

15-859(B) Machine Learning Theory

Lecture 1: intro, basic models and issues

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Admin

- Course web page. Textbook covers about 1/2 of course material.
- 5-6 hwk assignments. Exercises/problems.
- Small project: explore a theoretical question, try some experiments, or read a paper and explain the idea. Short writeup and possibly presentation. Small groups ok.
- Take-home exam (worth roughly 2 hwks).
- "volunteers" for hwk grading.

OK, let's get to it...

Machine learning can be used to...

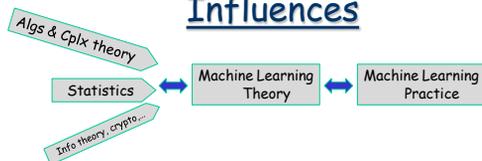
- recognize speech, faces,
- play games, steer cars,
- adapt programs to users,
- classify documents, protein sequences,...

Goals of machine learning theory:

develop and analyze models to understand:

- what kinds of tasks we can hope to learn, and from what kind of data,
- what types of guarantees might we hope to achieve,
- other common issues that arise.

Influences



Goals of machine learning theory:

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- what types of guarantees might we hope to achieve,
- other common issues that arise.

A typical setting

- Imagine you want a computer program to help you decide which email messages are **spam** and which are important.
- Might represent each message by n features. (e.g., return address, keywords, spelling, etc.)
- Take sample S of data, labeled according to whether they were/weren't **spam**.
- Goal of algorithm is to use data seen so far produce good prediction rule (a "hypothesis") $h(x)$ for future data.

The concept learning setting

E.g.,

	\$\$	meds	Mr.	bad spelling	known-sender	spam?
Y	N	Y	Y	N	Y	
N	N	N	Y	Y	N	
N	Y	N	N	N	Y	
Y	N	N	N	Y	N	
N	N	Y	N	Y	N	
Y	N	N	Y	N	Y	
N	N	Y	N	N	N	
N	Y	N	Y	N	Y	

Given data, some reasonable rules might be:

- Predict **SPAM** if \neg known AND ($$$$ OR meds)
- Predict **SPAM** if $$$ +$ meds - known > 0 .

•...

Big questions

- (A) How might we automatically generate rules that do well on observed data?
[algorithm design]
- (B) What kind of confidence do we have that they will do well in the future?
[confidence bound / sample complexity]

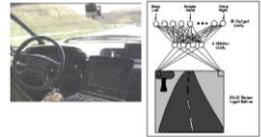
for a given learning alg. how much data do we need...

Power of basic paradigm

Many problems solved by converting to basic "concept learning from structured data" setting.

- E.g., document classification
 - convert to bag-of-words
 - Linear separators do well
- E.g., driving a car

- convert image into features.
- Use neural net with several outputs.



Natural formalization (PAC)

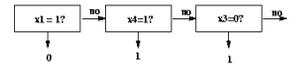
Email msg Spam or not?

- We are given sample $S = \{(x,y)\}$.
 - View labels y as being produced by some target function f .
- Alg does optimization over S to produce some hypothesis (prediction rule) h .
- Assume S is a random sample from some probability distribution D . Goal is for h to do well on new examples also from D .

I.e., $\Pr_D[h(x) \neq f(x)] < \epsilon$.

err(h)

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm A that will find a consistent DL if one exists.
2. Show that if S is of reasonable size, then $\Pr[\text{exists consistent DL } h \text{ with } \text{err}(h) > \epsilon] < \delta$.
3. This means that A is a good algorithm to use if f is, in fact, a DL.

If S is of reasonable size, then A produces a hypothesis that is Probably Approximately Correct.

How can we find a consistent DL?

x_1	x_2	x_3	x_4	x_5	label
1	0	0	1	1	+
0	1	1	0	0	-
1	1	1	0	0	+
0	0	0	1	0	-
1	1	0	1	1	+
1	0	0	0	1	-

if ($x_1=0$) then -, else
 if ($x_2=1$) then +, else
 if ($x_4=1$) then +, else -

Decision List algorithm

- Start with empty list.
- Find if-then rule consistent with data.
(and satisfied by at least one example)
- Put rule at bottom of list so far, and cross off examples covered. Repeat until no examples remain.

