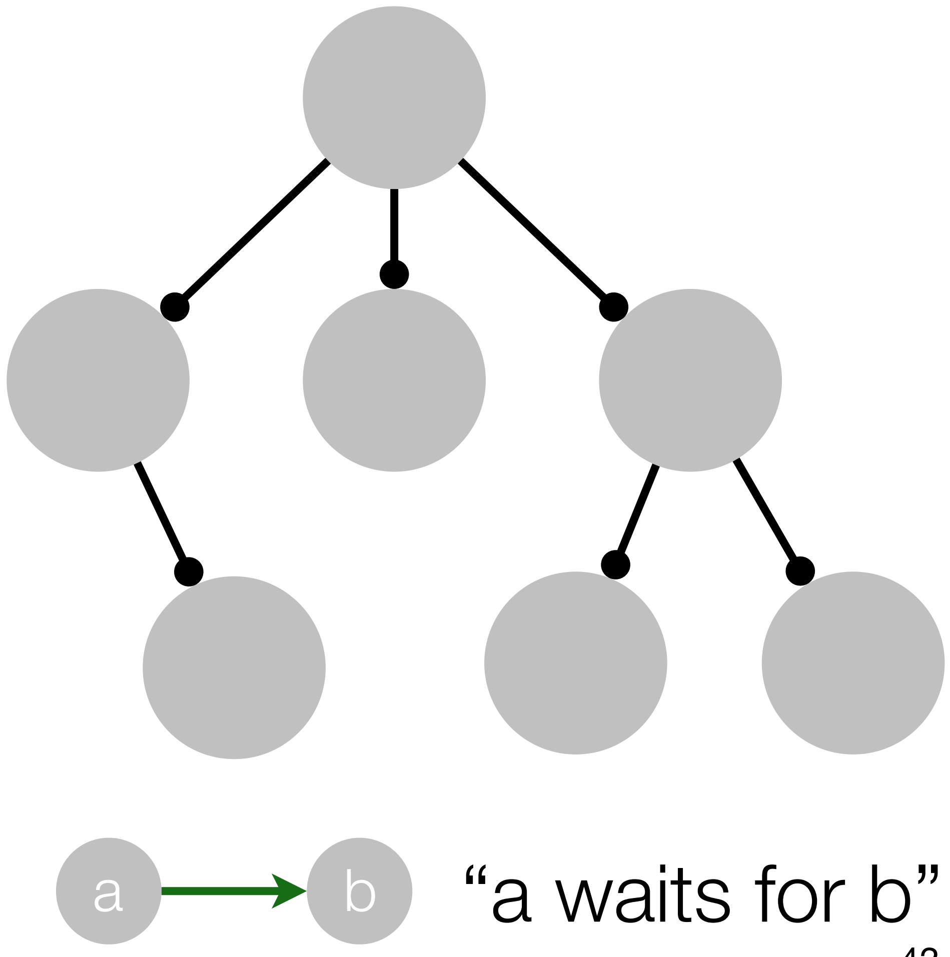

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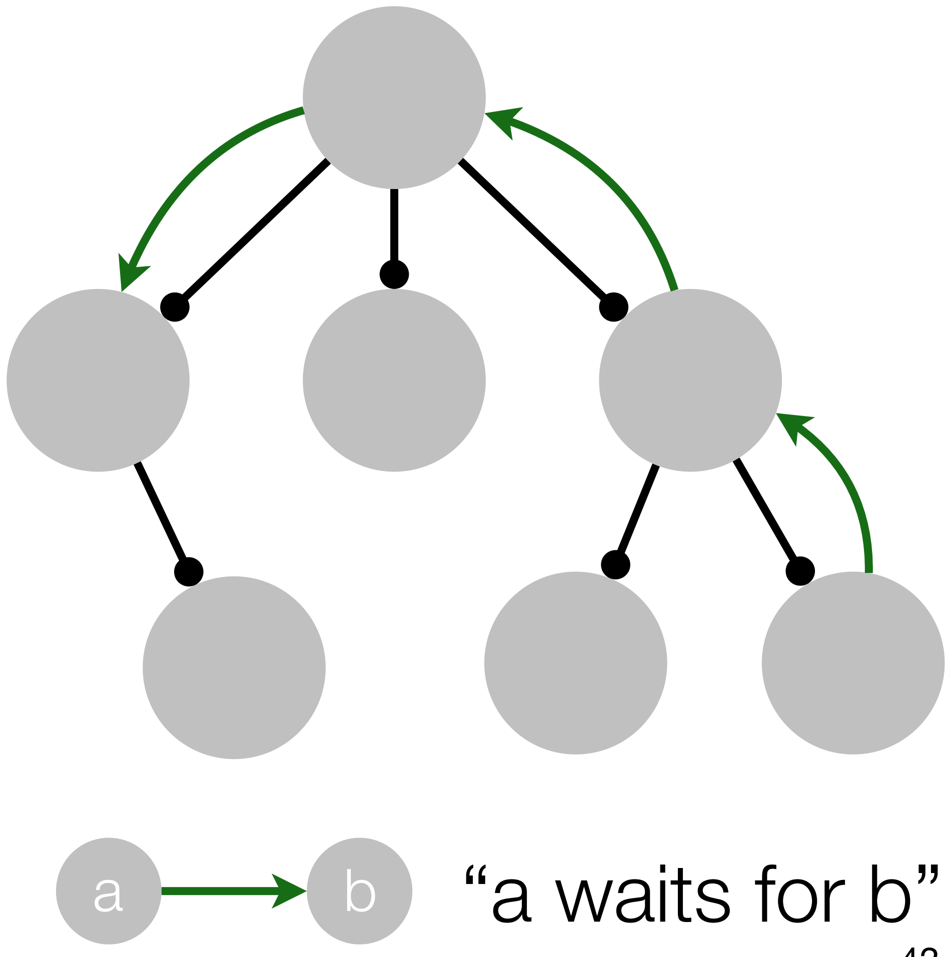

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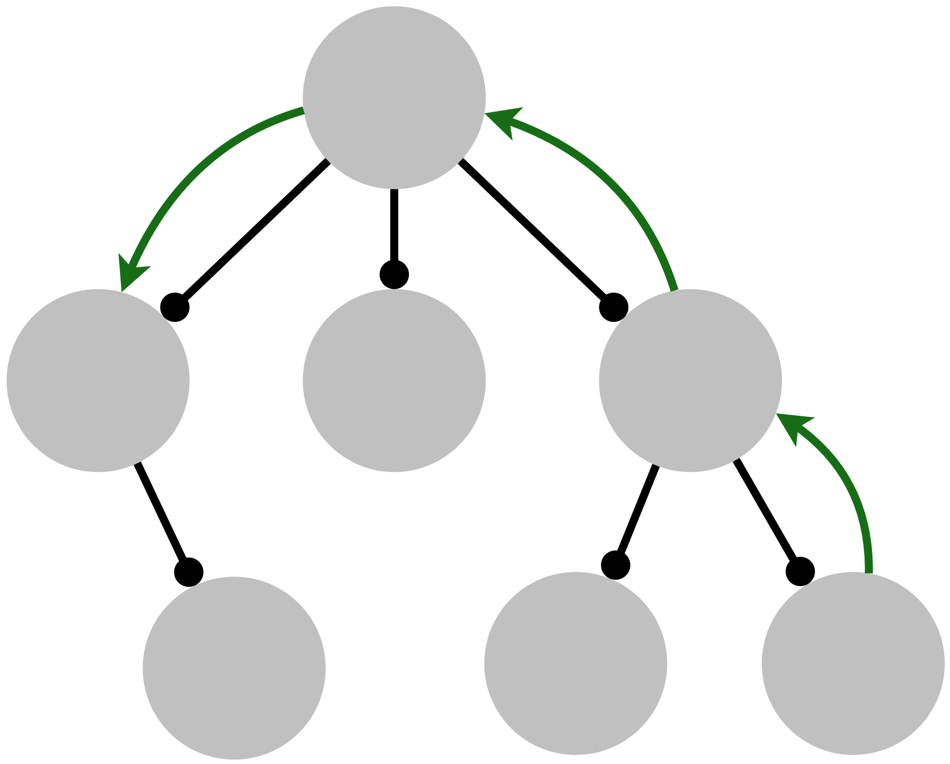

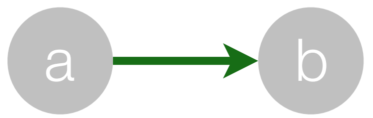

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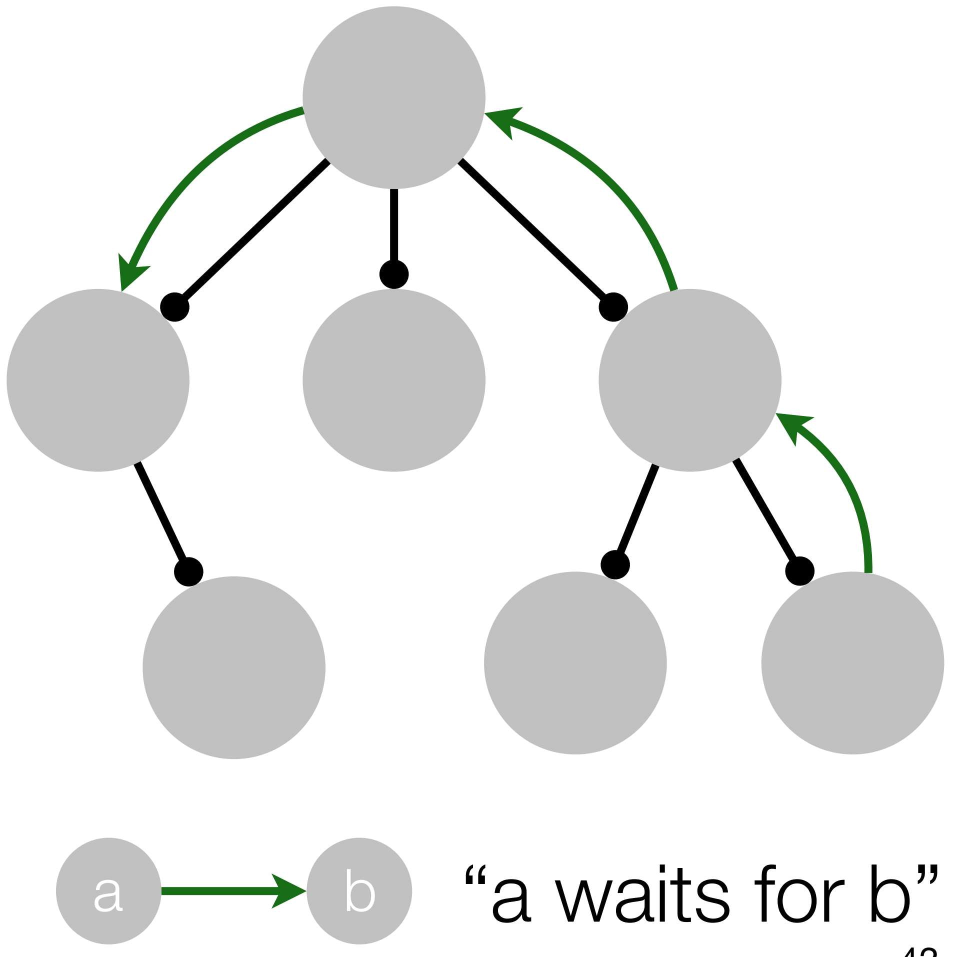

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➡ Unrestricted proposition, can be used arbitrarily often

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Ψ structural context (weakening and contraction)

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What is the computational meaning of “of course”?

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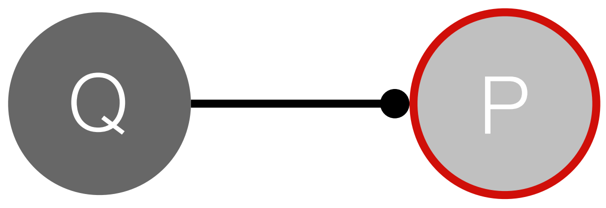

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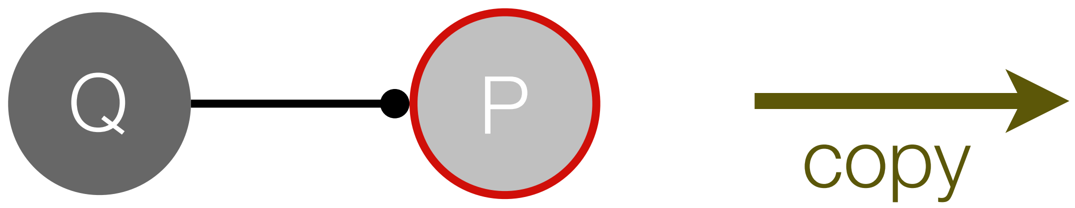

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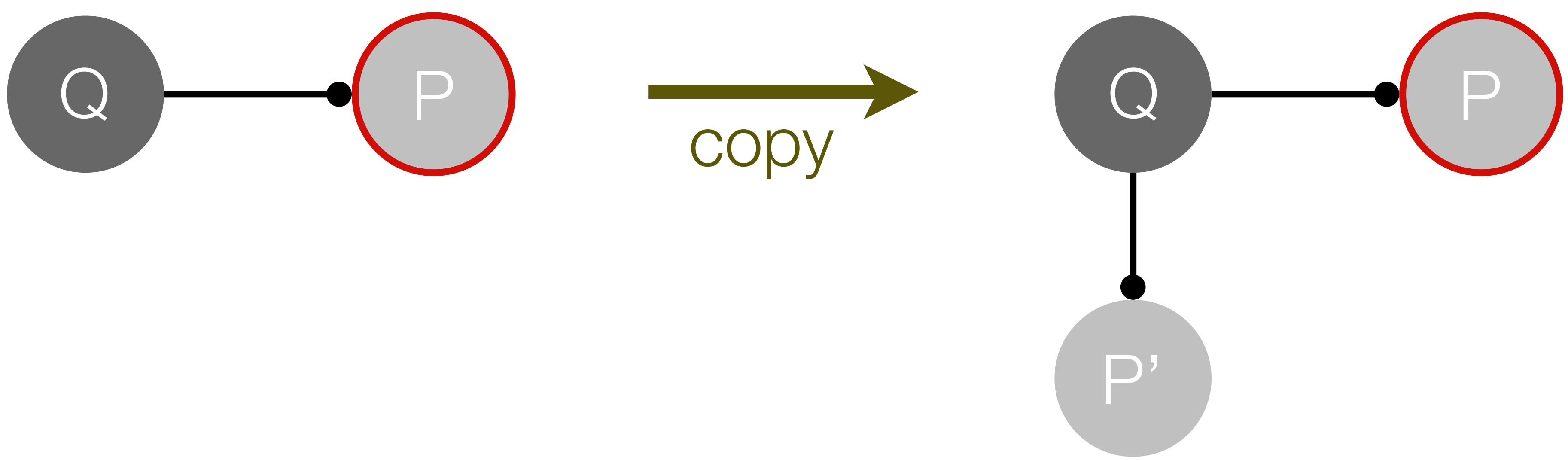

