

An Algorithms-based Intro to Machine Learning

Avrim Blum

[Based on a talk given at the National Academy of Sciences "Frontiers of Science" symposium]

Plan for today

- Machine Learning intro: models and basic issues
- An interesting algorithm for "combining expert advice"

Machine learning can be used to...

- recognize speech,
- identify patterns in data,
- steer a car,
- play games,
- adapt programs to users,
- improve web search, ...

From a scientific perspective: can we develop models to understand learning as a computational problem, and what types of guarantees might we hope to achieve?

A typical setting

- Imagine you want a computer program to help filter which email messages are **spam** and which are important.
- Might represent each message by **n** features. (e.g., **return address**, **keywords**, **spelling**, etc.)
- Take sample **S** of data, labeled according to whether they were/weren't **spam**.
- Goal of algorithm is to use data seen so far produce good prediction rule (a "hypothesis") $h(x)$ for future data.

The concept learning setting

E.g., money pills Mr. bad spelling known-sender | spam?

The concept learning setting

	money	pills	Mr.	bad	spelling	known-sender	spam?
a positive example	Y	N	Y	Y	N	Y	Y
a negative example	N	Y	N	N	Y	N	N
	Y	N	N	Y	N	Y	Y
	N	N	Y	N	N	N	N
	N	Y	N	Y	N	N	Y

Given data, some reasonable rules might be:

- Predict **SPAM** if \neg known AND (money OR pills)
- Predict **SPAM** if $\text{money} + \text{pills} - \text{known} > 0$.
- ...

Big questions

(A) How might we automatically generate rules that do well on observed data?

[algorithm design]

(B) What kind of confidence do we have that they will do well in the future?

[confidence bound / sample complexity]

for a given learning alg, how much data do we need...

Power of basic paradigm

Many problems solved by converting to basic "concept learning from structured data" setting.

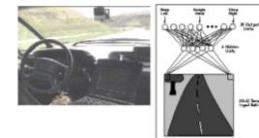
• E.g., document classification

- convert to bag-of-words
- Linear separators do well



• E.g., driving a car

- convert image into features.
- Use neural net with several outputs.



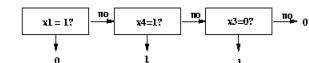
Natural formalization (PAC)

Email msg Spam or not?

- We are given sample $S = \{(x, y)\}$.
 - View labels y as being produced by some target function f .
- Alg does optimization over S to produce some hypothesis (prediction rule) h .
- Assume S is a random sample from some probability distribution D . Goal is for h to do well on new examples also from D .

I.e., $\Pr_D[h(x) \neq f(x)] < \varepsilon$.

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

1. Design an efficient algorithm A that will find a consistent DL if one exists.
2. Show that if S is of reasonable size, then $\Pr[\text{exists consistent DL } h \text{ with } \text{err}(h) > \varepsilon] < \delta$.
3. This means that A is a good algorithm to use if f is, in fact, a DL.

(a bit of a toy example since would want to extend to "mostly consistent" DL)

How can we find a consistent DL?

x_1	x_2	x_3	x_4	x_5	label
1	0	0	1	1	+
0	1	1	0	0	-
1	1	1	0	0	+
0	0	0	1	0	-
1	1	0	1	1	+
1	0	0	0	1	-

if ($x_1=0$) then -
 if ($x_2=1$) then +, else -
 if ($x_4=1$) then +, else -

Decision List algorithm

- Start with empty list.
- Find if-then rule consistent with data.
(and satisfied by at least one example)
- Put rule at bottom of list so far, and cross off examples covered. Repeat until no examples remain.

If this fails, then:

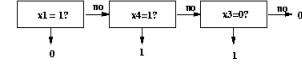
- No rule consistent with remaining data.
- So no DL consistent with remaining data.
- So, no DL consistent with original data.

OK, fine. Now why should we expect it to do well on future data?

