Recitation on Assignment 1: PAC Bounds

10-701/15-781: Machine Learning (Fall 2004)

September 29, 2004
1 PAC-learning

1.0.1 Definition

Suppose:

$x$ is a vector of $m$ boolean variables.
$y$ is a binary output.
Data is noiseless.

We are learning a hypothesis (function) $h : x \rightarrow y$.

The learned hypothesis $h'$ has $\text{trainerror}(h') = 0$ (in the simplest version of PAC and this homework).

We want to bound $P(\text{we learn } h' \text{ such that } \text{trueerror}(h') > \epsilon)$ in terms of training data size $m$. 
1.0.2 Derivation of Bounds

\[ P(\text{we learn } h' \text{ such that } \text{trueerror}(h') > \epsilon) \leq \]
\[ P(\text{the set } H \text{ contains } h' \text{ such that } \text{trueerror}(h') > \epsilon) = \]
\[ P(\exists h', h' \text{ has } \text{trainerror}(h') = 0 \text{ and } \text{trueerror}(h') > \epsilon) = \]
\[ P((\text{trainerror}(h_1) = 0 \land \text{testerror}(h_1) > \epsilon) \lor \]
\[ (\text{trainerror}(h_2) = 0 \land \text{testerror}(h_2) > \epsilon) \lor \]
\[ \vdots \]
\[ (\text{trainerror}(h_{|H|}) = 0 \land \text{testerror}(h_{|H|}) > \epsilon)) \leq \]
\[ \sum_{i=1}^{|H|} P(\text{trainerror}(h_i) = 0 \land \text{testerror}(h_i) > \epsilon) \leq \]
\[ \sum_{i=1}^{|H|} P(\text{trainerror}(h_i) = 0 | \text{testerror}(h_i) > \epsilon) \leq \]
\[ \sum_{i=1}^{|H|} (1 - \epsilon)^m = \]
\[ |H|(1 - \epsilon)^m \]
1.0.3 PAC Bounds

\[ \delta = P(\text{we learn } h' \text{ such that } \text{trueerror}(h') > \epsilon) \leq |H|(1 - \epsilon)^m \]

\[ m \geq \frac{0.69}{\epsilon}(\log_2 |H| + \log_2 \frac{1}{\delta}) \]