Auctions as Games: Equilibria and Efficiency
Near-Optimal Mechanisms

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Yesterday: Simple Auction Games

- item bidding games: second price simultaneous item auction
- Very simple valuations: unit demand or even single parameter
- Ad Auctions: Generalized Second Price

Today:

- More auction types
- More expressive valuations

Summary of problems

Full information single minded bidders

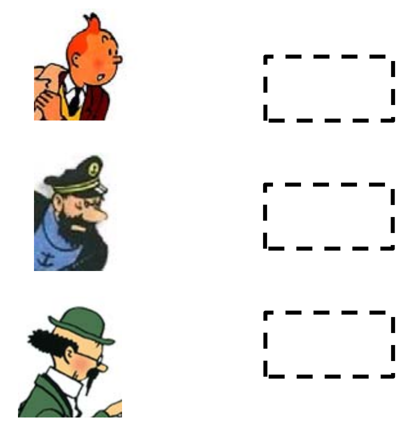
- v_{ij} = buyer i's value for house j
- $v_i(S) = \max_{j \in S} v_{ij}$

Bidding $b_{ij} > v_{ij}$ is dominated.

assume not done

GSP (AdAuction), also single parameter:

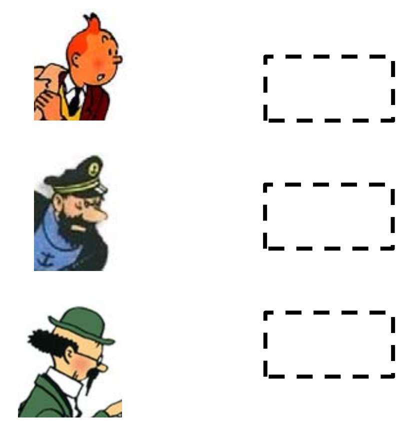
- $v_{kj} = v_k \gamma_k \alpha_j$



Summary of techniques

- Price of anarchy 2 based on: no-regret for bidding $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0 \ \forall j \neq j_i^*$
- Bound also applies to learning outcomes (see more Avrim Blum)

- Bayesian game (valuations from correlated distribution $\mathscr{F}$) price of anarchy of 4 based on no-regret for bidding $\frac{1}{2} v_i$
  – GSP
  – Single value auctions



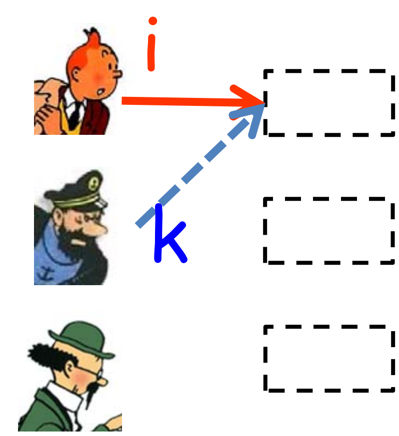
First Price vs Second Price?

Proof based on "player i has no regret about bidding $\frac{1}{2} v_i$" applies just as well for first price.

If player wins: price $\leq b_i \leq \frac{1}{2} v_i$
hence utility at least $\frac{1}{2} v_i$

- If he looses, all his items of interest, went to players with bid (and hence value) at least $\frac{1}{2} v_i$

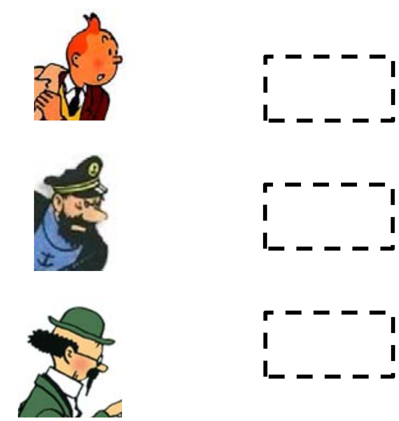
If i has value of opt, i or k has high value at Nash



First Price vs Second Price?

Proof based on "no-regret for bidding $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0\ \forall j \neq j_i^*$" no good, but similar proof applies with $b_{ij_i^*} = \frac{1}{2}v_{ij_i^*}$ and $b_{ij} = 0\ \forall j \neq j_i^*$"

- If player wins: price $\leq b_{ij_i^*} \leq \frac{1}{2}v_{ij_i^*}$
  hence utility at least $\frac{1}{2}v_{ij_i^*}$
- If he looses, his items of interest went to players with bid (and hence value) at least $\frac{1}{2}v_{ij_i^*}$



First Price Pure Nash

Theorem [Bikchandani GEB'99] Any valuation, first price pure Nash, socially optimal. Any combinatorial valuation.

Proof each item i was sold for a price p_i.

- price p is market equilibrium: all players maximizing $v_i(S) - \sum_{i \in S} p_i$ players

  otherwise bid $\boldsymbol{p_i^+}$ for items in $i \in S$

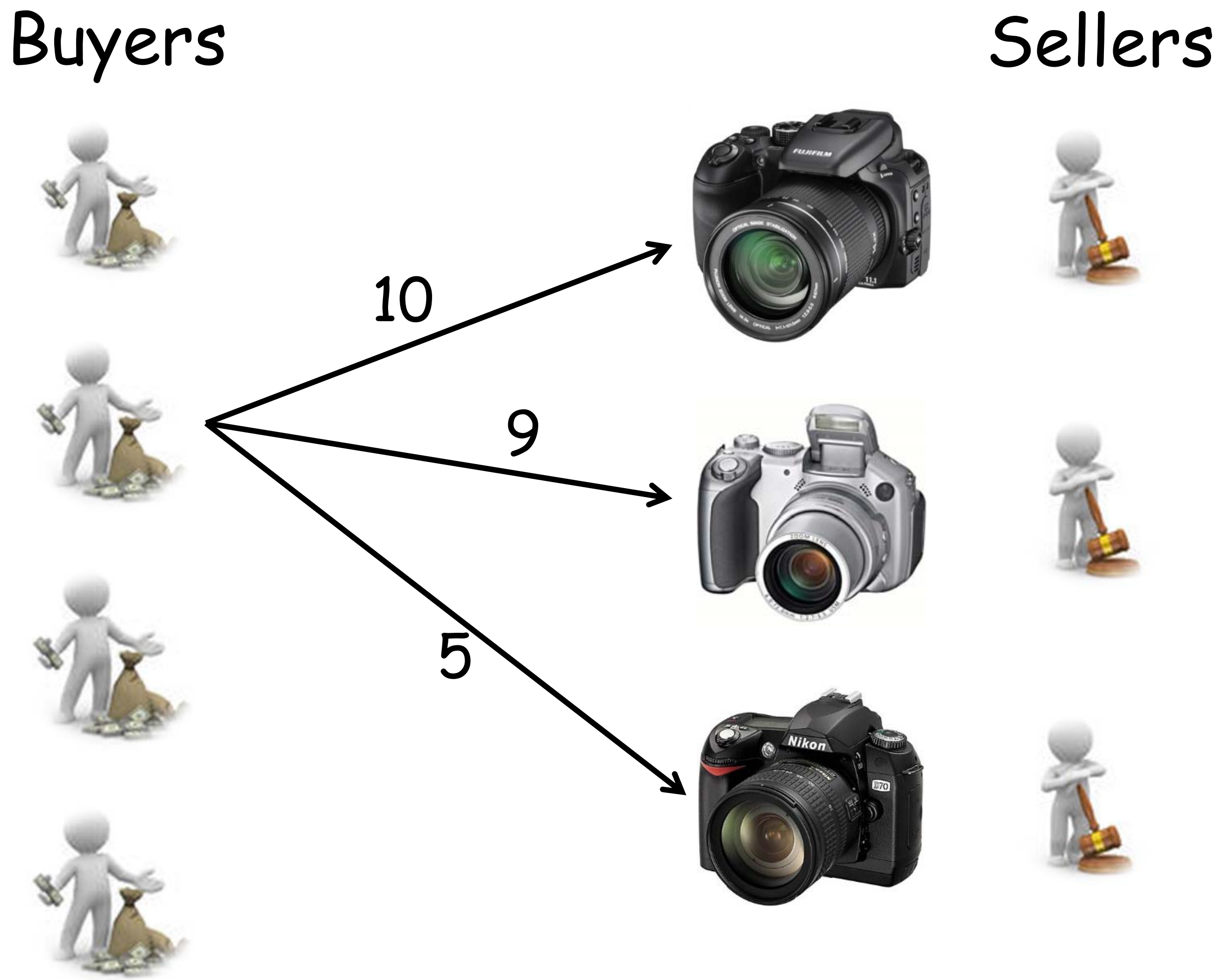
- market equilibrium is socially optimal