Confidence/sample-complexity

- Consider some DL h with $\text{err}(h) > \varepsilon$, that we're worried might fool us.
- Chance that h survives $|S|$ examples is at most $(1-\varepsilon)^{|S|}$.
- Let $|H| =$ number of DLs over n Boolean features. $|H| < (4n+2)^n$. (really crude bound)
- So, $\Pr[\text{some DL } h \text{ with } \text{err}(h) > \varepsilon \text{ is consistent}] < |H|(1-\varepsilon)^{|S|}$.
- This is < 0.01 for $|S| > (1/\varepsilon)[\ln(|H|) + \ln(100)]$ or about $(1/\varepsilon)[n \ln n + \ln(100)]$

Example of analysis: Decision Lists



Say we suspect there might be a good prediction rule of this form.

DONE 1. Design an efficient algorithm \mathbf{A} that will find a consistent DL if one exists.

DONE 2. Show that if $|S|$ is of reasonable size, then $\Pr[\text{exists consistent DL } h \text{ with } \text{err}(h) > \varepsilon] < \delta$.

3. So, if f is in fact a DL, then whp \mathbf{A} 's hypothesis will be approximately correct. "PAC model"

Confidence/sample-complexity

- What's great is there was nothing special about DLs in our argument.
- All we said was: "if there are not *too* many rules to choose from, then it's unlikely one will have fooled us just by chance."
- And in particular, the number of examples needs to only be proportional to $\log(|H|)$. (big difference between 100 and e^{100} .)

Occam's razor

William of Occam (~1320 AD):

"entities should not be multiplied unnecessarily" (in Latin)

Which we interpret as: "in general, prefer simpler explanations".

Why? Is this a good policy? What if we have different notions of what's simpler?

Occam's razor (contd)

A computer-science-ish way of looking at it:

- Say "simple" = "short description".
- At most 2^s explanations can be $< s$ bits long.
- So, if the number of examples satisfies:

Think of as
10x #bits to
write down h .

Then it's unlikely a bad simple explanation will fool you just by chance.

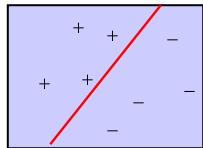
Occam's razor (contd)²

Nice interpretation:

- Even if we have different notions of what's simpler (e.g., different representation languages), we can both use Occam's razor.
- Of course, there's no guarantee there **will** be a short explanation for the data. That depends on your representation.

Further work

- Replace $\log(|H|)$ with "effective number of degrees of freedom".



- There are infinitely many linear separators, but not that many really different ones.
- Kernels, margins, more refined analyses....

Online learning

- What if we don't want to make assumption that data is coming from some fixed distribution? Or any assumptions on data?
- Can no longer talk about past performance predicting future results.
- **Can we hope to say anything interesting at all??**

Idea: regret bounds.

➤ Show that our algorithm does nearly as well as best predictor in some large class.

Using "expert" advice

Say we want to predict the stock market.

- We solicit n "experts" for their advice. (Will the market go up or down?)
- We then want to use their advice somehow to make our prediction. E.g.,

Expt 1	Expt 2	Expt 3	neighbor's dog	truth
down	up	up	up	up
down	up	up	down	down
...

Basic question: Is there a strategy that allows us to do nearly as well as best of these in hindsight?

[*"expert"* = someone with an opinion. Not necessarily someone who knows anything.]

Simpler question

- We have n "experts".
- One of these is perfect (never makes a mistake). We just don't know which one.
- Can we find a strategy that makes no more than $\lg(n)$ mistakes?

Answer: sure. Just take majority vote over all experts that have been correct so far.

➤ Each mistake cuts # available by factor of 2.

➤ Note: this means ok for n to be very large.

What if no expert is perfect?

Intuition: Making a mistake doesn't completely disqualify an expert. So, instead of crossing off, just lower its weight.