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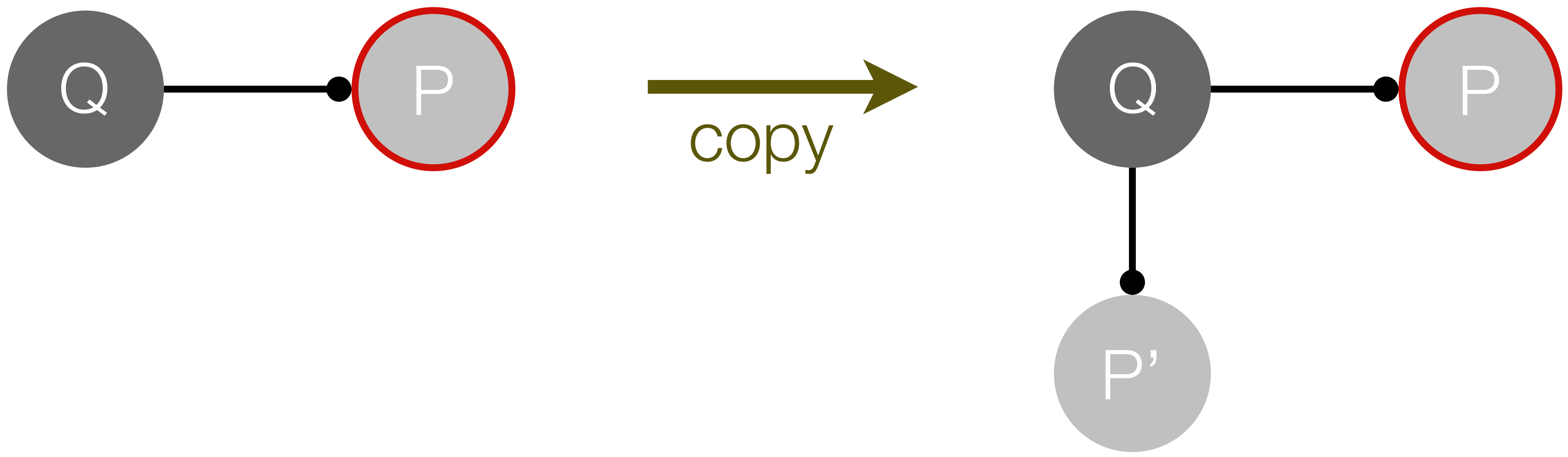

Copy: obtain linear copy from linear server process

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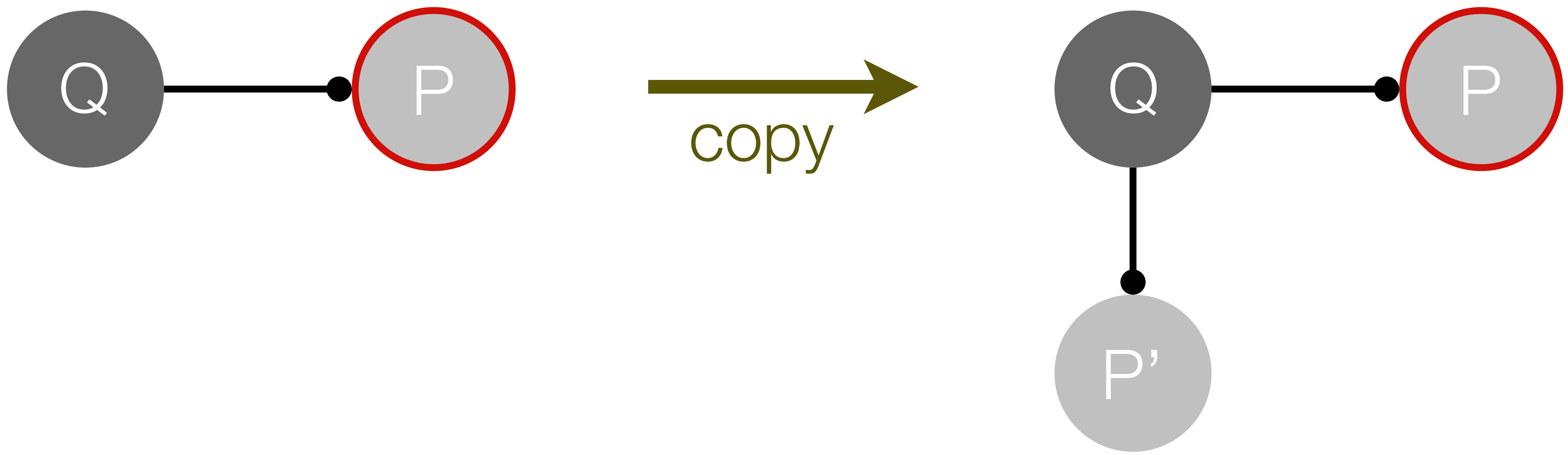

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- Corresponds to replication in the pi-calculus

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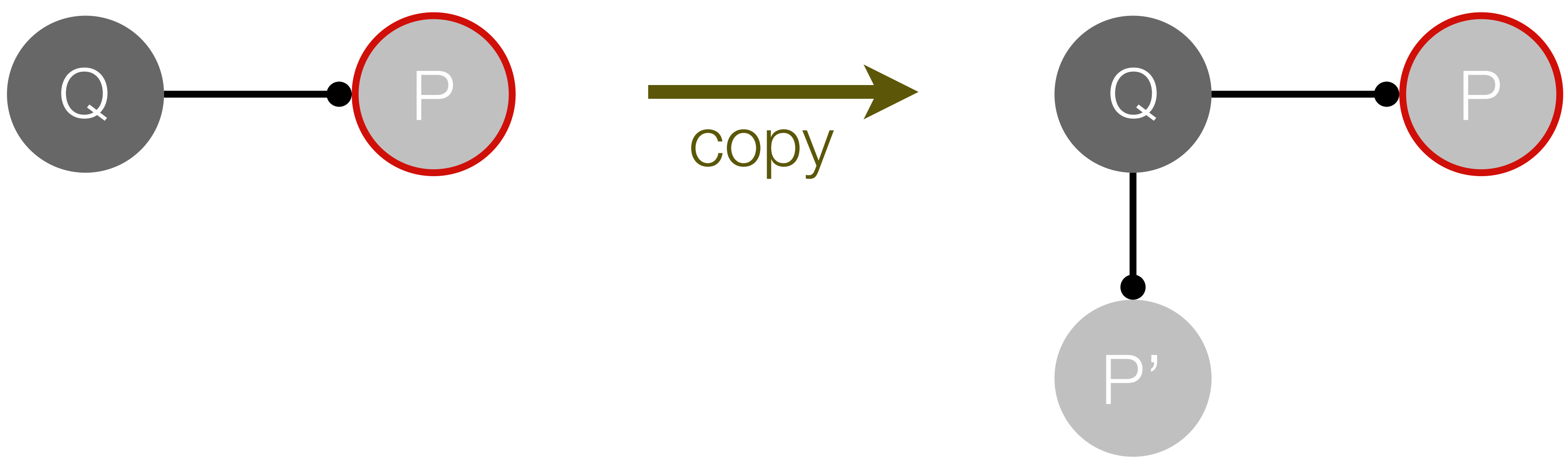

Copy: obtain linear copy from linear server process

Corresponds to replication in the pi-calculus

For some applications, a copying semantics is sufficient. For other applications, sharing is necessary.

Manifest sharing

Manifest sharing - key ideas [Balzer & Pfenning 2017]

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Acquire-release

- Multiple aliases (shared channels) to a process permitted, but communication requires exclusive access.

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- Enrich type structure with adjoint modalities to prescribe acquire and release points.

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- Invariant guaranteeing that a process is released to the same type at which it was previously acquired, should it be released at all.
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→ let's look at each in turn

Programmatic working of acquire-release

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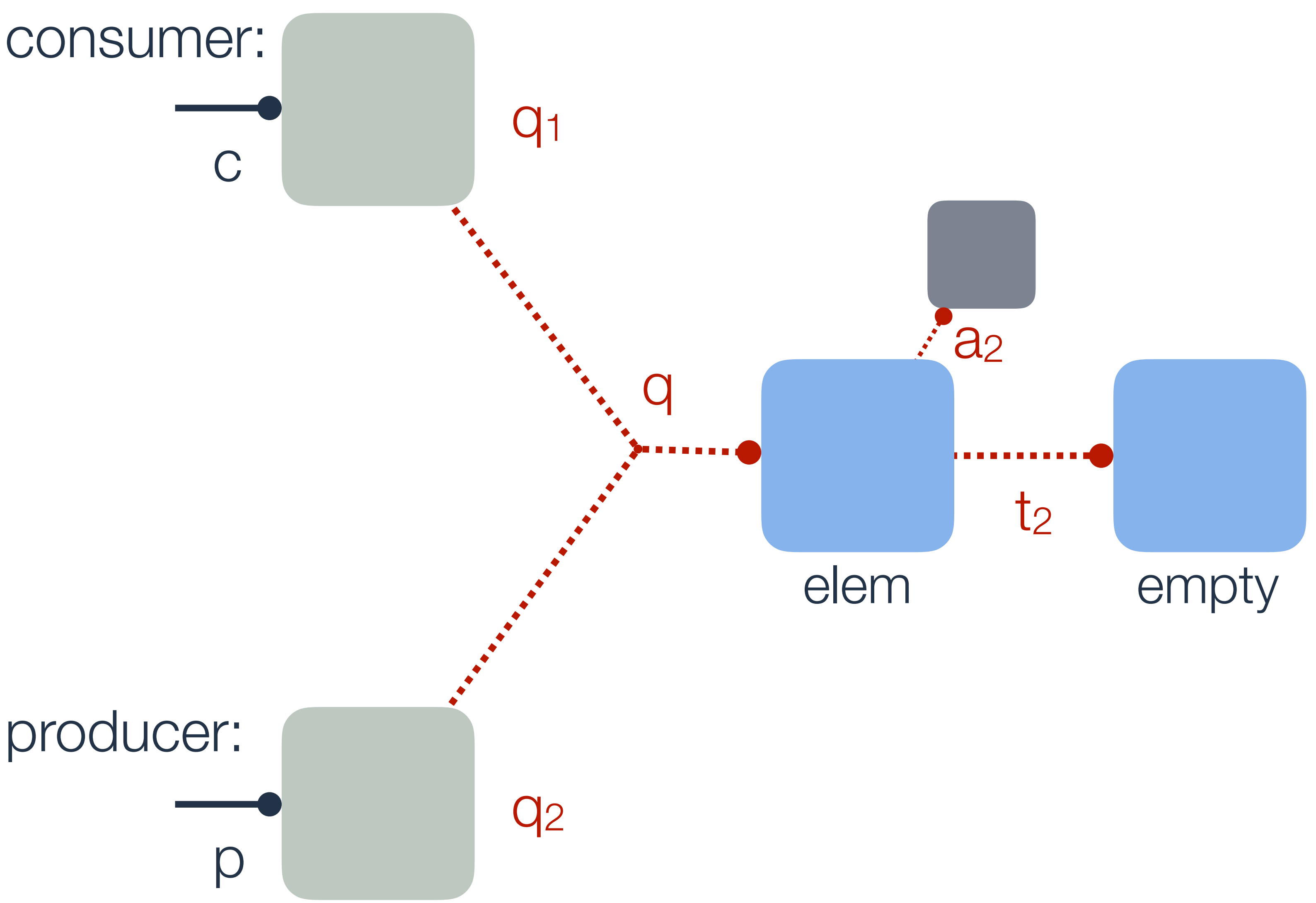
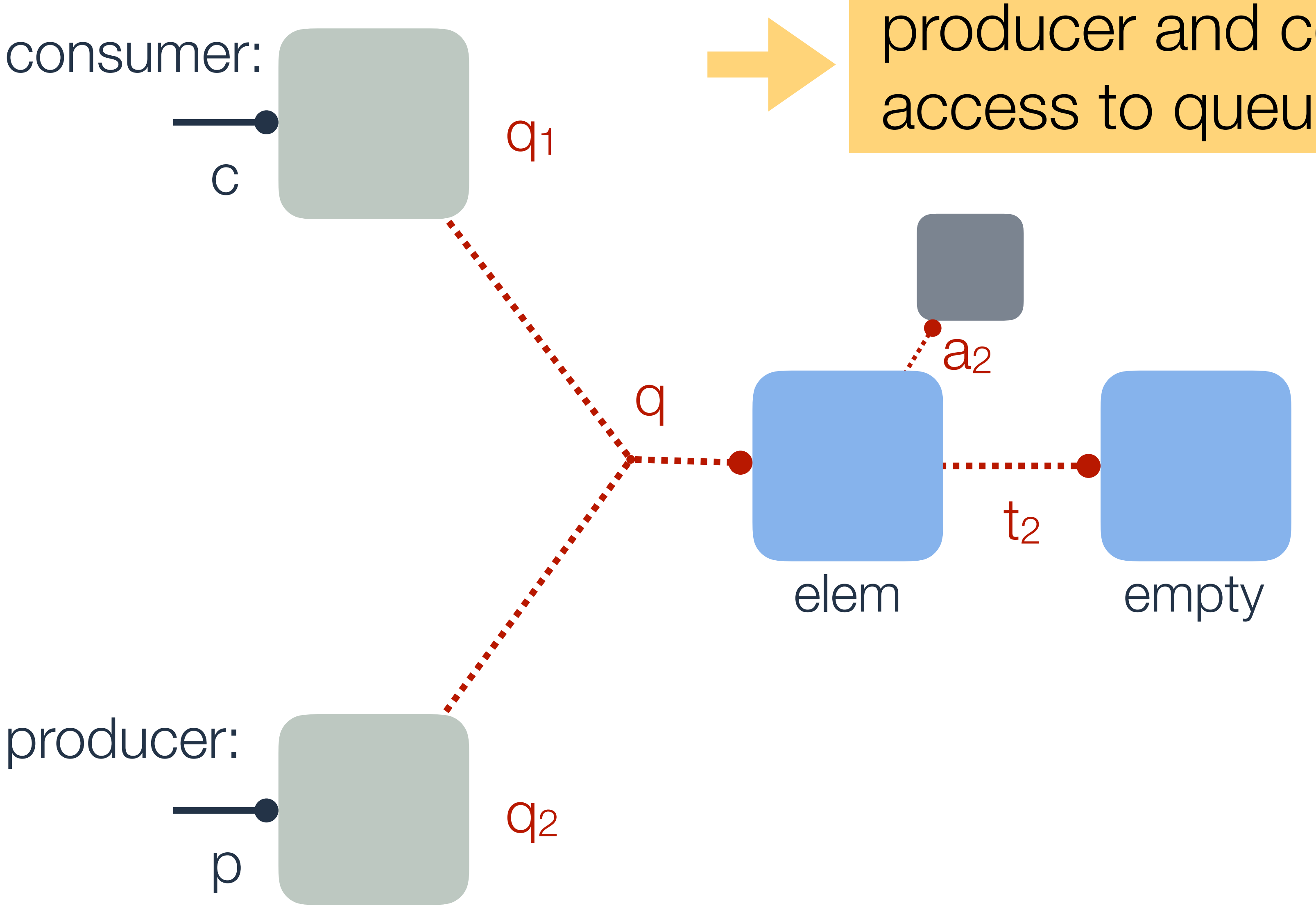