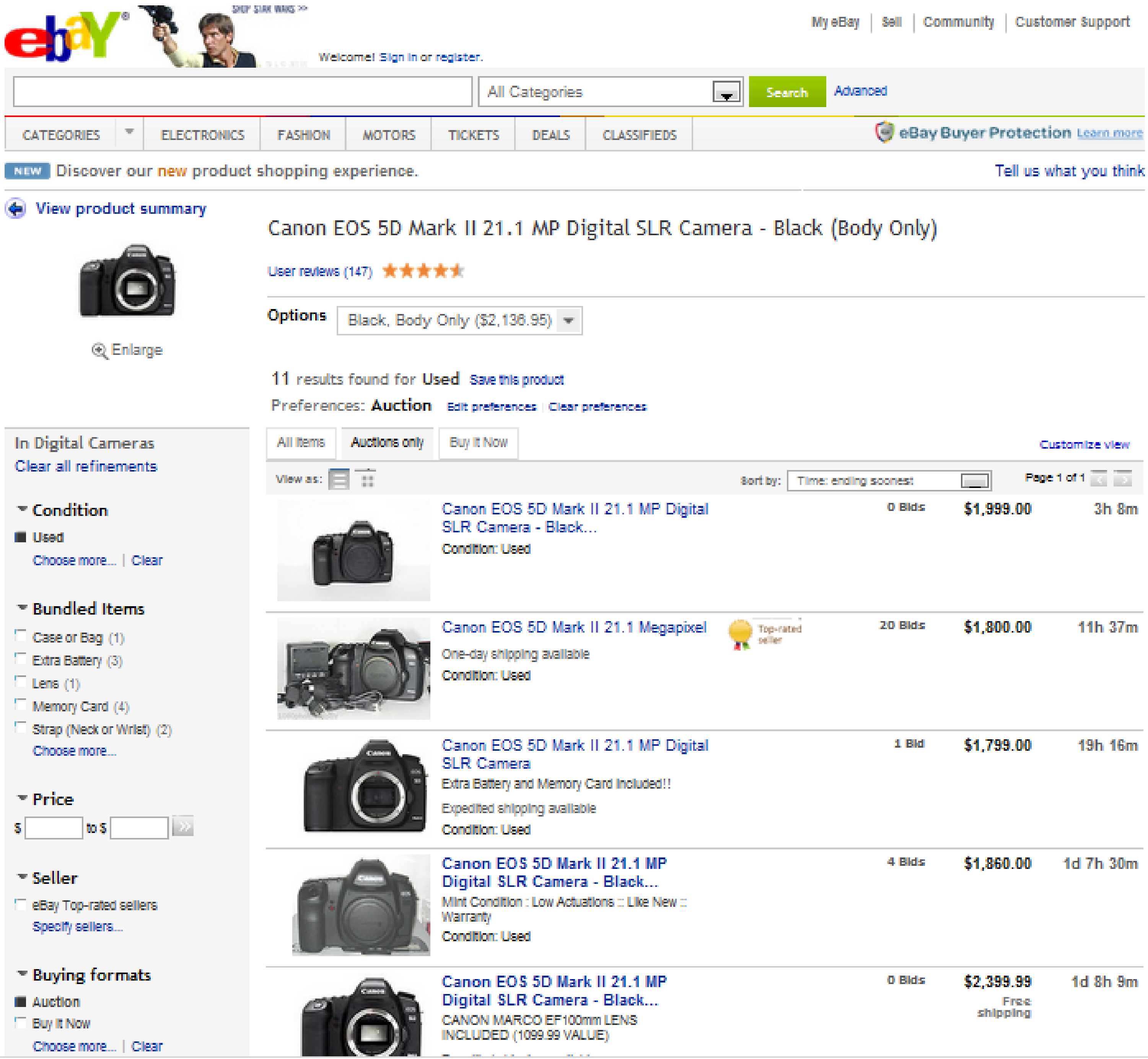
  $\{S_1, \dots, S_k\}$ Nash and $\{S_1^*, \dots, S_k^*\}$ alternate soln.

  $v_i(S_i) - \sum_{i \in S} p_i \geq v_i\ (S_i^*) - \sum_{i \in S'} p_i$

sum over all i $\quad \sum_i v_i(S_i) \geq \sum_i v_i(S_i^*)$

Sequential Game ( )
How important is simultaneous play?

