Auctions as Games: Equilibria and Efficiency
Near-Optimal Mechanisms

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Games and Quality of Solutions

• Rational selfish action can lead to outcome bad for everyone

Model:
• Value for each cow decreasing function of # of cows
• Too many cows: no value left

Tragedy of the Commons
Good Example: Routing Game

- Traffic subject to congestion delays
- cars and packets follow shortest path

Congestion game = cost (delay) depends only on congestion on edges
Simple vs Optimal

- Simple practical mechanism, that lead to good outcome.
- Optimal outcome is not practical
Simple vs Optimal

• Simple practical mechanism, that lead to good outcome.
• optimal outcome is not practical

Also true in many other applications:

• Need distributed protocol that routers can implement
• Models a distributed process

e.g. Bandwidth Sharing, Load Balancing,
Games with good Price of Anarchy

• Routing:
  • Cars or packets through the Internet

• Bandwidth Sharing:
  • Routers share limited bandwidth between processes

• Facility Location:
  • Decide where to host certain Web applications

• Load Balancing
  • Balancing load on servers (e.g. Web servers)

• Network Design:
  • Independent service providers building the Internet
Today Auction “Games”

Basic Auction: single item Vickrey Auction

Player utility $v_i - p_i$ — item value -price paid

Vickrey Auction (second price) - Truthful
- Efficient
- Simple

Extension VCG (truthful and efficient), but not so simple
Combinatorial Auctions

Buyers have values for any subset $S$: $v_i(S)$
user utility $v_i(S) - p_i$ — value – price paid

• Efficient assignment: $\max \sum_i v_i(S^*_i)$
over partitions $S^*_i$

• Payment: welfare loss of others
$p_i = \max \sum_{j \neq i} v_j(S_j) - \sum_{j \neq i} v_j(S^*_j)$

Truthful!
Truthful Auction

Special case: unit demand bidders:

\[ v_{ij} = \text{buyer i's value for house j} \]

\[ v_i(S) = \max_{j \in S} v_{ij} \]

Assignment: max value matching \( M^* = \{ (i, j^*_i) \} \)

\[ \max_{M^*} \sum_i v_{ij^*_i} \]

- \text{price} = \text{welfare loss of others}

\[ p_i = \max_{M = \{(k, j_k)\}} \sum_{k \neq i} v_{kj_k} - \sum_{k \neq i} v_{kj_k^*} \]
Truthful Auction

Special case: unit demand bidders:

Assignment: max value matching

$$\max_{M^*} \sum_i v_{ij_i^*}$$

price = welfare loss of others

$$p_i = \max_{M=\{(k,j_k)\}} \sum_{k \neq i} v_{kj_k} - \sum_{k \neq i} v_{kj_k^*}$$

- Requires computation and coordination
- pricing unintuitive
Auctions as Games

simpler auction game are better in many settings.

– analyze simple auctions
– understand which auctions well and which work less well

First idea: simultaneous second price
Auctions as Games

• Simultaneous second price?
  Christodoulou, Kovacs, Schapira ICALP'08
  Bhawalkar, Roughgarden SODA’10
• Greedy Algorithm as an Auction Game
  Lucier, Borodin, SODA’10
• AuAuctions (GSP)
  Paes-Leme, T FOCS’10, Lucier, Paes-Leme + CKKK EC’11
• First price?
  Hassidim, Kaplan, Mansour, Nisan EC’11
• Sequential auction?
  Paes Leme, Syrgkanis, T SODA’12, EC’12

Question: how good outcome to expect?
Simultaneous Second Price
unit demand bidders

• Is simultaneous second price truthful

No!

limited bidding language

How about Nash equilibria?
Nash equilibria of bidding games

Vickrey Auction - Truthful, efficient, simple (second price)

but has many bad Nash equilibria

Assume bid \leq value (higher bid is dominated)

**Theorem:** all Nash equilibria efficient: highest value winning
Simultaneous Second Price unit demand bidders

Bidding above the item value is dominated:
Assume $b_{ij} \leq v_{ij}$ all $i$&$j$.

Question:
How good are Nash equilibria?
Price of Anarchy

**Theorem** [Christodoulou, Kovacs, Schapira ICALP’08]

Total value $v(N) = \sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least $\frac{1}{2}$ of optimum $OPT = \max_{M^*} \sum_i v_{ij_i}^*$ (assuming $b_{ij} \leq v_{ij} \forall i \& j$).