Legend: linear channel x — shared channel y

Programmatic working of acquire-release

producer and consumer contend for access to queue



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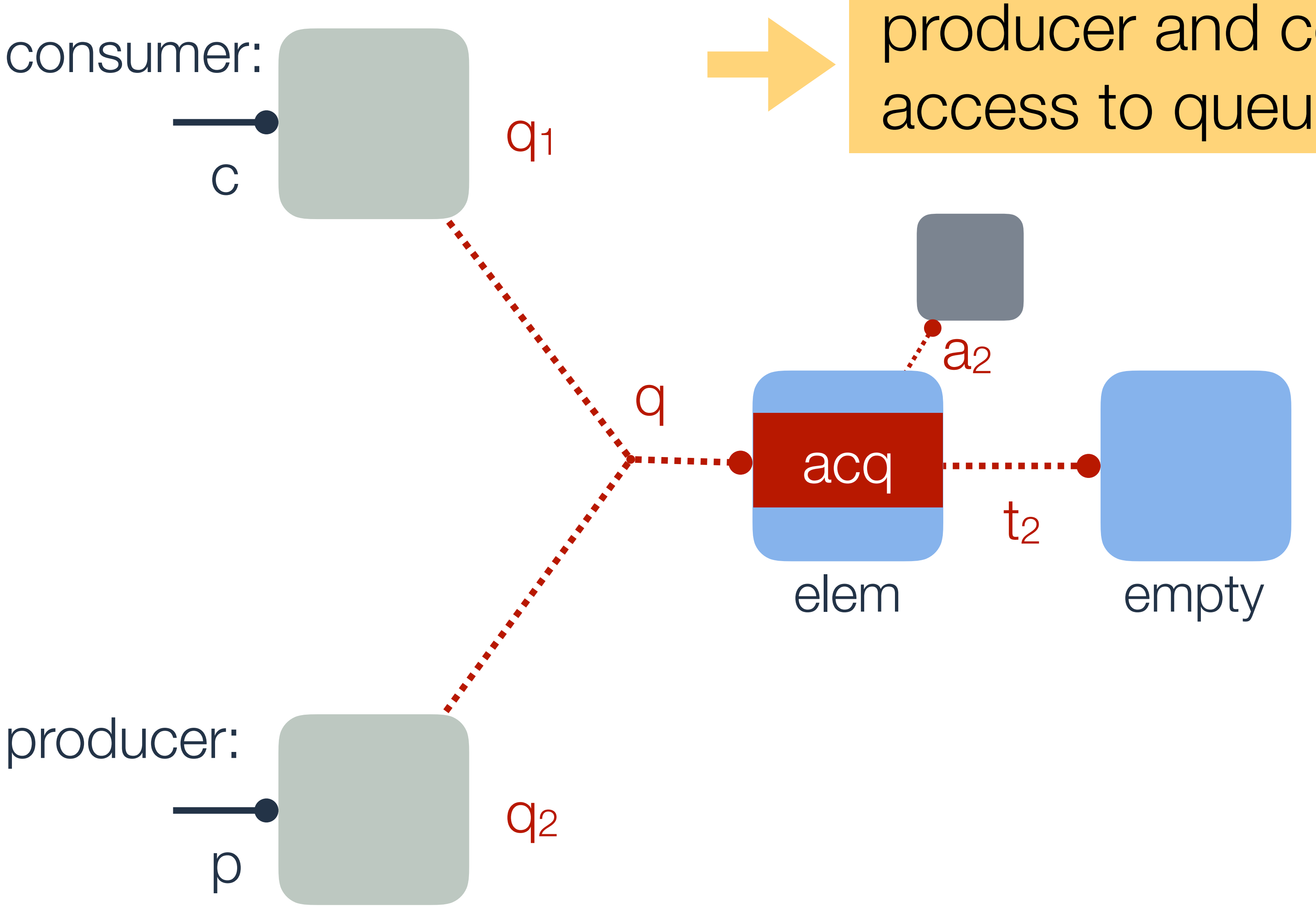

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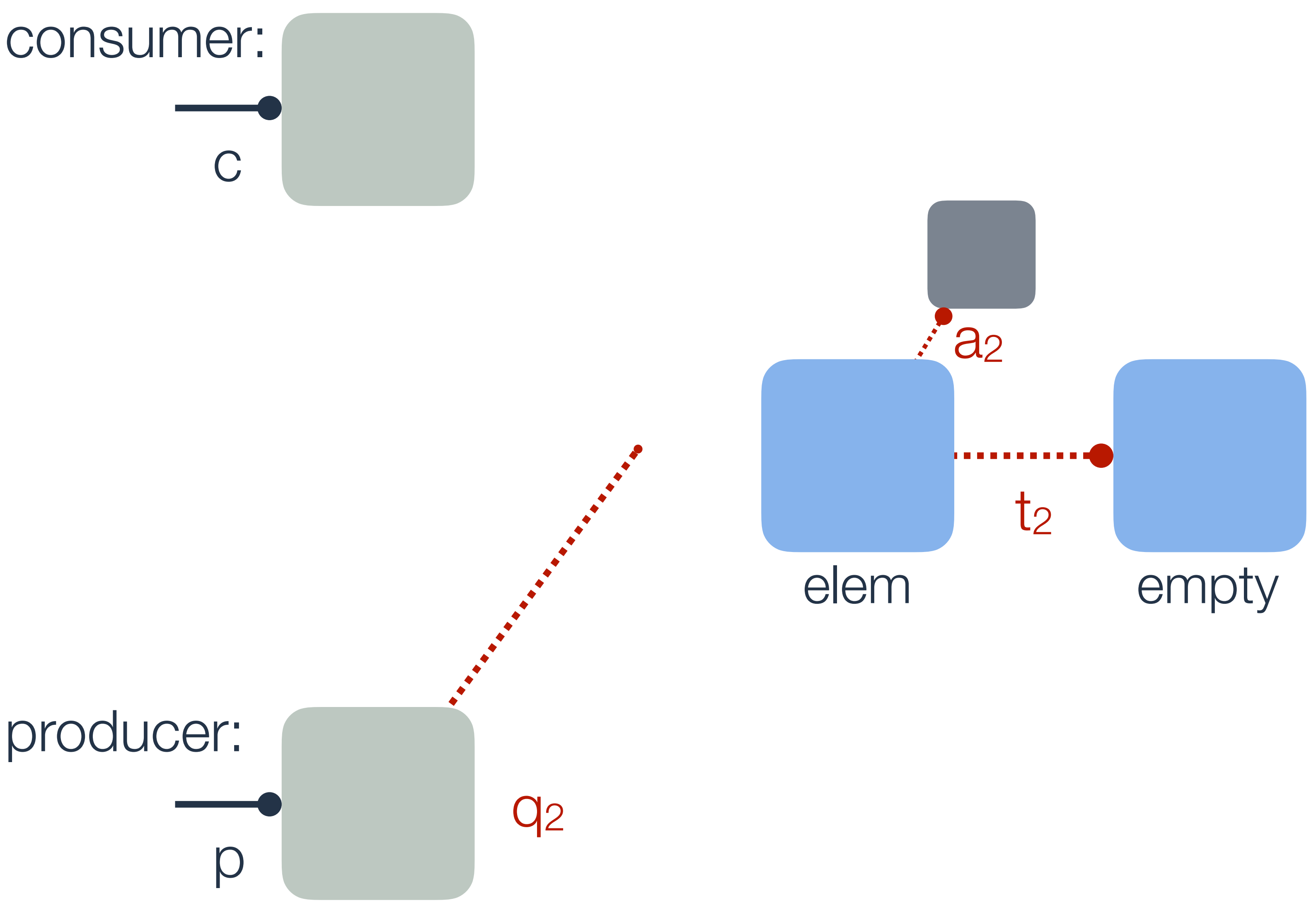

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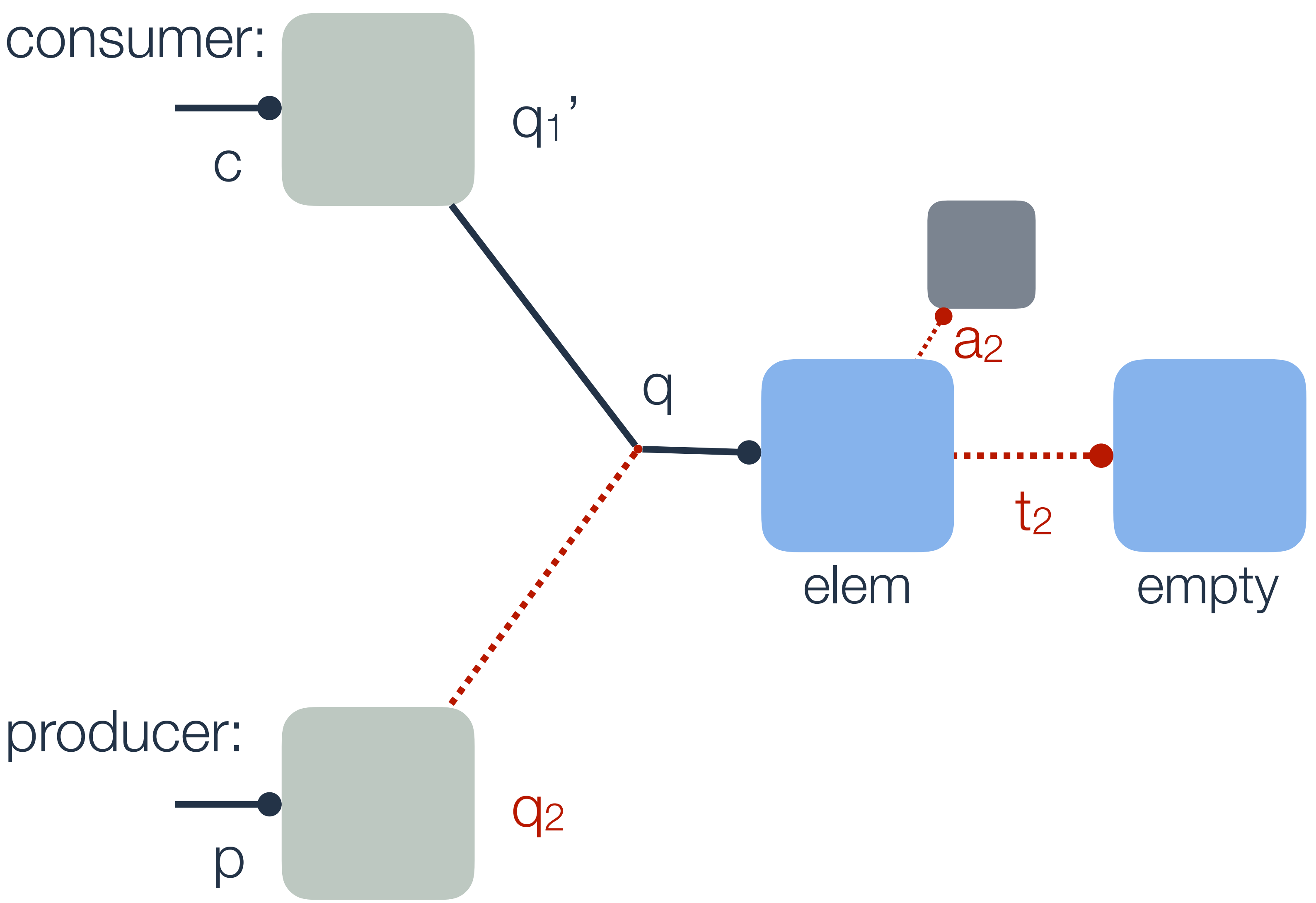