If this fails, then:

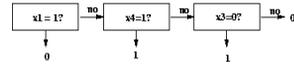
- No DL consistent with remaining data.
- So, no DL consistent with original data.

OK, fine. Now why should we expect it to do well on future data?

Confidence/sample-complexity

- Consider some DL h with $\text{err}(h) > \epsilon$, that we're worried might fool us.
 - Chance that h is consistent with S is at most $(1-\epsilon)^{|S|}$.
 - Let $|H|$ = number of DLs over n Boolean features. $|H| < n!4^n$. (for each feature there are 4 possible rules, and no feature will appear more than once)
- So, $\Pr[\text{some DL } h \text{ with } \text{err}(h) > \epsilon \text{ is consistent}] \leq |H|(1-\epsilon)^{|S|} \leq |H|e^{-\epsilon|S|}$.
- This is $< \delta$ for $|S| > (1/\epsilon)[\ln(|H|) + \ln(1/\delta)]$ or about $(1/\epsilon)[n \ln n + \ln(1/\delta)]$

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

- DONE** Design an efficient algorithm A that will find a consistent DL if one exists.
- DONE** Show that if $|S|$ is of reasonable size, then $\Pr[\text{exists consistent DL } h \text{ with } \text{err}(h) > \epsilon] < \delta$.
- So, if f is in fact a DL, then whp A 's hypothesis will be approximately correct. "PAC model"

PAC model more formally:

- We are given sample $S = \{(x,y)\}$.
 - Assume x 's come from some fixed probability distribution D over instance space.
 - View labels y as being produced by some target function f .
- Alg does optimization over S to produce some hypothesis (prediction rule) h . Goal is for h to do well on new examples also from D . I.e., $\Pr_D[h(x) \neq f(x)] < \epsilon$.

Algorithm PAC-learns a class of functions C if:

- For any given $\epsilon > 0, \delta > 0$, any target $f \in C$, any dist. D , the algorithm produces h of $\text{err}(h) < \epsilon$ with prob. at least $1-\delta$.
- Running time and sample sizes polynomial in relevant parameters: $1/\epsilon, 1/\delta, n$ (size of examples), $\text{size}(f)$.
- Require h to be poly-time evaluable. Learning is called "proper" if $h \in C$. Can also talk about "learning C by H ".

We just gave an alg to PAC-learn decision lists.

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Some notes:

- Can either view alg as requesting examples (button/oracle model) or just as function of S , with guarantee if S is suff. lg.
- "size(f)" term comes in when you are looking at classes where some fns could take $> \text{poly}(n)$ bits to write down. (e.g., decision trees, DNF formulas)

Confidence/sample-complexity

- What's great is there was nothing special about DLs in our argument.
- All we said was: "if there are not *too* many rules to choose from, then it's unlikely one will have fooled us just by chance."
- And in particular, the number of examples needs to only be proportional to $\log(|C|)$. (notice big difference between $|C|$ and $\log(|C|)$)

Occam's razor

William of Occam (~1320 AD):

"entities should not be multiplied unnecessarily" (in Latin)

Which we interpret as: "in general, prefer simpler explanations".

Why? Is this a good policy? What if we have different notions of what's simpler?

Occam's razor (contd)

A computer-science-ish way of looking at it:

- Say "simple" = "short description".
- At most 2^s explanations can be $< s$ bits long.
- So, if the number of examples satisfies:

Think of as 10x #bits to write down h.

$$|S| > (1/\epsilon)[s \ln(2) + \ln(1/\delta)]$$

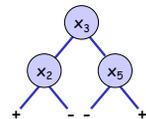
Then it's unlikely a bad simple explanation will fool you just by chance.

Occam's razor (contd)²

Nice interpretation:

- Even if we have different notions of what's simpler (e.g., different representation languages), we can both use Occam's razor.
- Of course, there's no guarantee there **will** be a short explanation for the data. That depends on your representation.