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$1,999.00
3h 8m
Canon EOS 5D Mark II 21.1 Megapixel
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One-day shipping available
Condition: Used
20 Bids
$1,800.00
11h 37m
Canon EOS 5D Mark II 21.1 MP Digital SLR Camera
Extra Battery and Memory Card Included!!
Expedited shipping available
Condition: Used
1 Bid
$1,799.00
19h 16m
Canon EOS 5D Mark II 21.1 MP Digital SLR Camera - Black...
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Condition: Used
4 Bids
$1,860.00
1d 7h 30m
Canon EOS 5D Mark II 21.1 MP Digital SLR Camera - Black...
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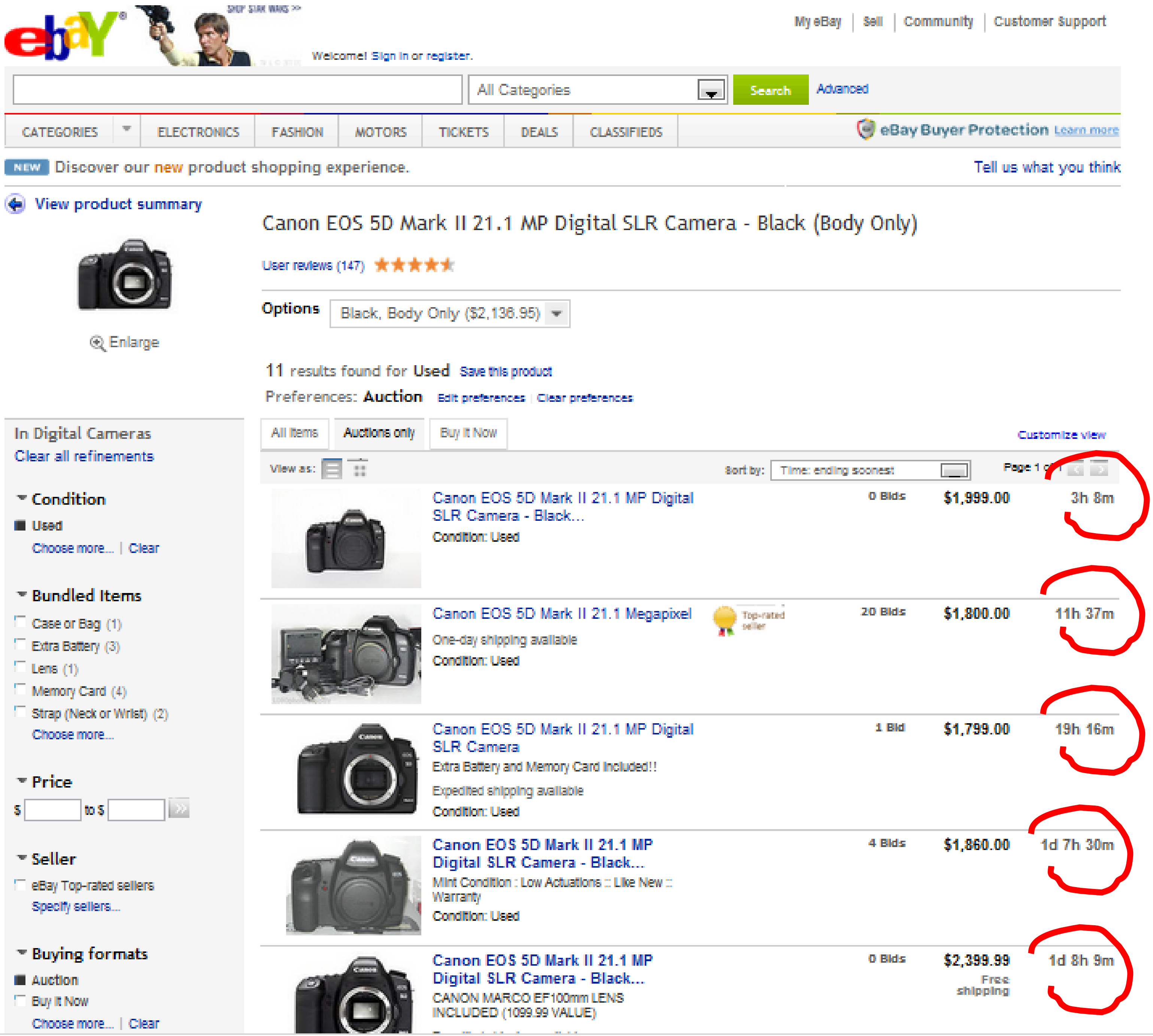
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EST : 3,000 EUR - 5,000 EUR

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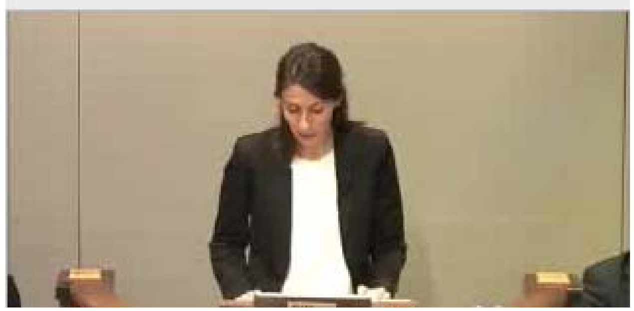


Current Lot

Second Price and Sequential Auctions

- Second price allows signaling
- Bidding above value is not dominated
- Can have unbounded price of anarchy both with
  - Additive valuations
  - Unit demand valuations (even after iterated elimination of dominated strategies)

Bad example for 2nd price

	k			k				
		…			…			
A	0		0	0	…	0	20	20
B						0	20	20
C							10	0
D	1			ϵ				
…								
Z			1			ϵ		

Sequential game

- Items are not available at the same time: sellers arrive sequentially

- Players are strategic and make decisions reasoning about the decisions of other players in the future

- Each player has unit demand valuation v_{ij} on the items

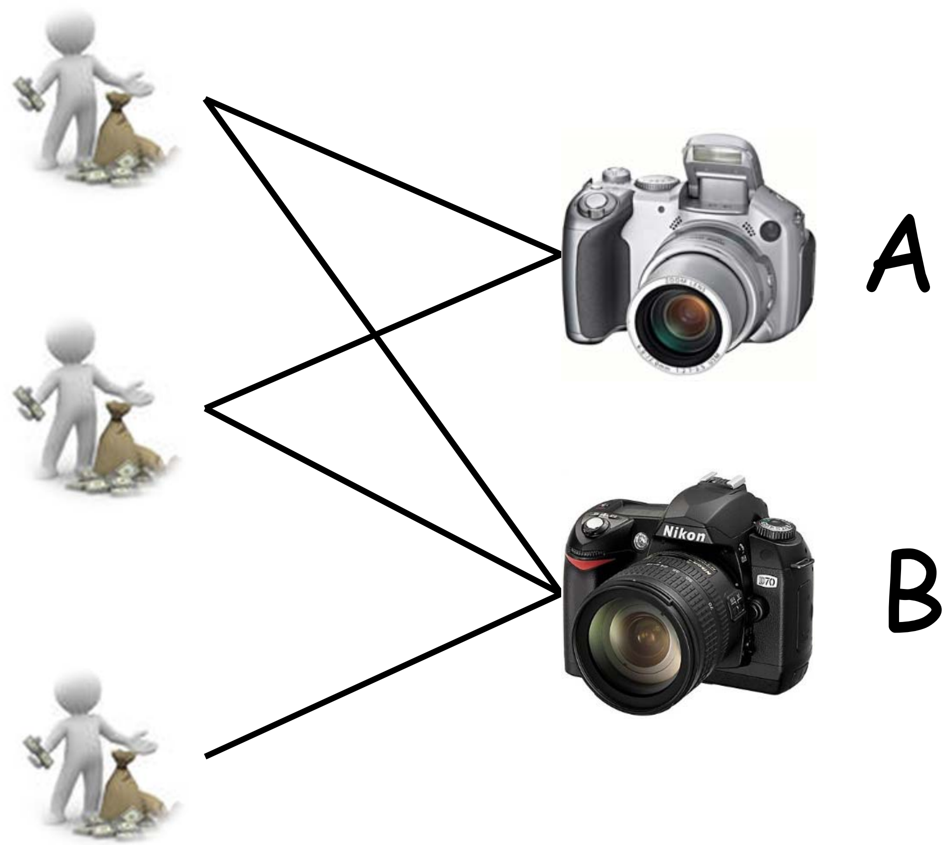
- First price auction
  - Full Information (Paes Leme, Syrgkanis, T. SODA'12)
  - Bayesian (Syrgkanis, T. EC'12)