**Proof** Consider the optimum $M^*$.
If $i$ won $j_i^*$ he has the same value as in $OPT$
Else, some other player $k$ won $j_i^*$
Current solution is Nash: $i$ cannot improve his utility by changing his bid
Price of Anarchy

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]

Total value \( v(N) = \sum_i v_{iji} \) at a Nash equilibrium \( N = \{(i, j_i)\} \) is at least \( \frac{1}{2} \) of optimum \( \text{OPT} = \max_{M^*} \sum_i v_{iji} \) (assuming \( b_{ij} \leq v_{ij} \ \forall \ i \& j \)).

Proof (cont.) player k won \( j_k = j_i^* \)

player i could bid \( b_{iji} = v_{iji}^* \) and \( b_{ij} = 0 \ \forall j \neq j_i^* \)
- If he wins he gets value \( v_{iji}^* - b_{kji}^* \)
- Else \( v_{iji}^* \leq b_{kji}^* \)

In either case

\[
 v_{iji} \geq v_{iji}^* - b_{kji}^* \geq v_{iji}^* - v_{kjk} \quad \text{(assuming} \ b_{ij} \leq v_{ij} \text{)}
\]

Sum over all players: \( \text{Nash} \geq \text{OPT} - \text{Nash} \)
Unit Demand Bidders: example

Nash value $19 + 1 = 20$
Bids 0, 1, 19, 0
OPT value $20 + 20 = 40$

Inequalities

$1 \geq 20 - 19$   winner of his item has high value at Nash
$19 \geq 20 - 1$   he has high value at Nash

Both “charging” to the same high value at OPT
Our questions

**Theorem** [Christodoulou, Kovacs, Schapira ICALP’08]

Total value $v(N) = \sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i,j_i)\}$ is at least $\frac{1}{2}$ of optimum $\text{OPT} = \max_{M^*} \sum_i v_{ij_i}^*$ (assuming $b_{ij} \leq v_{ij} \forall i \& j$).

✓ Quality of Nash Equilibria

- What if stable solution is not found?
  
  Is such a bound possible outside of Nash outcome?

- What if other player’s values are not known
  
  Is such a bound possible for a Bayesian game?

- Other games?
  
  Do bounds like this apply other kind of game?
Selfish Outcome (2)?

Is Nash the natural selfish outcome?

How do users coordinate on a Nash equilibrium, e.g., which do they choose?

- Does natural behavior lead to no Nash?
- Which Nash?
- Finding Nash is hard in many games...
- What is natural behavior?
  - Best response?
  - Noisy Best response (e.g. logit dynamic)
  - Learning?
  - Copying others?
Auctions and No-Regret Dynamics

Run Auction on (b_1^1, b_2^1, ..., b_n^1)

Run Auction on (b_1^t, b_2^t, ..., b_n^t)

Vanishingly small regret for any fixed strat x:

\[ \sum_t u_i(b_i^t, b_{-i}^t) \geq \sum_t u_i(x, b_{-i}^t) - o(T) \]
Learning:
see Avrim Blum starting Wednesday

Iterated play where users update play based on experience

Traditional Setting: stock market
m experts N options

Goal: can we do as well as the best expert?

Regret = average utility of single best strategy with hindsight - long term average utility.
No Regret Learning

Goal: can we do as well as the best expert? -as the single stock in hindsight?

Idea: if there is a real expert, we should find out who it is after a while.

No regret: too hard (would need to know expert at the start)

Goal: small regret compared to range of cost/benefit
Learning in Games

Goal: can we do (almost) as well as the best expert?

Games?

Focus on a single player: experts = strategies to play

Goal: learn to play the best strategy with hindsight

Best depends on others
Learning in Games

Focus on a single player: experts = strategies to play
Goal: learn to play the best strategy with hindsight

Best depends on others did

Example: matching pennies

With $q=(\frac{1}{2}, \frac{1}{2})$, best value with hindsight is 0.
Regret if our value $< 0$
Learning in Games

Focus on a single player:
experts = strategies to play

Goal: learn to play the best strategy with hindsight

Best depends on others did

Example: matching pennies

With \( q = (\frac{3}{4}, \frac{1}{4}) \), best value with hindsight is \( \frac{1}{2} \) (by playing top).
Regret if our value < \( \frac{1}{2} \)
Learning and Games
see Avrim Blum starting Wednesday

- **Regret** = average utility of single best strategy with hindsight - long term average utility.