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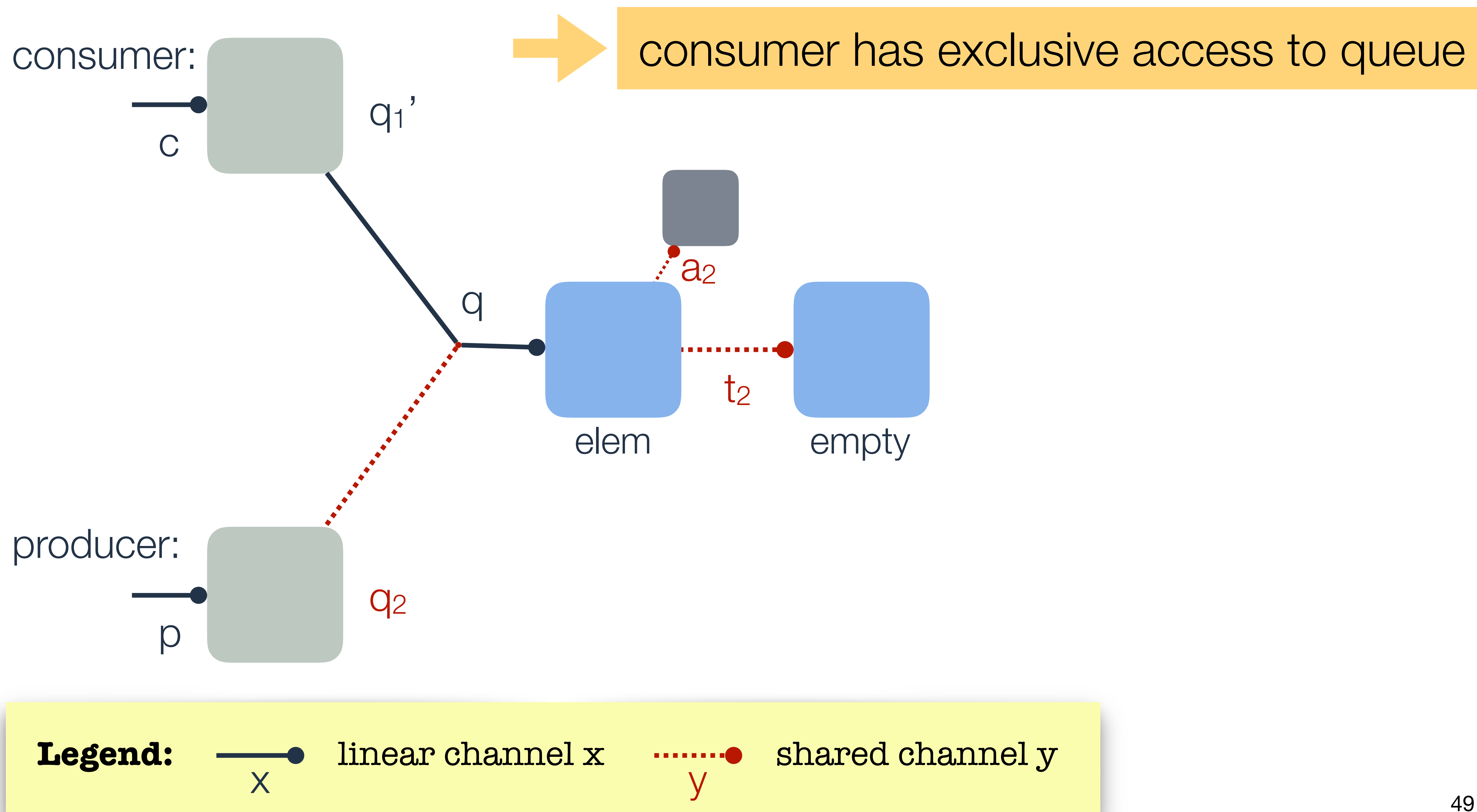
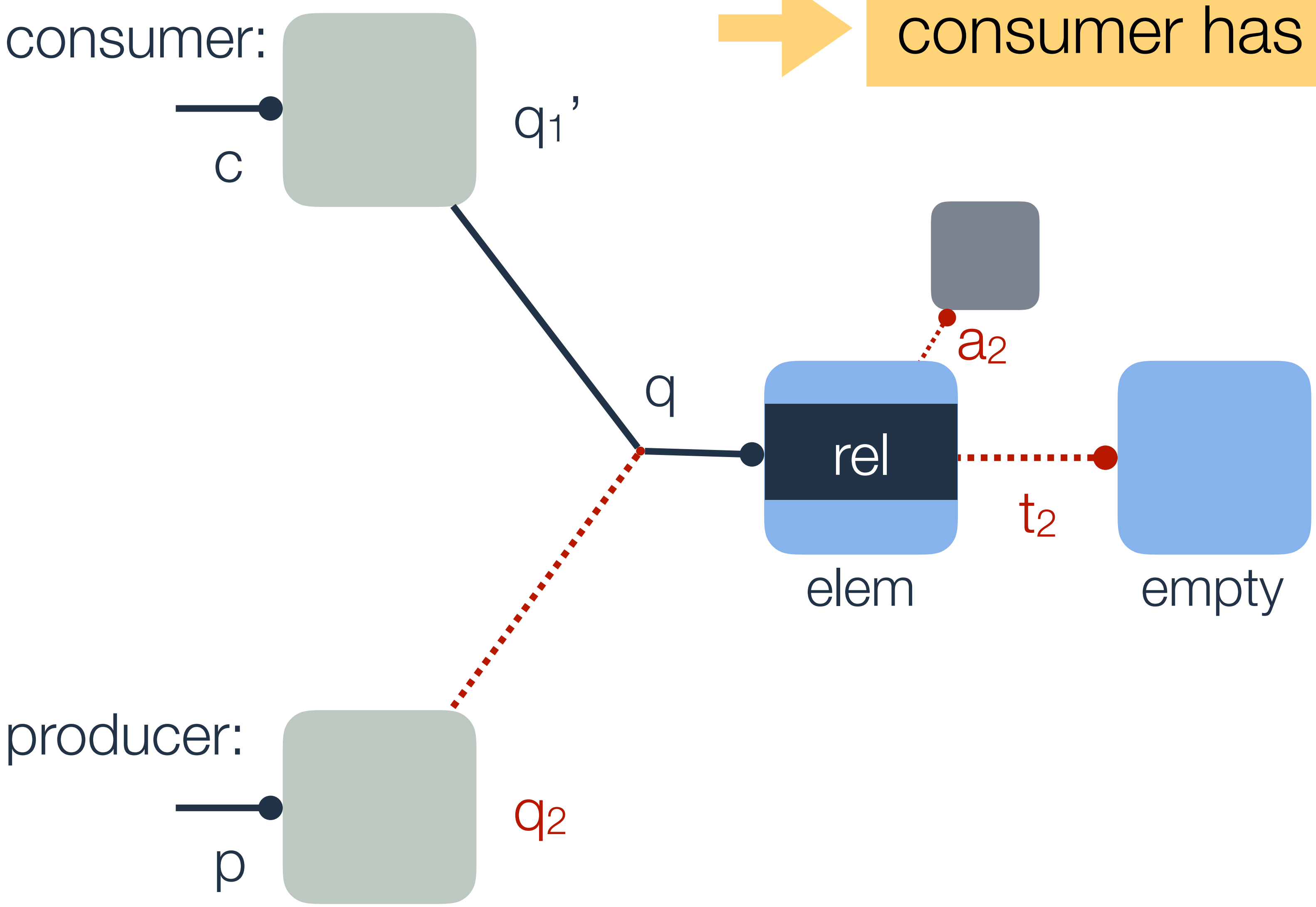


Programmatic working of acquire-release

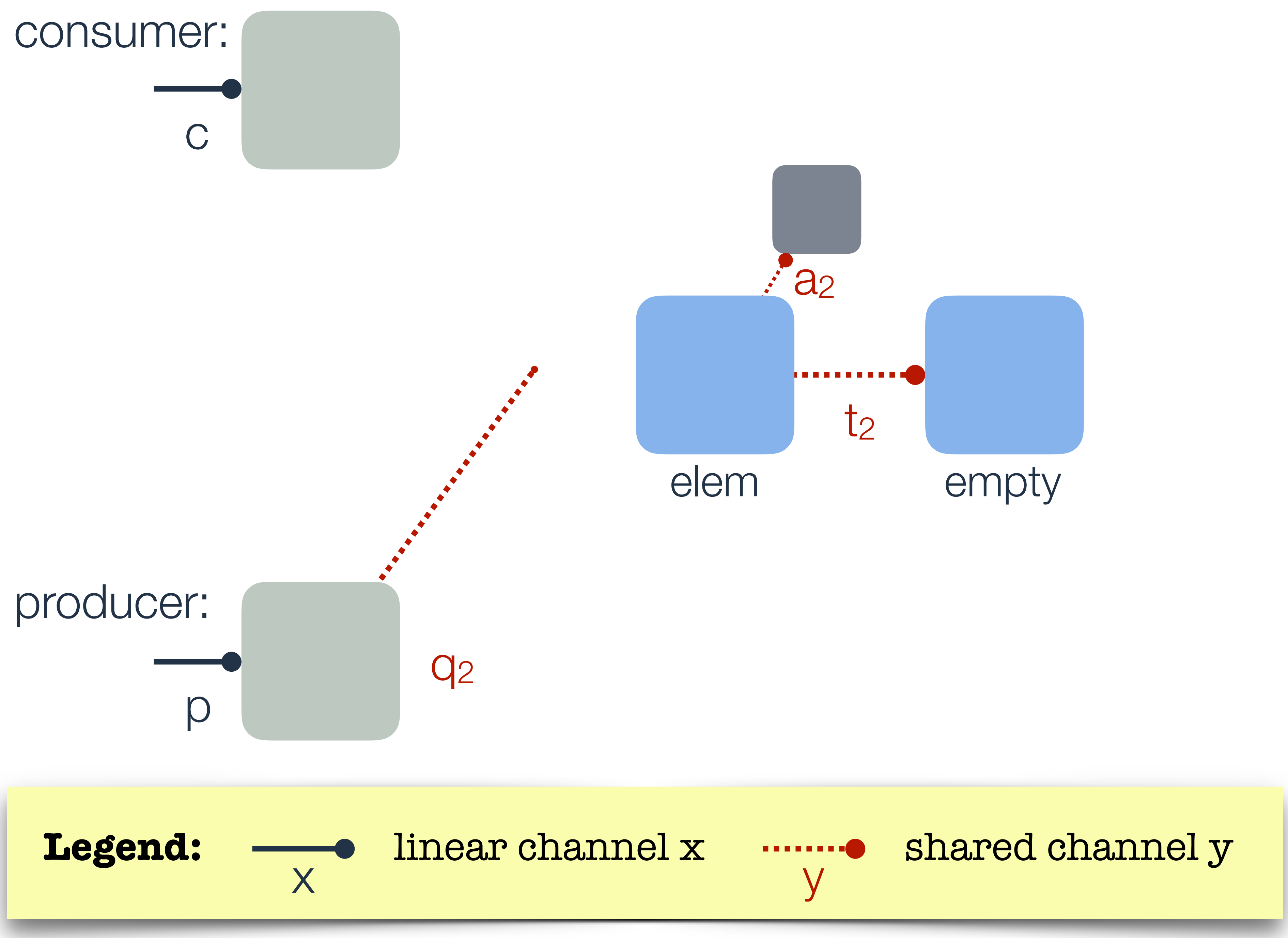
consumer has exclusive access to queue



Legend: linear channel x shared channel y

x y

Programmatic working of acquire-release



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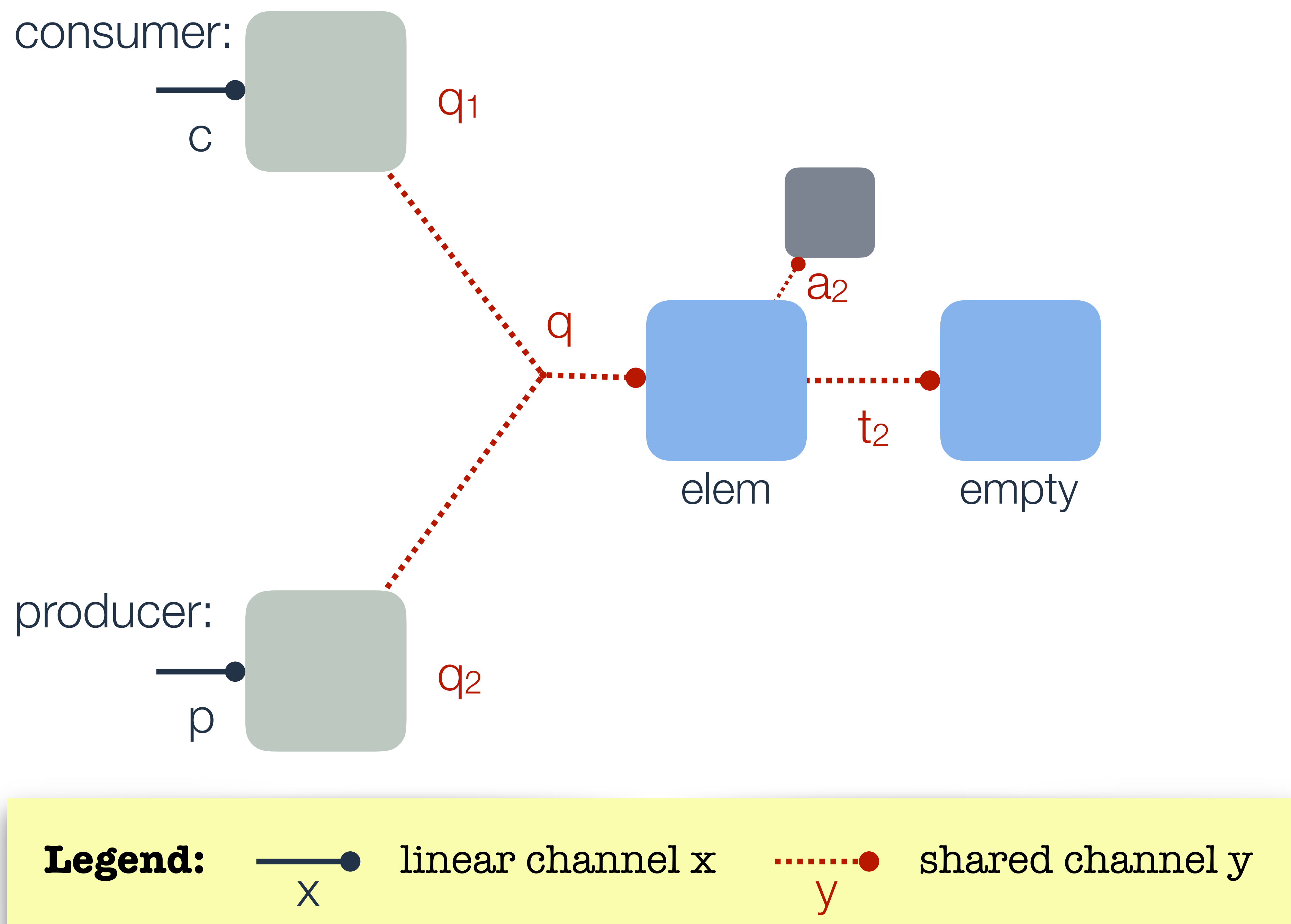
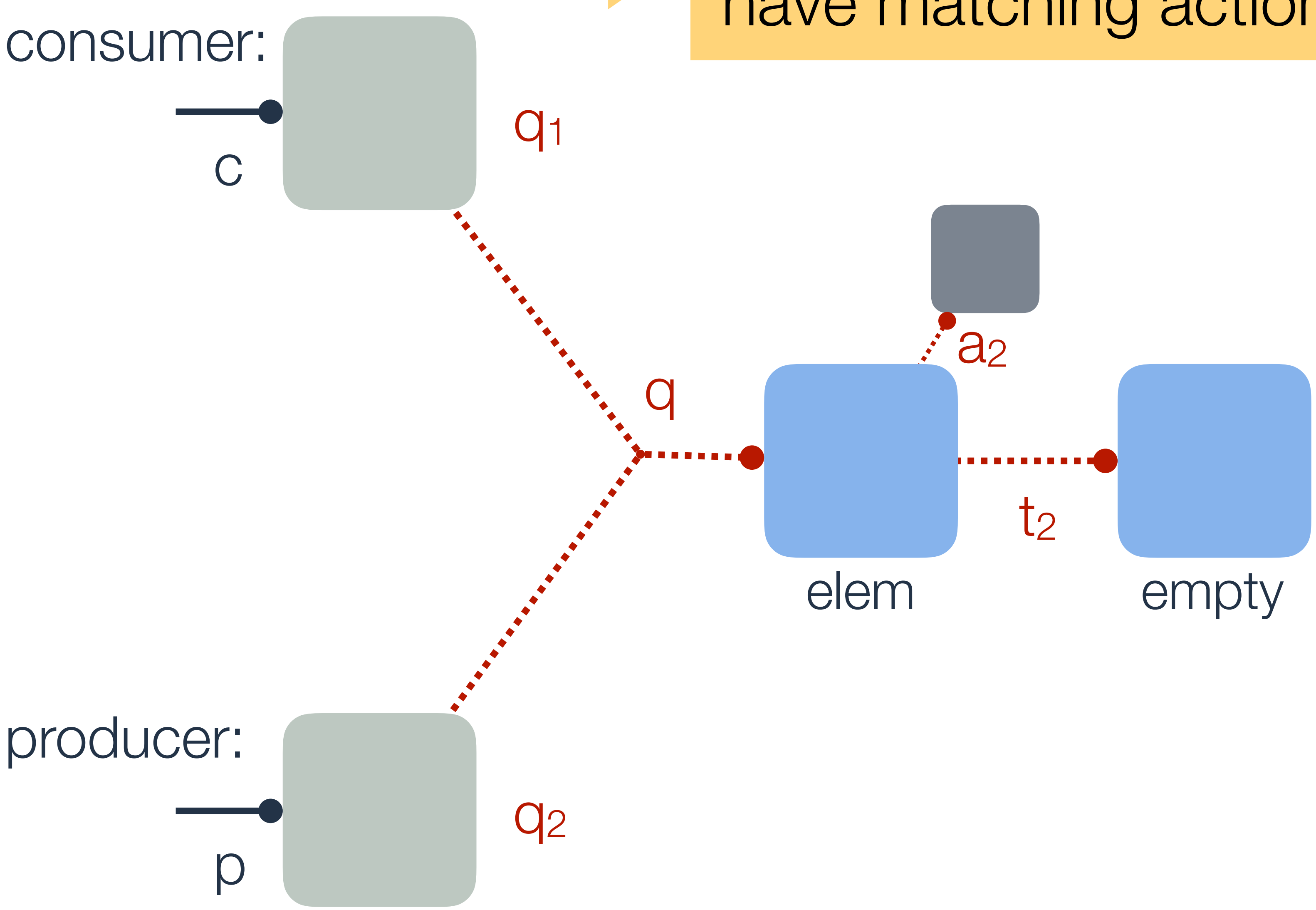


