

CS 598 (CRN 62819) Topics in Algorithms, Spring 2015

Homework # 1

Due: February 2, 2015

Homework is due in class (on paper) on the due date.

1. Chapter 2, Exercise 2.1, parts 1,2,3 (not part 4). In part 3, ignore “centered at the origin” (it’s irrelevant).
2. Consider drawing a random point \mathbf{x} on the surface of the unit sphere in R^d . What is the variance of x_1 (the first coordinate of \mathbf{x})? See if you can give an argument without doing any integrals.
3. Chapter 2, Exercise 2.51.
4. Consider a unit ball A centered at the origin and a unit ball B whose center is at distance s from the origin. Suppose that a random point x is drawn from the mixture distribution: “with probability $1/2$, draw at random from A ; with probability $1/2$, draw at random from B ”. Show that a separation $s = \omega(1/\sqrt{d-1})$ is sufficient so that $\Pr[x \in A \cap B] = o(1)$ (i.e., for any $\epsilon > 0$ there exists c such that if $s \geq c/\sqrt{d-1}$ then $\Pr[x \in A \cap B] < \epsilon$). In other words, this separation means that nearly all of the mixture distribution is identifiable.
5. Show how the bounds of Theorem 2.2 can be improved to $1 - O(\frac{1}{\gamma}e^{-\gamma^2/2})$ by either performing the integration or a more careful telescoping argument.
6. In d dimensional space, one can have at most d vectors that are all orthogonal to each other. However, one can have many more vectors that are all nearly orthogonal to each other. Show how you can have 10,000 nearly orthogonal vectors in $d = 1,000$ dimensional space using both arguments below:
 - (a) Pick 10,000 random vectors in the unit d -dimensional ball. Use Theorem 2.3 to argue that with non-zero probability they will all be nearly orthogonal. [You may assume that the $1 - O(1/n)$ bound in the theorem is greater than 0 for $n = 10,000$]
 - (b) Pick 10,000 perfectly orthogonal vectors in 10,000-dimensional space and then randomly project them down to a 1,000-dimensional space. Use Theorem 2.9 and the Johnson-Lindenstrauss Lemma.

Give concrete bounds in your arguments (i.e., concrete upper and lower bounds on the pairwise angles). They don’t have to be tight, just reasonable.