Nash = strategy for each player so that players have no regret

**Hart & Mas-Colell:** general games $\rightarrow$ Long term average play is (coarse) correlated equilibrium

Simple strategies guarantee vanishing regret.
(Coarse) correlated equilibrium

*Coarse correlated equilibrium:* probability distribution of outcomes such that for all players expected utility $\geq$ exp. utility of any fixed strategy

Correlated eq. & players independent = Nash

Learning:
Players update independently, but correlate on shared history
Quality of learning outcome

**Theorem** Unit demand bidders, the total value $v(N) = \sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least $\frac{1}{2}$ of optimum $\text{OPT} = \max_{M^*} \sum_i v_{ij_i}$ (assuming $b_{ij} \leq v_{ij} \forall i & j$).

How about outcome of no-regret learning (coarse correlated equilibria)?

Same bound applies!

Idea: proof was based on “player i has no regret about one strategy”

bid $b_{ij_i}^* = v_{ij_i}^*$ and $b_{ij} = 0 \forall j \neq j_i^*$

outcome of no-regret learning: no regret about any strategy!
Quality of learning outcome

**Theorem** Unit demand bidders, the total value $E[v(N)] = E[\sum_i v_{iji}]$ expected value at an outcome distribution $D = \{(i, j_i)\}$ with no regret is $\geq \frac{1}{2}$ of $OPT = \max_{M^*} \sum_i v_{ij_i}^*$ (assuming $b_{ij} \leq v_{ij} \ \forall \ i \& j \& \text{all bids}$).

**Proof:** player $i$ has no regret about one strategy bid $b_{ij_i}^* = v_{ij_i}^*$ and $b_{ij} = 0 \ \forall j \neq j_i^*$

Price of $j_i^*$ is a bid by an other player $\leq \text{value } v_{ij_i} = \text{value for player } i$

$b_j = \text{bid winning item } j \leq v(j) = \text{value for winner}$

$$E(v_{ij_i}) \geq v_{ij_i}^* - E(b_{ij_i}^*) \geq v_{ij_i}^* - E(v(j_i^*))$$

Sum over all player $E_D(SW) \geq OPT - E_D(SW)$
Our questions

**Theorem** [Christodoulou, Kovacs, Schapira ICALP'08]

Total value \( v(N) = \sum_i v_{ij_i} \) at a Nash equilibrium \( N = \{(i,j_i)\} \) is at least \( \frac{1}{2} \) of optimum \( \text{OPT} = \max_{M^*} \sum_i v_{ij_i^*} \) (assuming \( b_{ij} \leq v_{ij} \ \forall \ i & j \)).

✓ Quality of Nash Equilibria

✓ What if stable solution is not found?
  
  Is such a bound possible outside of Nash outcome?

• What if other player’s values are not known
  
  Is such a bound possible for a Bayesian game?

• Other games?
  
  Do bounds like this apply other kind of game?
Bayesian Auction games

Valuations $v$ drawn from distribution $F$

For simplicity assume for now

- single value $v_i$ for items of interest
- $(v_1, ..., v_n) \in F$ drawn from a joint distribution

- $\text{OPT } i_j^*$ random
- Depends on information $i$ doesn’t have!
Bayesian Price of Anarchy

**Theorem** Unit demand bidders, the total value \( v(N) = \sum_i v_{ij_i} \) at a Nash equilibrium \( N = \{(i,j_i)\} \) is at least \( \frac{1}{2} \) of optimum \( \text{OPT} = \max_{M^*} \sum_i v_{ij_i^*} \) (assuming \( b_{ij} \leq v_{ij} \forall i,j) \).

How about outcome of Bayesian game?

proof was based on “player i has no regret about one strategy”

- \( b_{ij_i^*} = v_{ij_i^*} \) and \( b_{ij} = 0 \ \forall j \neq j_i^* \)

- **Optimal item** \( j_i^* \) depends on others
- **Player can have no regret about any fixed item j, but not about** \( j_i^* \)
Bayesian Price of Anarchy

**Theorem** Unit demand single parameter bidders, total expected value $E(v(N)) = E(\sum_{i \in N} v_i)$ at an equilibrium distr. $N = \{(i,j)\}$ (assuming $b_i \leq v_i \forall i$) is at least $\frac{1}{4}$ of the $\text{OPT} = E(\max_M \sum_{i \in M} v_i)$ assuming auction guarantees max one assigned item

proof “player $i$ has no regret about bidding $\frac{1}{2}v_i$”

- If player wins: price $\leq b_i \leq \frac{1}{2}v_i$
  hence utility at least $\frac{1}{2}v_i$
- If he looses, all his items of interest, went to players with bid (and hence value) at least $\frac{1}{2}v_i$.