Programmatic working of acquire-release

Note: acquire and release messages by client have matching actions by provider



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Programmatic working of acquire-release

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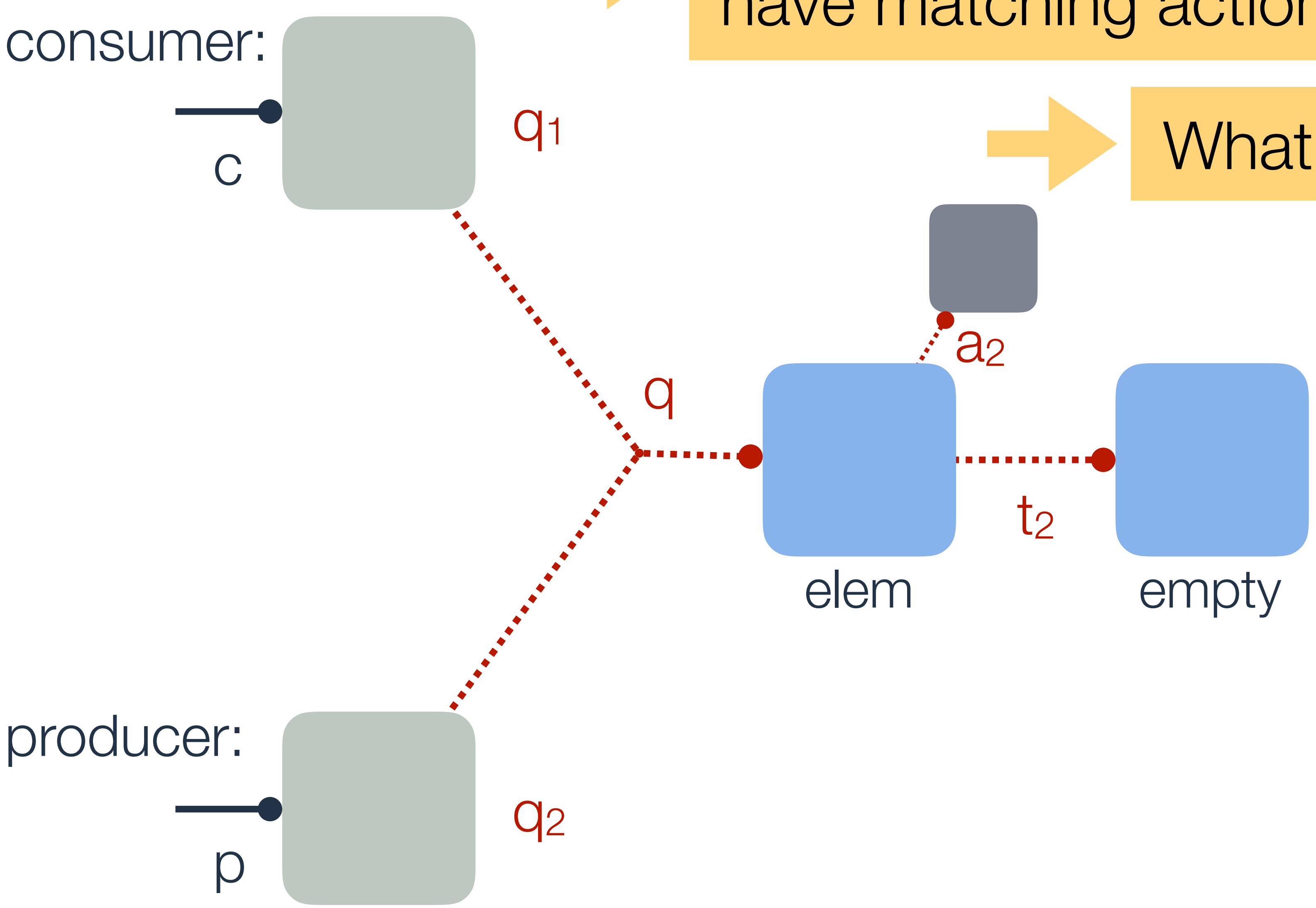
What should be the type of q?



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Manifestation of acquire-release in typing

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release release

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release release

Legend: process in linear phase process in shared phase

Adjoint stratification of session types[1]

1 Generalization of [Benton 1994; Reed 2009; Pfenning and Griffith 2015]

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stratify session types into a linear and shared layer, s.t. S > L

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$$A_{\mathsf{S}} \quad \triangleq$$

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- stratify session types into a linear and shared layer, s.t. S > L
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Adjoint stratification of session types[1]

- stratify session types into a linear and shared layer, s.t. S > L
- connect layers with modalities going back and forth

$$
\begin{array}{lll}
+ & & \\
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \\
- & &
\end{array}
$$

- support of communication of shared channels, using Π for receive and $\exists$ for send[2]

1 Generalization of [Benton 1994; Reed 2009; Pfenning and Griffith 2015]

2 Based on [Cervesato et al. 2002; Watkins et al. 2001]

Adjoint stratification of session types[1]

stratify session types into a linear and shared layer, s.t. S > L

connect layers with modalities going back and forth

+

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$\begin{aligned} A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq\ & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\ & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \end{aligned}$$

-

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Adjoint stratification of session types[1]

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-

Shared queue

+

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid$$
$$\&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}$$

-

Shared queue

+

$$
\begin{array}{lcl}
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

-

$$
\begin{array}{lll}
\mathsf{queue}\ A_{\mathsf{S}} = & \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}. & \mathsf{queue}\ A_{\mathsf{S}}, \\
 & \quad \mathsf{deq} : \oplus\{\mathsf{none} : & \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\quad \mathsf{queue}\ A_{\mathsf{S}}\}\}
\end{array}
$$

Shared queue

+

$$
\begin{array}{lcl}
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
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 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

-

$$
\begin{array}{lll}
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 & \quad \mathsf{deq} : \oplus\{\mathsf{none} : & \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \ \mathsf{queue}\ A_{\mathsf{S}}\}\}
\end{array}
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Shared queue

+

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\begin{array}{lcl}
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\end{array}
$$

-

$$
\begin{array}{l}
\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}. \quad \mathsf{queue}\ A_{\mathsf{S}}, \\
\qquad\qquad \mathsf{deq} : \oplus\{\mathsf{none} : \quad \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}. \quad \mathsf{queue}\ A_{\mathsf{S}}\}\}
\end{array}
$$

Shared queue

+

$$
\begin{array}{lcl}
A_{\mathsf{S}} & \triangleq & \uparrow^{\mathsf{S}}_{\mathsf{L}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow^{\mathsf{S}}_{\mathsf{L}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

-

$$
\begin{array}{l}
\mathsf{queue}\ A_{\mathsf{S}} = \uparrow^{\mathsf{S}}_{\mathsf{L}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow^{\mathsf{S}}_{\mathsf{L}} \mathsf{queue}\ A_{\mathsf{S}}, \\
\qquad\qquad\qquad \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow^{\mathsf{S}}_{\mathsf{L}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow^{\mathsf{S}}_{\mathsf{L}} \mathsf{queue}\ A_{\mathsf{S}}\}\}
\end{array}
$$

Shared queue

+

-

$$
\begin{array}{lcl}
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\end{array}
$$

$$
\begin{array}{l}
\mathsf{queue}\ A_{\mathsf{S}} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}, \\
\qquad \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}
\end{array}
$$

$\uparrow_{\mathsf{L}}^{\mathsf{S}}$ and $\downarrow_{\mathsf{L}}^{\mathsf{S}}$ prescribe acquire-release points, resp.

Equi-synchronizing session type

Equi-synchronizing session type

Invariant guaranteeing that a process is released to the same type at which it was previously acquired, should it be released at all.

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➡ Guarantees session fidelity w/o run-time checking upon acquire

Equi-synchronizing session type

Invariant guaranteeing that a process is released to the same type at which it was previously acquired, should it be released at all.

Guarantees session fidelity w/o run-time checking upon acquire

Examples of equi-synchronizing session types:

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}^{\mathsf{S}}_{\mathsf{L}}\ \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}^{\mathsf{S}}_{\mathsf{L}}\ \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

Equi-synchronizing session type

Invariant guaranteeing that a process is released to the same type at which it was previously acquired, should it be released at all.

Guarantees session fidelity w/o run-time checking upon acquire

Examples of equi-synchronizing session types:

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} &= \uparrow_{\mathsf{L}}^{\mathsf{S}} \&\{\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}, \\ &\qquad\qquad \mathsf{deq} : \oplus\{\mathsf{none} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

$$\begin{aligned}\mathsf{queue}\ A &= \&\{\mathsf{enq} : A \multimap \mathsf{queue}\ A, \\ &\qquad \mathsf{deq} : \oplus\{\mathsf{none} : \mathbf{1},\ \mathsf{some} : A \otimes \mathsf{queue}\ A\}\}\end{aligned}$$

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Invariant guaranteeing that a process is released to the same type at which it was previously acquired, should it be released at all.

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$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\, \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathsf{queue}\ A_{\mathsf{S}},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

$$\begin{aligned}\mathsf{queue}\ A = \&\{&\mathsf{enq} : A \multimap \mathsf{queue}\ A,\\ &\mathsf{deq} : \oplus\{\mathsf{none} : \mathbf{1},\ \mathsf{some} : A \otimes \mathsf{queue}\ A\}\}\end{aligned}$$

Non-equi-synchronizing:

$$\begin{aligned}\mathsf{queue}\ A_{\mathsf{S}} = {\uparrow}_{\mathsf{L}}^{\mathsf{S}}\, \&\{&\mathsf{enq} : \Pi x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathsf{queue}\ A_{\mathsf{S}},\\ &\mathsf{deq} : \oplus\{\mathsf{none} : {\downarrow}_{\mathsf{L}}^{\mathsf{S}}{\uparrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathbf{1},\ \mathsf{some} : \exists x{:}A_{\mathsf{S}}.\ {\downarrow}_{\mathsf{L}}^{\mathsf{S}}\,\mathsf{queue}\ A_{\mathsf{S}}\}\}\end{aligned}$$

Typing judgments

Typing judgments

$$
\begin{array}{lcl}
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

+

-

Typing judgments

+

-

$$
\begin{array}{lll}
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$	shared process P, providing session of type A_{S} along x_{S}, using channels in Γ
$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$	linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ
Γ	shared (structural) context
Δ	linear context

Typing judgments

+

$$
\begin{array}{lcl}
A_{\mathsf{S}} & \triangleq & \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}} \\
A_{\mathsf{L}}, B_{\mathsf{L}} & \triangleq & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\
 & & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}
\end{array}
$$

-

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$ — shared process P, providing session of type A_{S} along x_{S}, using channels in Γ

$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$ — linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ

Γ — shared (structural) context

Δ — linear context

Typing judgments

+

$$A_{\mathsf{S}} \triangleq {\uparrow}_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$\begin{aligned} A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq\ & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\ & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid {\downarrow}_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \end{aligned}$$

-

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$	shared process P, providing session of type A_{S} along x_{S}, using channels in Γ
$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$	linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ
Γ	shared (structural) context
Δ	linear context

Typing judgments

+	A_{S}	$\triangleq$	$\uparrow^{\mathsf{S}}_{\mathsf{L}} A_{\mathsf{L}}$
	$A_{\mathsf{L}}, B_{\mathsf{L}}$	$\triangleq$	$\oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid$
-			$\&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow^{\mathsf{S}}_{\mathsf{L}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}}$

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$	shared process P, providing session of type A_{S} along x_{S}, using channels in Γ
$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$	linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ
Γ	shared (structural) context
Δ	linear context

Typing judgments

+

A_S	$\triangleq$	$\uparrow_L^S A_L$
A_L, B_L	$\triangleq$	$\oplus\{\overline{l : A_L}\} \mid A_L \otimes B_L \mid \mathbf{1} \mid \exists x{:}A_S.\, B_L \mid$
		$\&\{\overline{l : A_L}\} \mid A_L \multimap B_L \mid \downarrow_L^S A_S \mid \Pi x{:}A_S.\, B_L$

-

$\Gamma \vdash_\Sigma P :: (x_S : A_S)$ shared process P, providing session of type A_S along x_S, using channels in Γ

$\Gamma;\ \Delta \vdash_\Sigma P :: (x_L : A_L)$ linear process P, providing session of type A_L along x_L, using channels in Γ and Δ