Incomplete Information and Efficiency

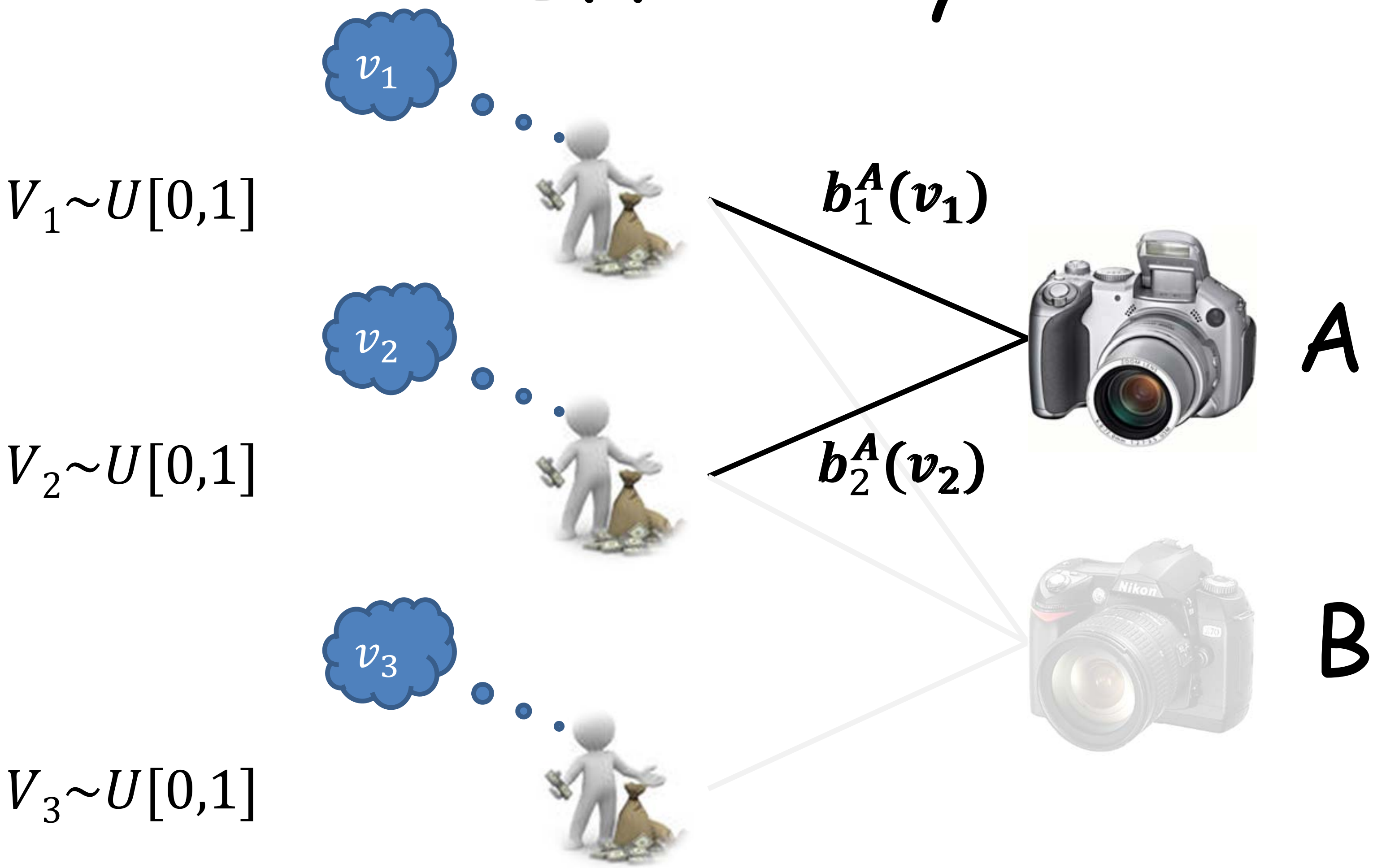
$V_1 \sim U[0,1]$

$V_2 \sim U[0,1]$

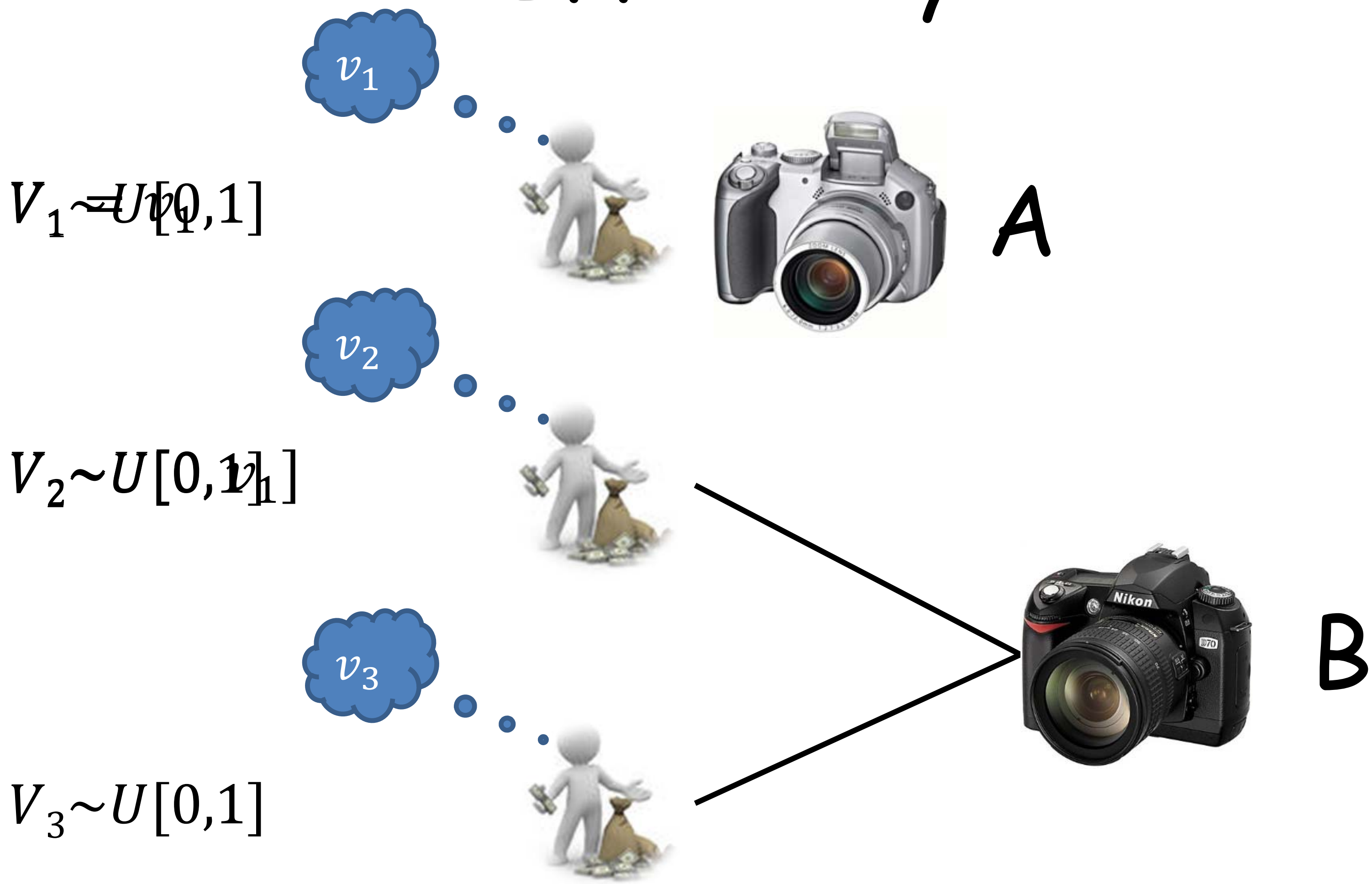
$V_3 \sim U[0,1]$



Incomplete Information and Efficiency

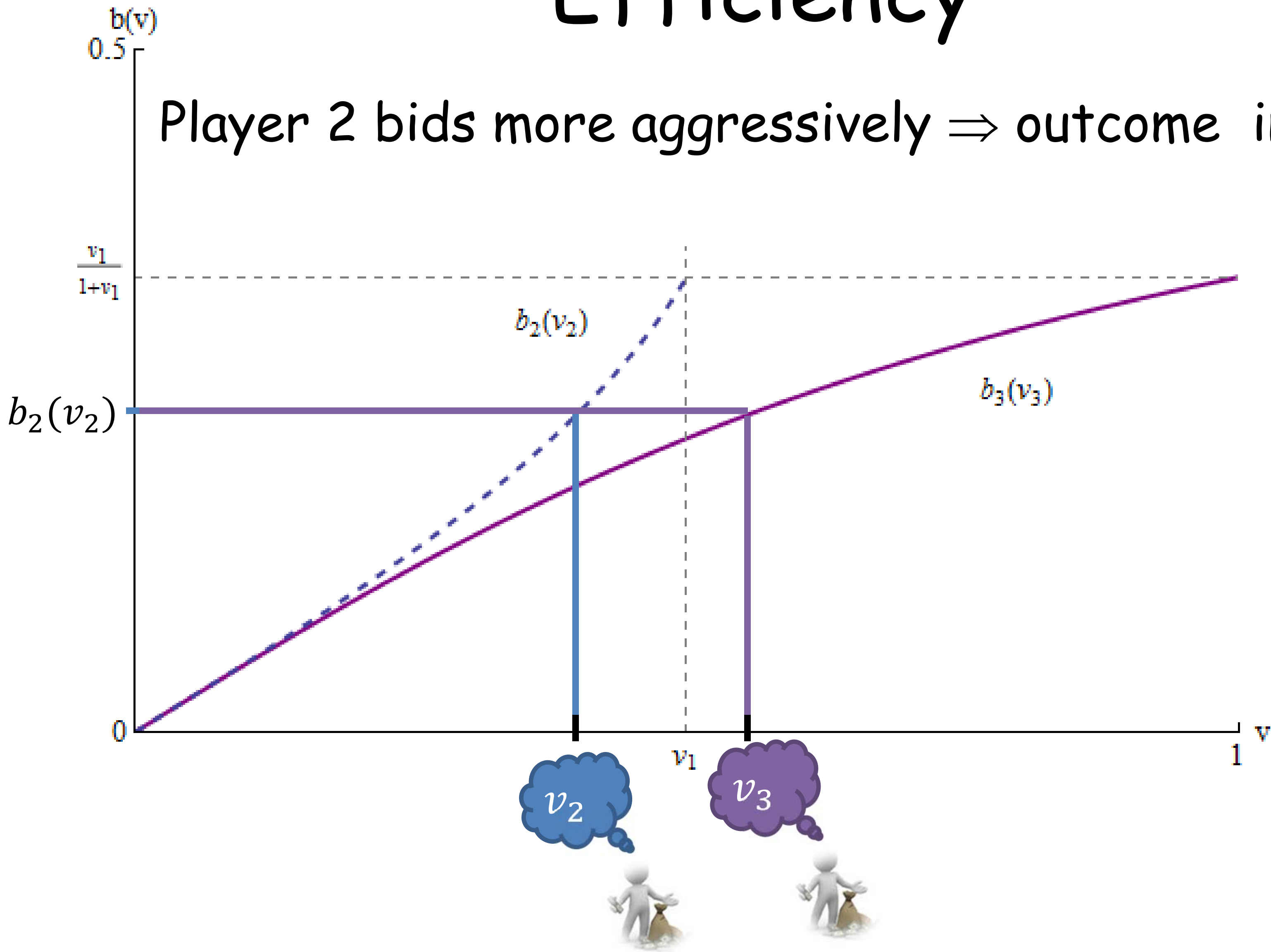


Incomplete Information and Efficiency

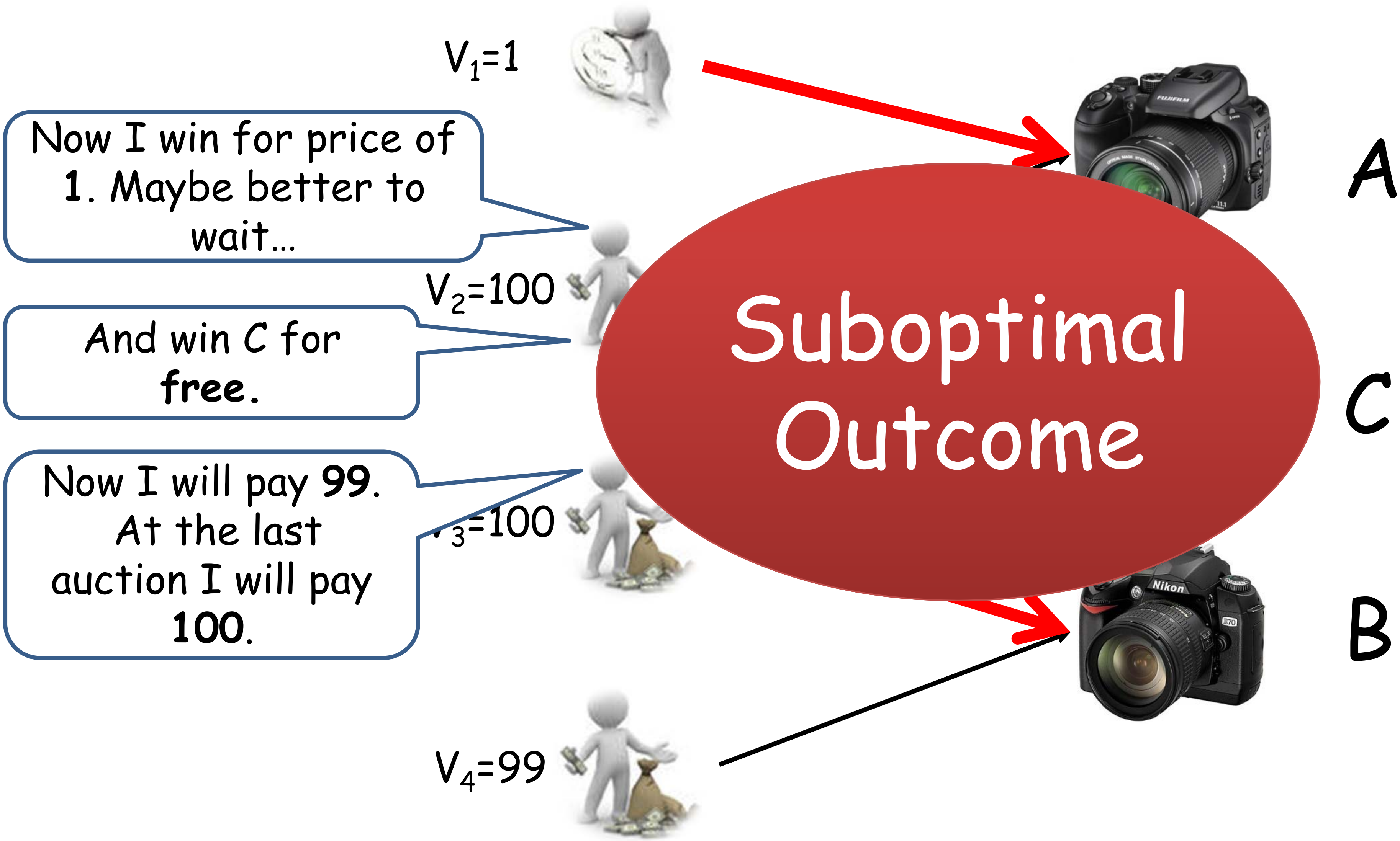


Incomplete Information and Efficiency

Player 2 bids more aggressively ⇒ outcome inefficient



Example



V_1=1
Now I win for price of **1**. Maybe better to wait…
V_2=100
And win C for **free**.
Now I will pay **99**. At the last auction I will pay **100**.
V_3=100
V_4=99
A
C
B
Suboptimal Outcome

Formal model

- A bidding strategy is a bid for each item for each possible history of play on previous items
  - Can depend only on information known to player:
  - Identity of winner, maybe also winner's price.