Weighted Majority Alg:

- Start with all experts having weight 1.
- Predict based on weighted majority vote.
- Penalize mistakes by cutting weight in half.

weights	1	1	1	1			prediction	correct
predictions	Y	Y	Y	N	Y	Y		
weights	1	1	1	.5				
predictions	Y	N	N	Y	N	Y		
weights	1	.5	.5	.5				

Analysis: do nearly as well as best expert in hindsight

- $M = \#$ mistakes we've made so far.
- $m = \#$ mistakes best expert has made so far.
- $W = \text{total weight (starts at } n\text{)}.$
- After each mistake, W drops by at least 25%. So, after M mistakes, W is at most $n(3/4)^M$.
- Weight of best expert is $(1/2)^m$. So,

$$(1/2)^m \leq n(3/4)^M$$

$$(4/3)^M \leq n2^m$$

$$M \leq 2.4(m + \lg n)$$

So, if m is small, then M is pretty small too.

Randomized Weighted Majority

- 2.4($m + \lg n$) not so good if the best expert makes a mistake 20% of the time. Can we do better? Yes.
- Instead of taking majority vote, use weights as probabilities. (e.g., if 70% on up, 30% on down, then pick 70:30) Idea: smooth out the worst case.
- Also, generalize $\frac{1}{2}$ to $1 - \varepsilon$.

Solves to: $M \leq \frac{-m \ln(1 - \varepsilon) + \ln(n)}{\varepsilon} \approx (1 + \varepsilon/2)m + \frac{1}{\varepsilon} \ln(n)$

M = expected #mistakes
 $M \leq 1.39m + 2 \ln n - \varepsilon = 1/2$

$M \leq 1.15m + 4 \ln n - \varepsilon = 1/4$

$M \leq 1.07m + 8 \ln n - \varepsilon = 1/8$

Analysis

- Say at time t we have fraction F_t of weight on experts that made mistake.
- So, we have probability F_t of making a mistake, and we remove an εF_t fraction of the total weight.
 - $W_{\text{final}} = n(1 - \varepsilon F_1)(1 - \varepsilon F_2) \dots$
 - $\ln(W_{\text{final}}) = \ln(n) + \sum_t [\ln(1 - \varepsilon F_t)] \leq \ln(n) - \varepsilon \sum_t F_t$
 (using $\ln(1-x) < -x$)
 $= \ln(n) - \varepsilon M$.
 ($\sum F_t = E[\# \text{mistakes}]$)
- If best expert makes m mistakes, then $\ln(W_{\text{final}}) > \ln((1 - \varepsilon)^m)$.
- Now solve: $\ln(n) - \varepsilon M > m \ln(1 - \varepsilon)$.

$$M \leq \frac{-m \ln(1 - \varepsilon) + \ln(n)}{\varepsilon} \approx (1 + \varepsilon/2)m + \frac{1}{\varepsilon} \log(n)$$

What can we use this for?

- Can use to combine multiple algorithms to do nearly as well as best in hindsight.
 - E.g., do nearly as well as best strategy in hindsight in repeated play of matrix game.
- Extension: "sleeping experts". E.g., one for each possible keyword. Try to do nearly as well as best "coalition".
- More extensions: "bandit problem", movement costs.

Other models

Some scenarios allow more options for algorithm.

- "Active learning": have large unlabeled sample and alg may choose among these.
 - E.g., web pages, image databases.
- Or, allow algorithm to construct its own examples. "Membership queries"
 - E.g., features represent variable-settings in some experiment, label represents outcome.
 - Gives algorithm more power.

Other models

- A lot of ongoing research into better algorithms, models that capture additional issues, incorporating Machine Learning into broader classes of applications.

Additional notes

- Some courses at CMU on machine learning:
 - 10-601 Machine Learning
 - 10-701/15-781 Machine Learning
 - 15-859(B) Machine Learning Theory. See <http://www.machinelearning.com>.