Γ shared (structural) context

Δ linear context

Typing judgments

+

$$A_{\mathsf{S}} \triangleq \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}$$

$$\begin{aligned} A_{\mathsf{L}}, B_{\mathsf{L}} \triangleq\ & \oplus\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \otimes B_{\mathsf{L}} \mid \mathbf{1} \mid \exists x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \mid \\ & \&\{\overline{l : A_{\mathsf{L}}}\} \mid A_{\mathsf{L}} \multimap B_{\mathsf{L}} \mid \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \mid \Pi x{:}A_{\mathsf{S}}.\, B_{\mathsf{L}} \end{aligned}$$

-

$\Gamma \vdash_{\Sigma} P :: (x_{\mathsf{S}} : A_{\mathsf{S}})$	shared process P, providing session of type A_{S} along x_{S}, using channels in Γ
$\Gamma;\ \Delta \vdash_{\Sigma} P :: (x_{\mathsf{L}} : A_{\mathsf{L}})$	linear process P, providing session of type A_{L} along x_{L}, using channels in Γ and Δ
Γ	shared (structural) context
Δ	linear context

Typing of acquire

Typing of acquire

$$\frac{\Gamma, x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}{\uparrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{L})$$

Typing of acquire

$$\frac{\Gamma, \boxed{x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}};\ \Delta, \boxed{x_{\mathsf{L}} : A_{\mathsf{L}}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, \boxed{x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\text{T-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\text{L})$$

Typing of acquire

$$\frac{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

$$\frac{\Gamma;\ \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept}\ x_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} : \uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})}\ (\mathrm{T}\text{-}\uparrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{R})$$

Typing of acquire

$$\frac{\Gamma, x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta, x_{\mathsf{L}} : A_{\mathsf{L}} \vdash_{\Sigma} Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma, x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}};\ \Delta \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{acquire}\ x_{\mathsf{S}}\ ; Q_{x_{\mathsf{L}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}{\uparrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{L})$$

$$\frac{\Gamma;\ \cdot \vdash_{\Sigma} P_{x_{\mathsf{L}}} :: (x_{\mathsf{L}} : A_{\mathsf{L}})}{\Gamma \vdash_{\Sigma} x_{\mathsf{L}} \leftarrow \mathsf{accept}\ x_{\mathsf{S}}\ ; P_{x_{\mathsf{L}}} :: (x_{\mathsf{S}} :\uparrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{L}})}\ (\mathrm{T}\text{-}{\uparrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{R})$$

Typing of release

Typing of release

$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}};\ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma;\ \Delta, x_{\mathsf{L}} : \downarrow^{\mathsf{S}}_{\mathsf{L}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release}\ x_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\downarrow^{\mathsf{S}}_{\mathsf{L}}\mathrm{L})$$

Typing of release

$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}};\ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma;\ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release}\ x_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}\downarrow_{\mathsf{L}}^{\mathsf{S}}\mathrm{L})$$

Typing of release

$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}};\ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma;\ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release}\ x_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}{\downarrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{L})$$

$$\frac{\Gamma \vdash_{\Sigma} P_{x_{\mathsf{S}}} :: (x_{\mathsf{S}} : A_{\mathsf{S}})}{\Gamma;\ \cdot \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{detach}\ x_{\mathsf{L}}\ ; P_{x_{\mathsf{S}}} :: (x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}})}\ (\mathrm{T}\text{-}{\downarrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{R})$$

Typing of release

$$\frac{\Gamma, x_{\mathsf{S}} : A_{\mathsf{S}};\ \Delta \vdash_{\Sigma} Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}{\Gamma;\ \Delta, x_{\mathsf{L}} : \downarrow_{\mathsf{L}}^{\mathsf{S}} A_{\mathsf{S}} \vdash_{\Sigma} x_{\mathsf{S}} \leftarrow \mathsf{release}\ x_{\mathsf{L}}\ ; Q_{x_{\mathsf{S}}} :: (z_{\mathsf{L}} : C_{\mathsf{L}})}\ (\mathrm{T}\text{-}{\downarrow_{\mathsf{L}}^{\mathsf{S}}}\mathrm{L})$$

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Let’s program in Concurrent C0!

What about type safety?

What about type safety?

Preservation (aka session fidelity)

What about type safety?

Preservation (aka session fidelity)

- acquire-release and equi-synchronizing invariant guarantee that every linear process has a unique client

What about type safety?

Preservation (aka session fidelity)

- acquire-release and equi-synchronizing invariant guarantee that every linear process has a unique client
- state-altering exchange only along linear channels

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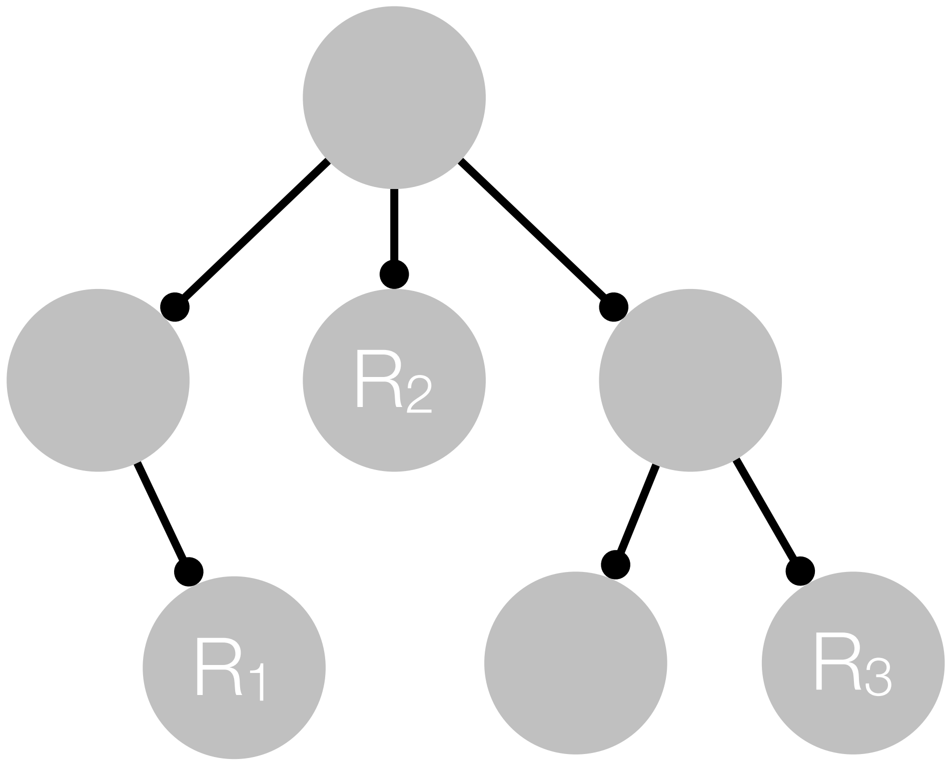

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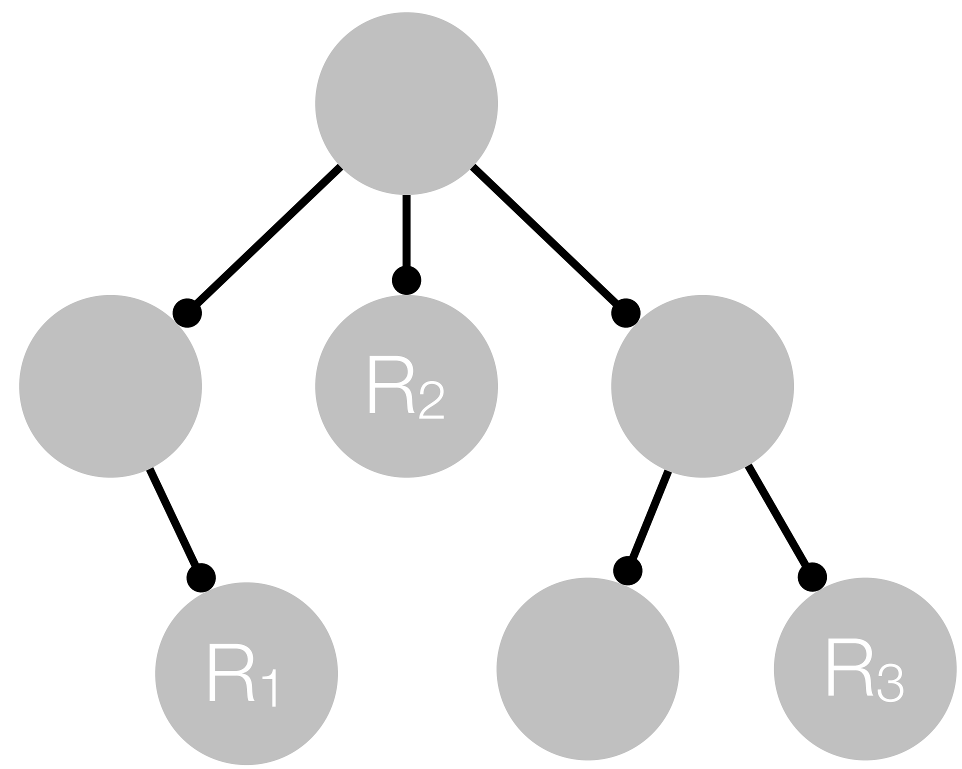

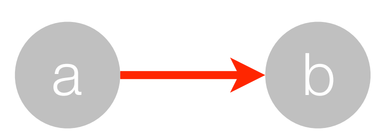

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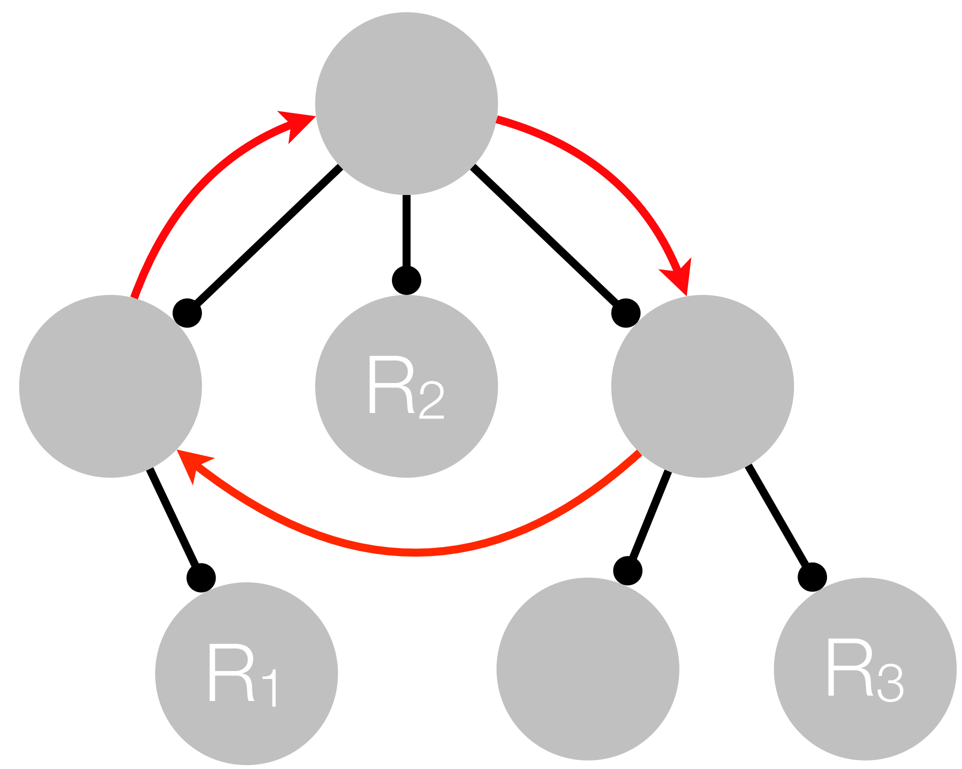

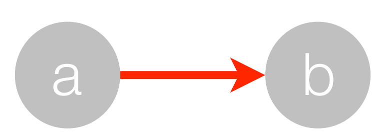

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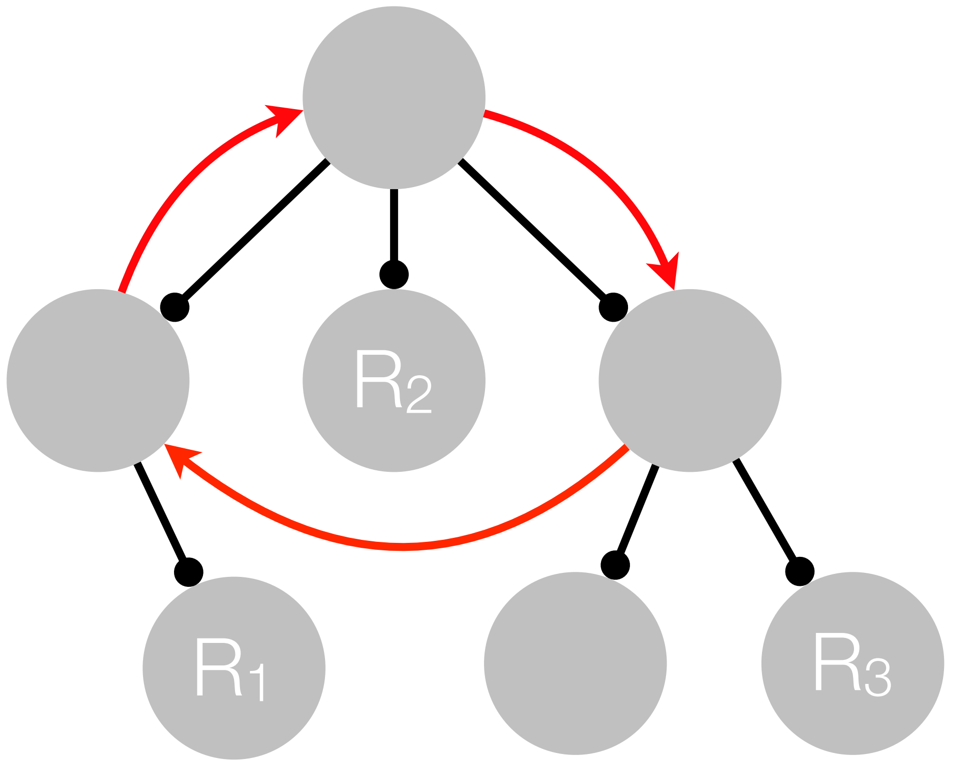