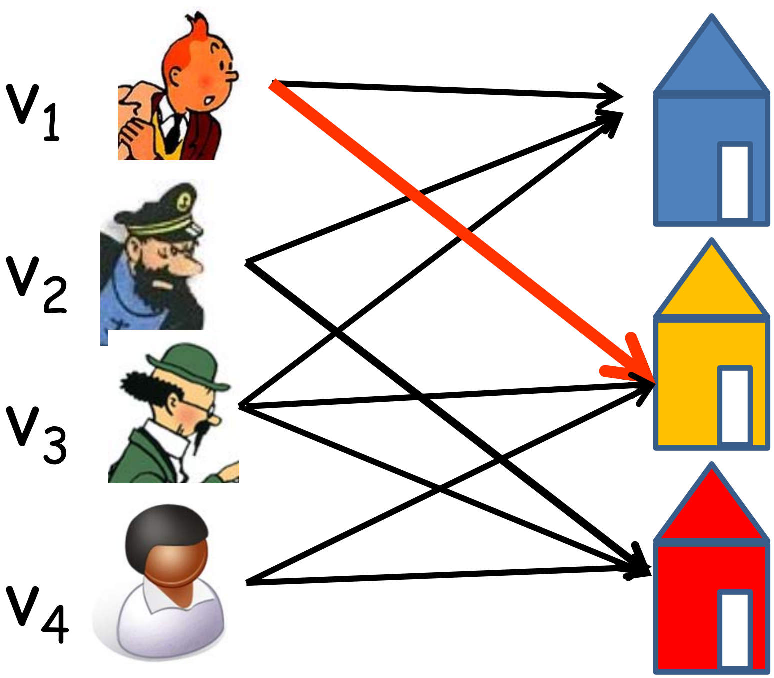
- Solution concept:
  Subgame Perfect Equilibrium
  = **Nash** in each subgame

Bayesian Sequential Auction games

Valuations v drawn from distribution $\mathcal{F}$

For simplicity assume for now

- single value v_i for items of interest
- $(v_1, \ldots, v_n) \in \mathcal{F}$ drawn from a joint distribution



- OPT i_j^* random
- Depends on information i doesn't have!
- Deviating in early auctions may change behavior of others later

Sequential Bayesian Price of Anarchy

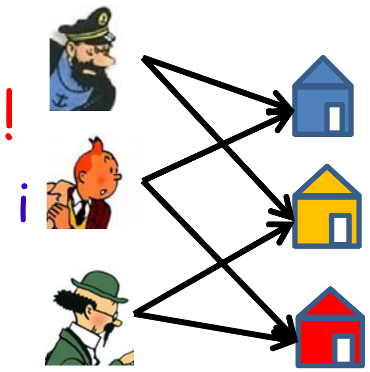
Theorem In first price sequential auction for unit demand single parameter bidders from correlated distributions.

The total value v(N)=$\sum_{i \in N} v_i$ at a Bayesian Nash equilibrium Distribution D of $N = \{(i, j_i)\}$ is at least $\frac{1}{4}$th of optimum expected value of OPT (assuming $b_i \leq v_i$ ∀ i).

proof based player i bidding $\frac{1}{2}v_i$ on all items of interest.

Deviation only noticeable if winning!

- If player wins: hence utility = $\frac{1}{2}v_i$
- If he looses, his items of interest valued at least $\frac{1}{2}v_i$ by others.



In either case $\frac{1}{2}v_{ij_i^*} \geq v_{ij_i} + v(j_i^*)$

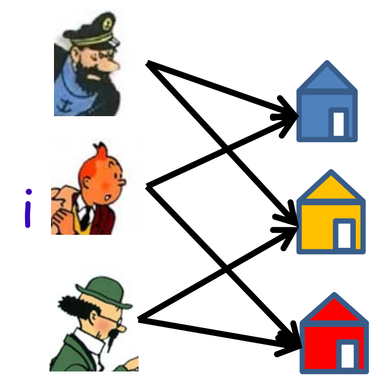
Sum over player, and take expectation over v∈$\mathscr{F}$

$\frac{1}{2}$OPT≥ E(v(N)+ E(v(N))

Bayesian Price of Anarchy

Theorem Unit demand single parameter bidders, the total expected value E(v(N))=$E(\sum_{i \in N} v_i)$ at an equilibrium distribution $N = \{(i,j)\}$ (assuming $b_i \leq v_i \forall$ i) is at least $\frac{1}{4}$ of the expected optimum OPT=$E(\max_{M} \sum_{i \in M} v_i)$

proof "player i has no regret about bidding $\frac{1}{2}$ v_i on all items of interest"