Decision trees



- Decision trees over $\{0,1\}^n$ not known to be PAC-learnable.
- Given any data set S , it's easy to find a consistent DT if one exists. How?
- Where does the DL argument break down?
- Simple heuristics used in practice (ID3 etc.) don't work for all $c \in C$ even for uniform D .
- Would suffice to find the (apx) smallest DT consistent with any dataset S , but that's NP-hard.

More examples

Other classes we can PAC-learn: (how?)

- Monomials [conjunctions, AND-functions]
 - $x_1 \wedge x_4 \wedge x_6 \wedge x_9$
- 3-CNF formulas (3-SAT formulas)
- OR-functions, 3-DNF formulas
- k-Decision lists (each if-condition is a conjunction of size k), k is constant.

Given a data set S , deciding if there is a consistent 2-term DNF formula is NP-complete. Does that mean 2-term DNF is hard to learn?

More examples

Hard to learn C by C , but easy to learn C by H , where $H = \{2\text{-CNF}\}$.

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If computation-time is no object, then any class is PAC-learnable

- Occam bounds \Rightarrow any class is learnable if computation time is no object:
 - Let $s_1=10, \delta_1 = \delta/2$. For $i=1,2,\dots$ do:
 - Request $(1/\delta)[s_i + \ln(1/\delta_i)]$ examples S_i .
 - Check if there is a function of size at most s_i consistent with S_i . If so, output it and halt.
 - $s_{i+1} = 2s_i, \delta_{i+1} = \delta_i/2$.
 - At most $\delta_1 + \delta_2 + \dots \leq \delta$ chance of failure.
 - Total data used: $O((1/\epsilon)[\text{size}(f)+\ln(1/\delta)\ln(\text{size}(f))])$.

1st terms sum to $O(\text{size}(f))$ by telescoping. 2nd terms sum to:
 $\ln(\frac{2}{\delta}) + \ln(\frac{2}{\delta}) + \dots + \ln(\frac{\text{size}(f)}{\delta}) \leq \ln(\text{size}(f) \ln(\frac{\text{size}(f)}{\delta})) = \ln^2(\text{size}(f)) + \ln(\text{size}(f)) \ln(\frac{1}{\delta})$

More about the PAC model

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- Running time and sample sizes polynomial in relevant parameters: $1/\epsilon, 1/\delta, n, \text{size}(f)$.
- Require h to be poly-time evaluable. Learning is called "proper" if $h \in C$. Can also talk about "learning C by H ".
- What if your alg only worked for $\delta = \frac{1}{2}$, what would you do?
- What if it only worked for $\epsilon = \frac{1}{4}$, or even $\epsilon = \frac{1}{2}-1/n$? This is called **weak-learning**. Will get back to later.
- **Agnostic learning** model: Don't assume anything about f . Try to reach error $\text{opt}(H) + \epsilon$.

More about the PAC model

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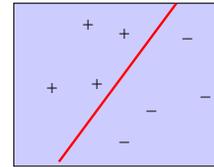
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Drawbacks of model:

- In the real world, labeled examples are much more expensive than running time. **Poly(size(f)) not enough.**
- "Prior knowledge/beliefs" might be not just over form of target but other relations to data.
- Doesn't address other kinds of info (**cheap unlabeled data, pairwise similarity information**).
- Only considers "one shot" learning.

Extensions we'll get at later:

- Replace $\log(|H|)$ with "effective number of degrees of freedom".



- There are infinitely many linear separators, but not that many really different ones.
- Other more refined analyses.

Some classic open problems

Can one efficiently PAC-learn...

- an intersection of 2 halfspaces? (**2-term DNF trick doesn't work**)
- $C=\{\text{fns with only } O(\log n) \text{ relevant variables}\}$? (or even $O(\log \log n)$ or $\omega(1)$ relevant variables)? **This is a special case of DTs, DNFs.**
- Monotone DNF over uniform D ?
- Weak agnostic learning of monomials.