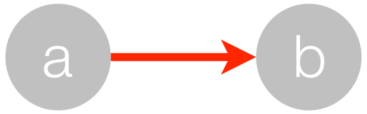

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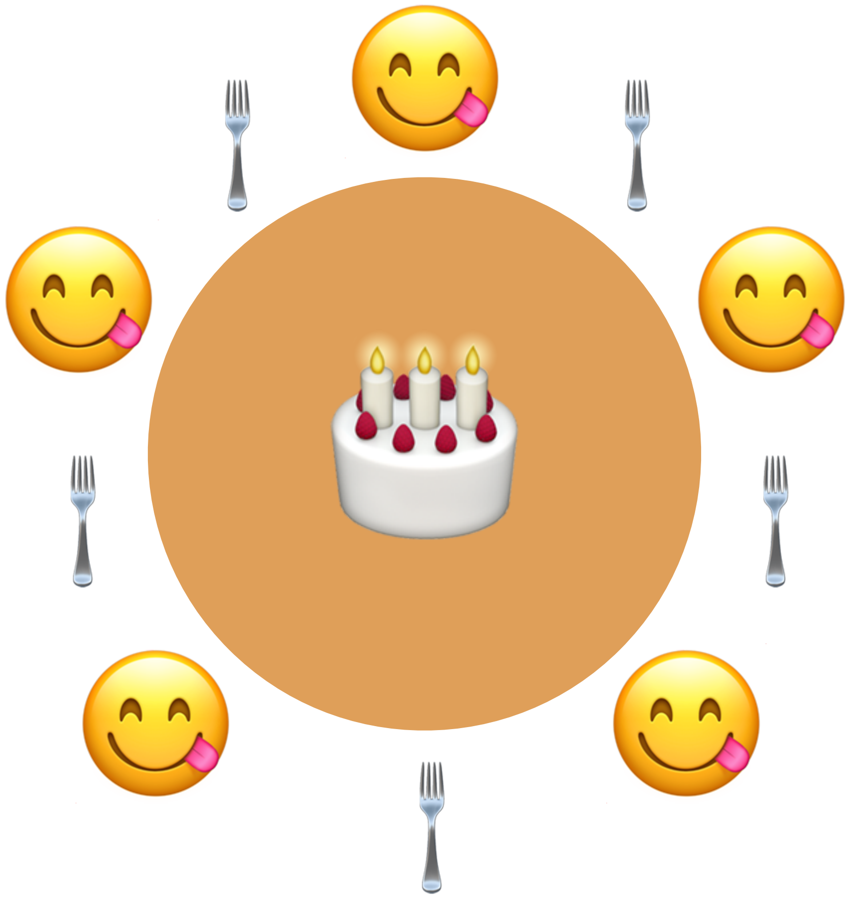

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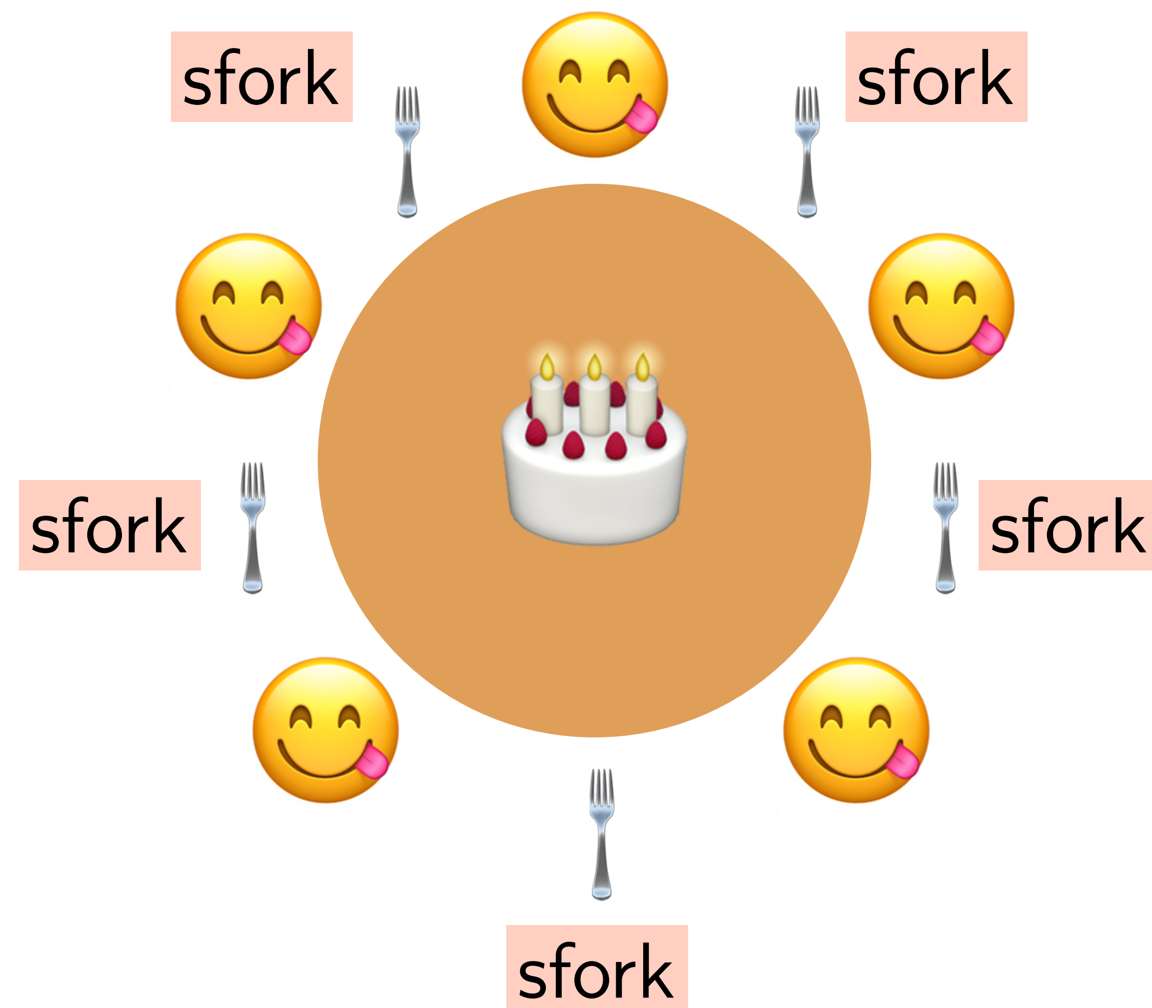

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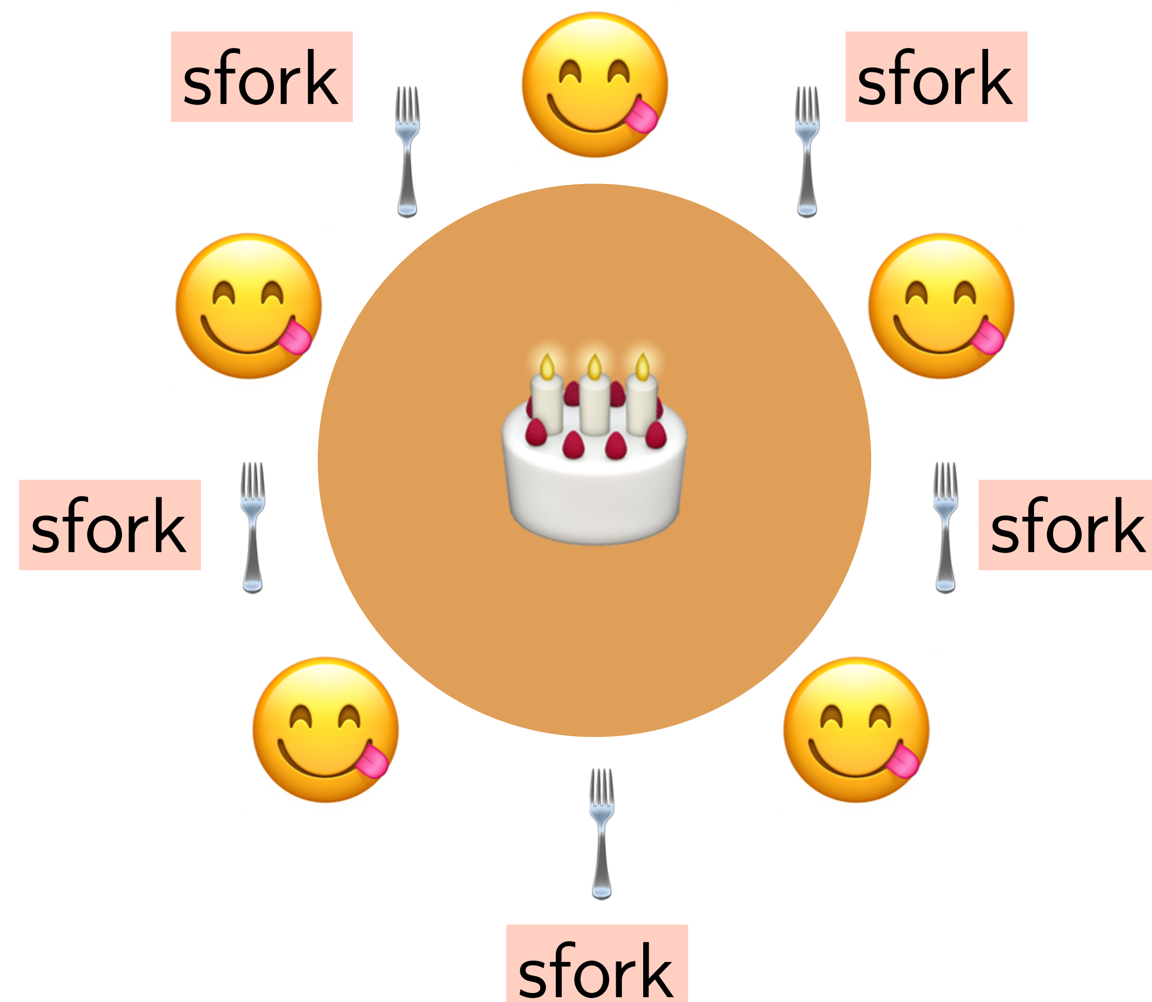

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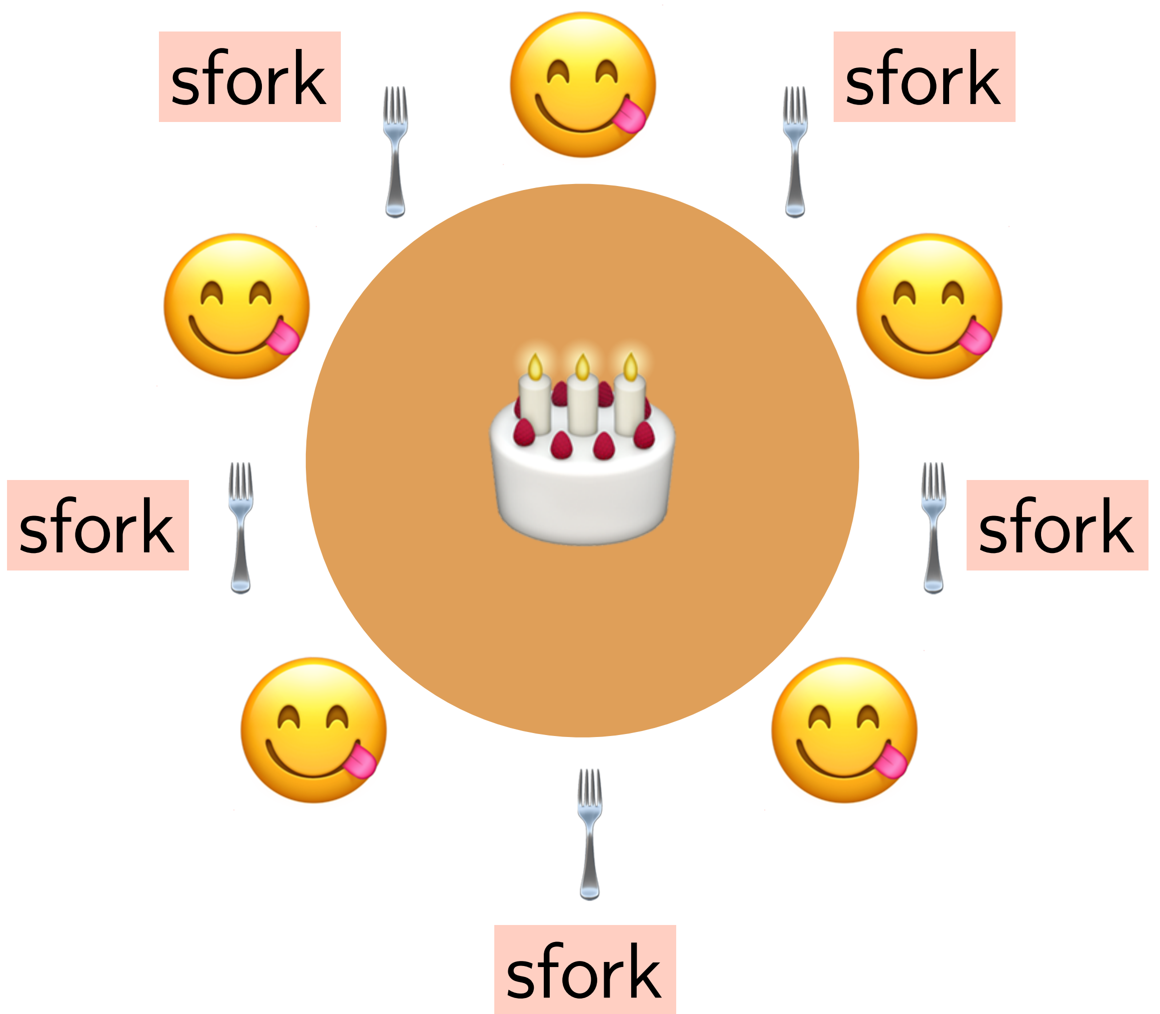

Progress theorem:

blocked process: linear process attempting to acquire

configuration is stuck only if all processes are blocked

Curry-Howard isomorphism revisited

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Linear session types w/o manifest sharing:

- linear propositions — session types
- proofs — processes
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→ deadlock: failure of proof construction

Recovering expressiveness of pi-calculus for session-typed concurrency

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→ encoding of untyped asynchronous π-calculus into SILL_S and proof of its operational and observational correspondence

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$$\mathsf{coin} = \uparrow_{\mathsf{L}}^{\mathsf{S}} \oplus\{\mathsf{head} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{coin}, \ \mathsf{tail} : \downarrow_{\mathsf{L}}^{\mathsf{S}} \mathsf{coin}\}\}$$

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Processes:

head: coin	sends "head" and recurs as tail process
tail: coin	sends "tail" and recurs as head process
filpper: $\mathbf{1} \leftarrow \mathsf{coin}$	reads ("flips") given coin once and terminates
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Characteristics of untyped, asynchronous π-calculus:

- Arity: a π-calculus channel may connect arbitrarily many processes
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unordered “buffer”

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translate $\mathbf{P}$ in terms of processes