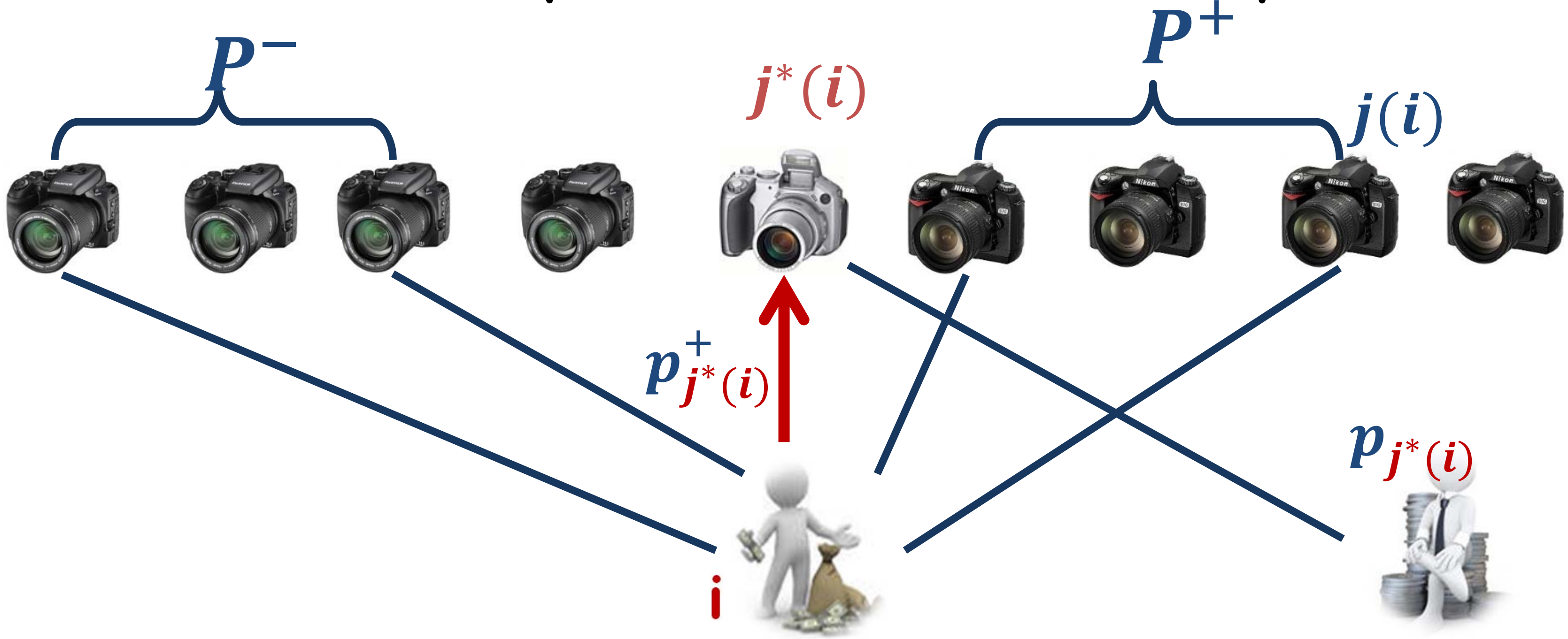


Simple strategy: no regret about this one strategy is all that we need for quality bound!

Applies for learning outcome, and Bayesian Nash with correlated bidder types.

Full info Sequential Auction with unit demand bidders

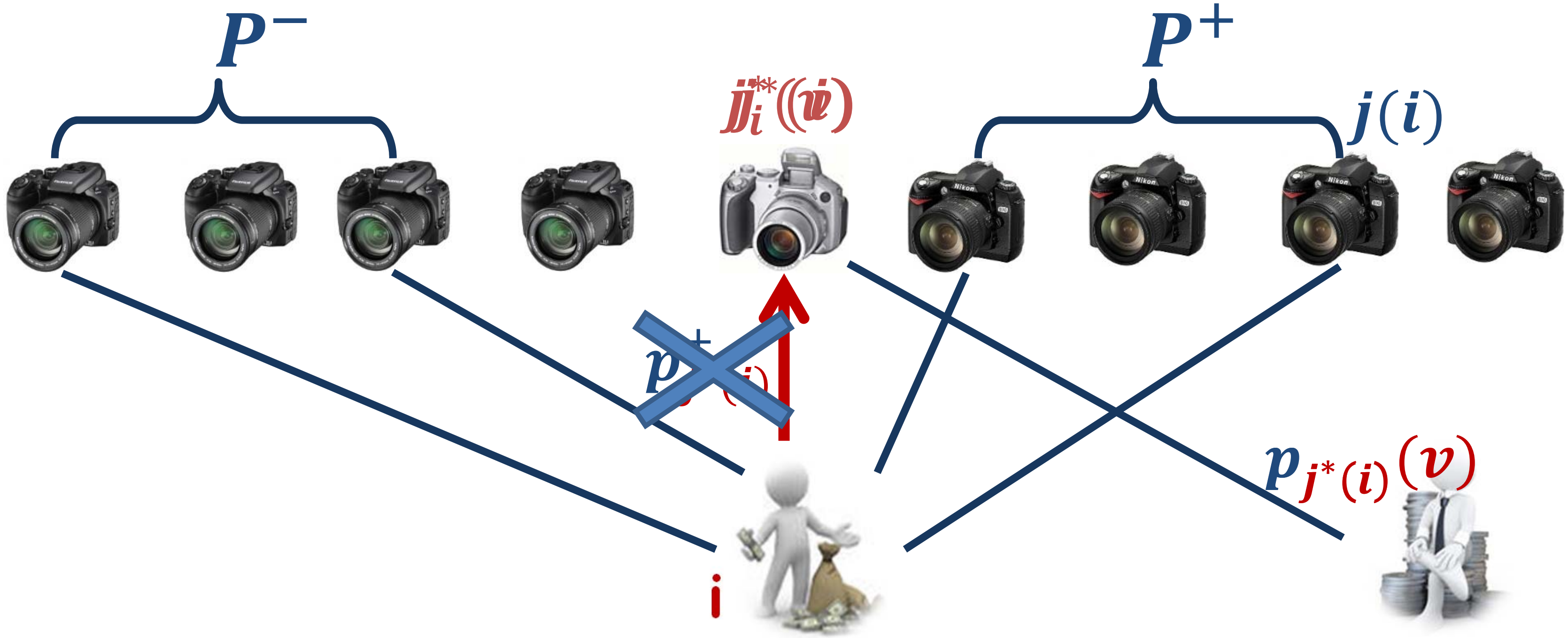
Thm: Value of any Nash at least ½ of optimum



$$v_{ij^*(i)} - p_{j^*(i)} - P^- \leq v_{ij(i)} - P^-$$

Summing for all i: $OPT \leq 2\,SPE$

Bayesian Sequential Auction?



$$v_{ij^*(i)} - p_{j^*(i)} - P^- \leq v_{ij(i)} - P^-$$

Summing for all i: $OPT \leq 2\,SPE$

Complications of Incomplete Information

- $j_i^*(v)$ depends on other players' values which you don't know
- Bidding becomes correlated at later stages of the game since players condition on history

Simultaneous Item Auctions

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]

Unit demand bidders, assuming values drawn independently v_i from $\mathcal{F}_i$, and $b_{ij} \leq v_{ij} \forall$ i&j

the total expected value E(v(N))=$\mathrm{E}\left(\sum_{i \in N} v_{i j_i}\right)$ at an equilibrium distribution $N = \{(i,j)\}$ is at least ½ of the expected optimum OPT=$E\left(\max_M \sum_{(i,j) \in \mathrm{M}} v_{ij}\right)$.

Proof? The assigned item in optimum j_i^* depends on v_{-i} hence not known to i.

Not a possible bid to consider

Simultaneous Item Auctions (proof)

Sample valuations of other players w_{-i} from $\mathcal{F}_{-i}$,
Use (v_i, w_{-i}) to determine j_i^*

- bid $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0 \; \forall j \neq j_i^*$

- Nash's value of j_i^* is v(j_i^*). Exp. cost of item j_i^*

$$\leq E_v \left(v(j_i^*) | \mathrm{v_i} \right)$$

- i's utility for given v_i

$$E_w\left(v_{ij_i^*}\right) - E_w E_{v_{-i}}(v(j_i^*)|v_i)$$

- Use Nash for i

$$E_{v_{-i}}\left(v_{ij_i}\right) \geq E_w\left(v_{ij_i^*}\right) - E_w E_{v_{-i}}(v(j_i^*)|v_i)$$

Simultaneous Item Auctions (proof2)

Use Nash for i

$$E_{v_{-i}}(v_{ij_i}) \geq E_w(v_{ij_i^*}) - E_w E_{v_{-i}}(v(j_i^*)|v_i)$$

- Take expectation over

$$E_v(v_{ij_i}) \geq E_v E_w(v_{ij_i^*}) - E_w E_v(v(j_i^*))$$

  - lhs sum over i: $\sum_i E_v(v_{ij_i}) = Nash(\text{SW})$
  - rhs term 1: $E_v E_w(v_{ij_i^*}) = \mathrm{E}_{\mathrm{v_i}} \mathrm{E}_{\mathrm{w_{-i}}}(\mathrm{v}_{ij_i^*}) = E_v\ (v_{ij_i^*})$
  - Sum over i: $\sum_i E_v E_w(v_{ij_i^*}) = OPT(\text{SW})$ (use indep)
  - Last term sum over i:

$$\sum_i E_w\ E_v\ (v(j_i^*)) = \sum_j E_w E_v(v(j))$$

$$= \sum_j E_v(v(j)) = Nash(SW)$$

Bayesian second Price of Anarchy

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]
Unit demand bidders, assuming values drawn independently v_i from $\mathcal{F}_i$, and $b_{ij} \le v_{ij} \forall$ i&j

the total expected value E(v(N))=$\mathrm{E}\left(\sum_{i \in N} v_{i j_i}\right)$ at an equilibrium distribution $N = \{(i,j)\}$ is at least ½ of the expected optimum OPT=$E\left(\max_M \sum_{(i,j) \in \mathrm{M}} v_{ij}\right)$.

Proof: In expectation over v and w

Nash(SW) ≥OPT(SW)-Nash(SW)

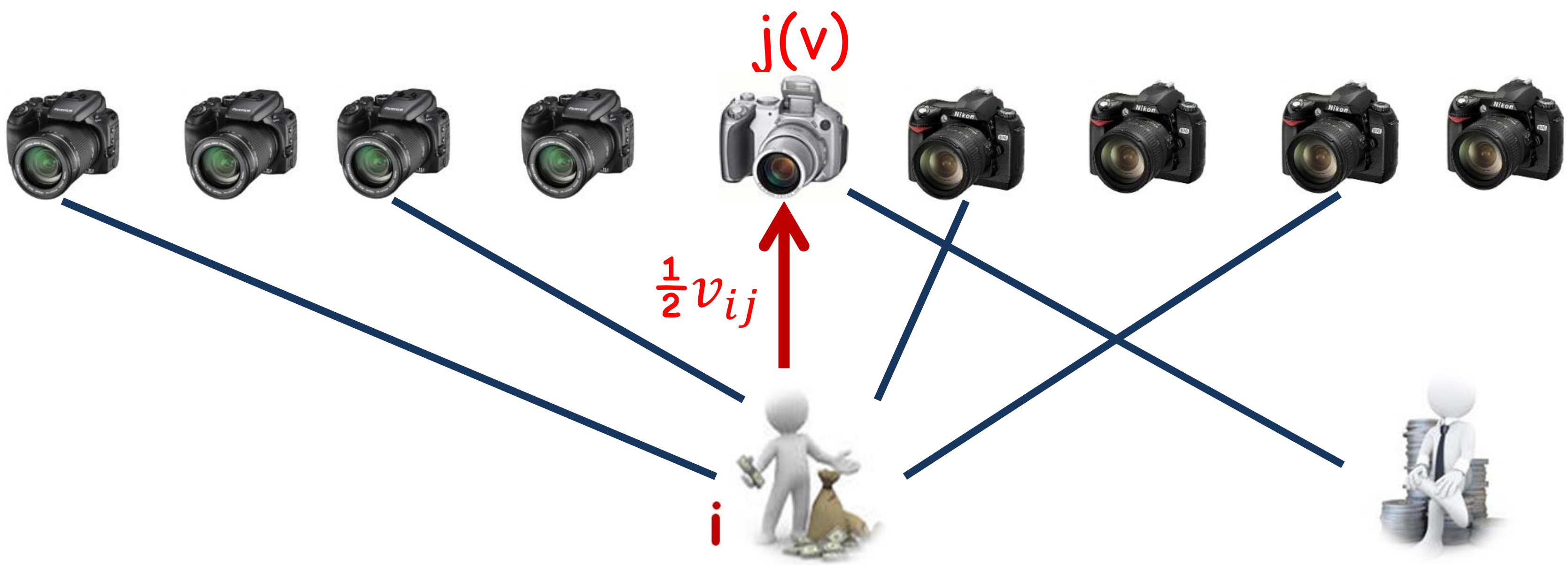
Bayesian Sequential Auction

Try similar idea (idea 1):

Sample valuations of other players w_{-i} from $\mathfrak{F}_{-i}$,

Use (v_i, w_{-i}) to determine $j = j_i^*$

- Bid as before till j comes up, then bid $\frac{1}{2}v_{ij}$ for j



Bayesian Sequential Auction (idea 1)

- If i wins item j then he gets utility at least:

$$v_{ij} - \frac{v_{ij}}{2} - P_{ij}^{-}(v, v_{-i}) = \frac{v_{ij}}{2} - P_{ij}^{-}(v, v_{-i})$$

- If he doesn't then the winning bid must be at least:

$$p_j^{-i}(v_i, v_{-i}) \geq \frac{v_{ij}}{2}$$

- In any case utility from the deviation is at least:

$$\frac{v_{ij}}{2} - P_i(v_i, v_{-i}) - p_j^{-i}(v_i, v_{-i})$$

Correlated Bidding

- $p_j^{-i}(v_i, v_{-i})$ depends implicitly on your bid through the history of play

- When player i arrives at $j_i^*(v_i, w_{-i})$ he doesn't "face" the expected equilibrium price but a "biased" price

- Will not allow us to claim that:
  - "either bidder already gest high value or expected price of some item is high"

The Bluffing Deviation

- Player **draws a random sample w_i from his value** and a random sample w_{-i} of the other players' values
- He plays **as if he was of type w_i** until item

$$\mathrm{j} = j_i^*(v_i, w_{-i})$$

- Then he bids

$$\frac{v_{ij}}{2}$$

The Bluffing Deviation

The utility from the deviation is at least:

$$\frac{v_{ij}}{2} - P_i(w_i, v_{-i}) - p_j^{-i}(w_i, v_{-i})$$

Summing for all players and taking expectation

Note: price for j independent of v_i

$$\frac{1}{2}\boldsymbol{OPT} - \boldsymbol{Rev}(\boldsymbol{SPE}) - \boldsymbol{Rev}(\boldsymbol{SPE}) \leq \boldsymbol{Util}(\boldsymbol{SPE})$$

$$\frac{1}{2}\boldsymbol{OPT} \leq \boldsymbol{2SPE}$$

Simple Auction Games

Examples of simple games

- Item bidding first and second price
- Generalized Second Price

Simple valuations: unit demand

Results: Bounding outcome quality

- Nash,
- Bayesian Nash,
- learning outcomes

Overbidding assumptions

- We used: unit demand bidders
  - assume $b_{ij} \leq v_{ij}$
  - Bidding $b_{ij} > v_{ij}$ is dominated by $b_{ij} = v_{ij}$
- more general 2nd price results use
  - assume $\sum_{j \in S} b_{ij} \leq v_i(S)$
  - A best respond in this class always exists!
- First price: no such assumption is needed
- Sequential Auction: overbidding may be very useful/natural

The Dining Bidder Example

	k					
		…				
 ***A***	1	…	1	0	$10-\frac{k\epsilon}{2}$	0
 ***B***	1	…	1	0	0	$10-\frac{k\epsilon}{2}$
 ***C***	ϵ	…	ϵ	20	$20-\text{fish}-\text{bread}$	
 ***D***	ϵ	…	ϵ	$20-\text{bread}-\frac{\epsilon}{2}$	0	

References and Better results

- [Christodoulou, Kovacs, Schapira ICALP'08] Price of anarchy of 2 assuming conservative bidding, and fractionally subadditive valuations, independent types
- [Bhawalkar, Roughgarden SODA'10] subaddivite valuations,
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- [Syrgkanis, T EC'12] Bayesian Price of Anarchy for sequential auction, better bounds of 3 and 3.16