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Ongoing and future work

Deadlock-freedom [Balzer et al. 2019]

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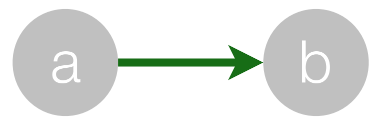
- waiting to synchronize: 

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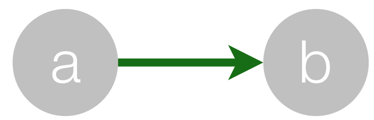
- waiting to synchronize: 

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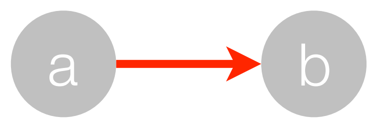
- waiting to release: 

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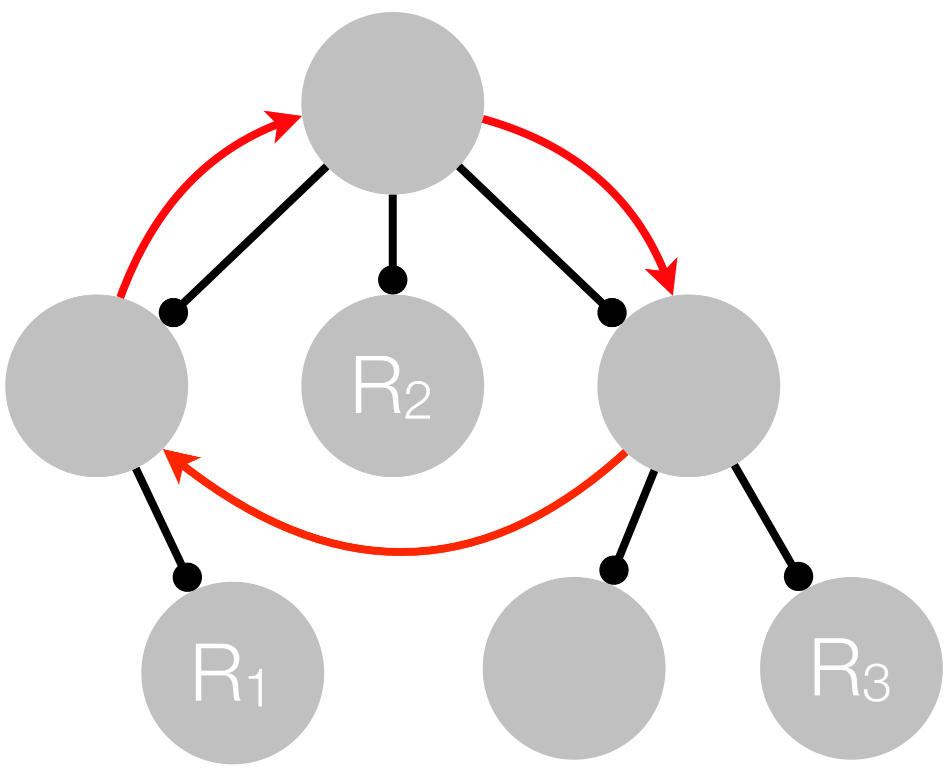

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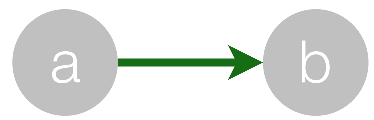
- waiting to synchronize: 

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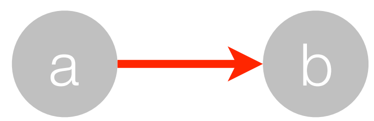
- waiting to release: 

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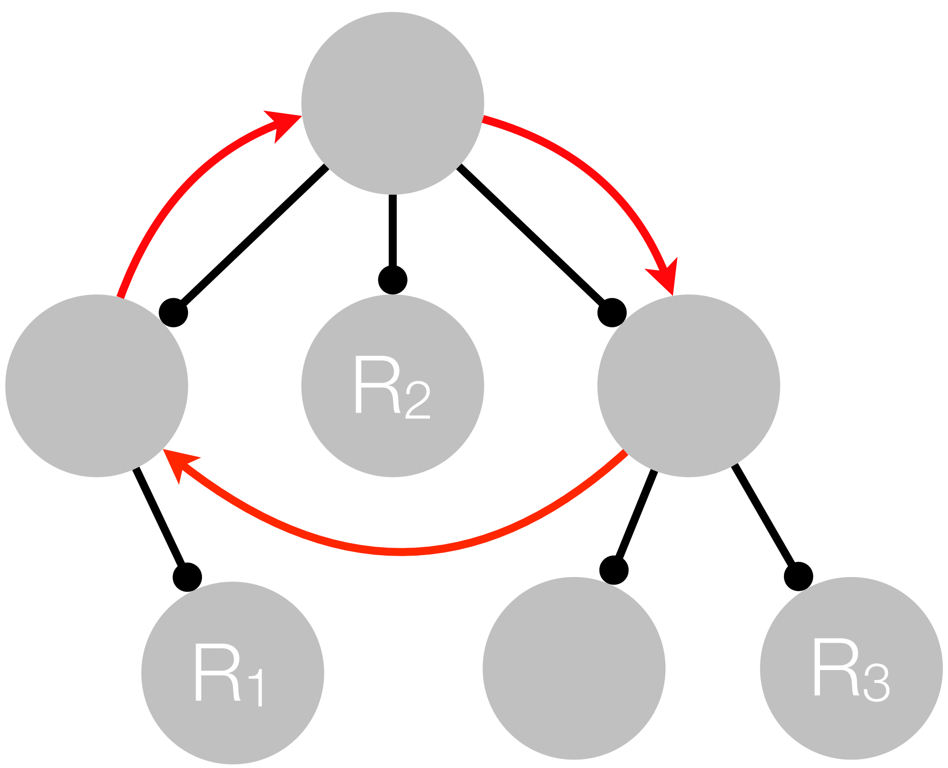

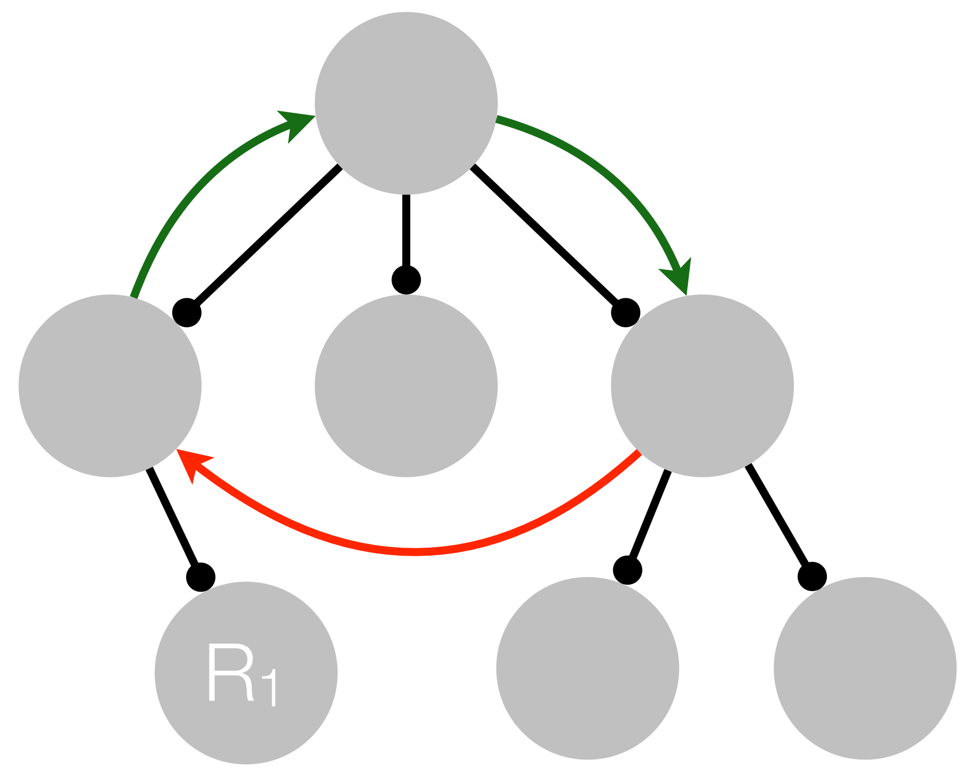

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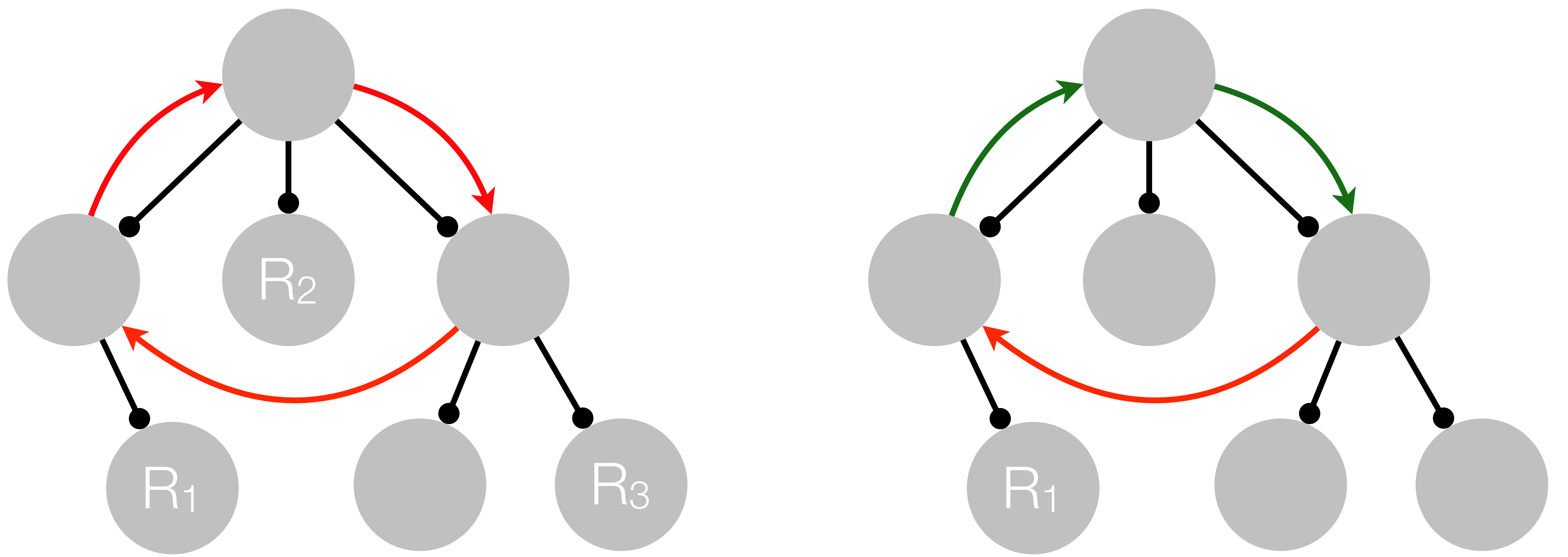

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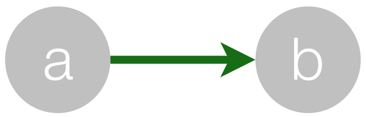
- waiting to synchronize: 

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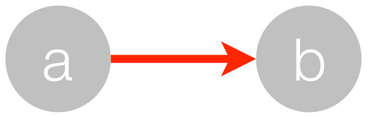
- waiting to release: 

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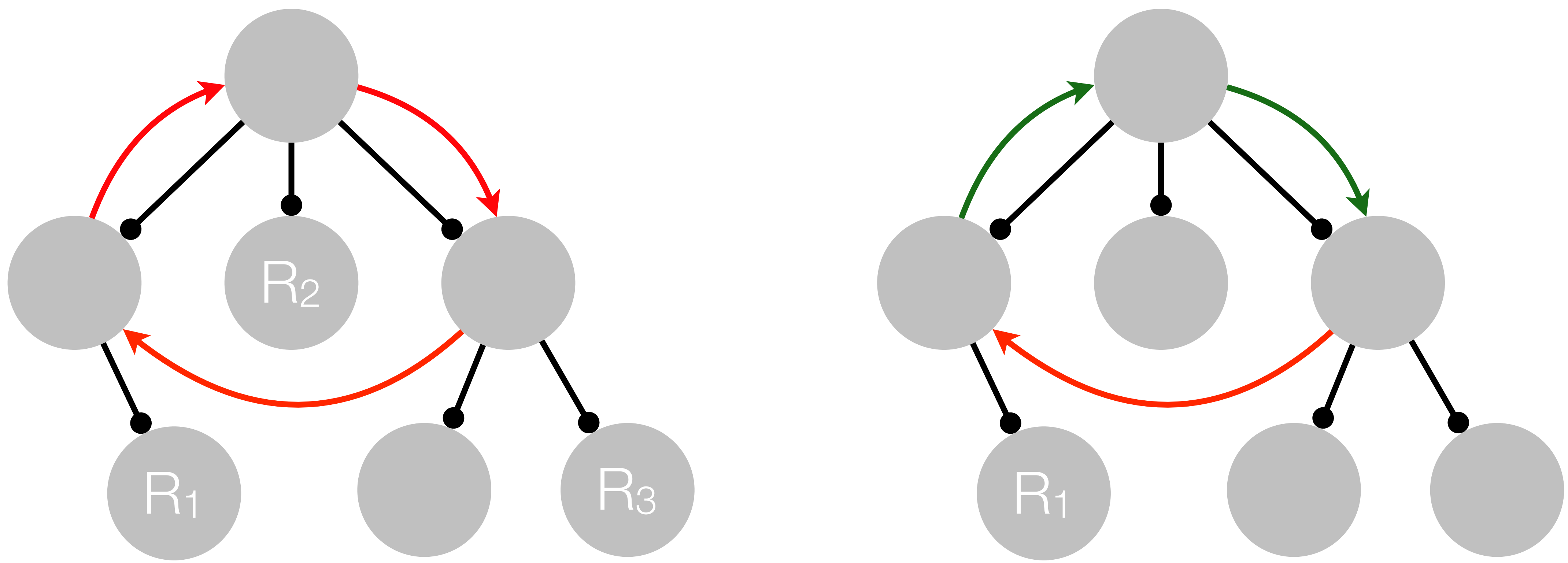

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see “manifest deadlock-freedom” [Balzer et al. 2019]

Session types for Rust (with Mozilla Research)

- Development of a library for Manifest Sharing in Rust
- Explore application to Servo

Digital contracts (with Das & Hoffmann & Pfenning)

- Development of Nomos, a new digital contract language
- Static guarantees:
  - protocol enforcement (shared session types)
  - control over resource usage (resource analysis)
  - tracking of assets (linear type system)

Summary

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