In either case

$$\frac{1}{2}v_{ij^*} \geq v_{ij} + b_{j^*} \geq v_{ij} + v(j^*)$$

Sum over player, and take expectation over $v \in \mathcal{F}$

$$\frac{1}{2}\text{OPT} \geq E(v(N) + E(v(N)))$$
Our questions

**Theorem** [Christodoulou, Kovacs, Schapira ICALP’08]

Total value $v(N) = \sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least $\frac{1}{2}$ of optimum $OPT = \max_{M^*} \sum_i v_{ij_i}^*$ (assuming $b_{ij} \leq v_{ij} \forall i, j$).

- Quality of Nash Equilibria

- What if stable solution is not found?
  
  Is such a bound possible outside of Nash outcome?

- What if other player’s values are not known
  
  Is such a bound possible for a Bayesian game?

- Other games?
  
  Do bounds like this apply other kind of game?
AdAuction

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Online Ads

Online auctions:
• Display ads
• Search Ads

Powerful ad:
customized by information about user
Search term, History of user, Time of the day,
Geographic Data, Cookies, Budget
• Millions of ads each minute, and all different!
• Needs a simple and intuitive scheme
Model of Sponsored Search

Ordered slots, higher is better

Advertisers:
Hilton, RailEurope, CentralBudapestHotels, DestinationBudapest, RacationRentals.com, Travelzoo.com, TravelYahoo.com, BudgetPlace.com
Selling one Ad Slot

Prospective advertisers

- Hilton Hotel Budapest
  - Our best rates guaranteed online.
  - Book at the official Hilton site.
  - $2

- Discover Budapest
  - And The Many Hungarian Cities.
  - Find Places Trains Can Take You.
  - $5

- Budapest Central Hotel
  - Central Location, Great Rates
  - Fantastic B&B, Book Online!
  - $7

- Budapest Hotel Apartments
  - Hotels, apartments, pension, hostel
  - Book & stay Budapest room from 25€!
  - $3

- Budapest Vacation Listings
  - Thousands of Vacation Home Choices
  - Book Directly from Owners & Save!
  - $4

Pays $5

Vickrey Auction

- Truthful
- Efficient
- Simple
- ...

Bids on click value

$\alpha = \text{click rate}$
Keyword Auction = Matching Problem

Version 1
• n ads and n slots
• Each advertiser has a value $v_k$ per click
• Each slot has click through rate $\alpha_j$

• Value of slot j for k
  $v_{kj} = v_k \cdot \alpha_j$

$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$
Maximizing welfare (matching)

- $n$ advertisers and $n$ slots
- Each advertiser has a value $v_i$
- Click through rate is $\alpha_j$
- $\max \sum_j \alpha_j v_j = \text{total value}$

Assume:

\begin{align*}
v_1 &\geq v_2 \geq \ldots \geq v_n \\
\alpha_1 &\geq \alpha_2 \geq \ldots \geq \alpha_n
\end{align*}
VCG for AdAuctions

• n advertisers and n slots

Assignment: max total value $\sum_i \alpha_i v_i$

Price paid
$p_i =$ welfare loss of others

$p_i = \sum_{j>i} (\alpha_{j-1} - \alpha_j) v_j$

Assume:
$v_1 \geq v_2 \geq ... \geq v_n$
$\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_n$
**Generalized Second Price (GSP)**

- Users bid per click
- Sort by bid
- **Charge next lower bid for each click**

Recall:

Analogous rule for lower slots

Sort by

\[ b_{\pi(1)} \geq b_{\pi(2)} \geq ... \geq b_{\pi(n)} \]
Is GSP truthful?

Is bidding $b_k = v_k$ Nash equilibrium for the bidders?

Example:
Bidder 1’s value if telling the truth
$(9-5) \cdot 1 = 4$
If bidding $b_1 < 5$
$(9-1) \cdot 0.9 = 7.2$

Sort by bid value
$b_1 > b_2 > b_3 > b_4 > \ldots$

Charge next price $p = b_{k+1}$

Value to bidder $k$
$(v_k - b_{k+1}) \cdot \alpha_k$
Measuring efficiency

Social welfare

\[ \text{Social welfare} = \text{click} \cdot \text{value} = \sum_i v_i \alpha_{\sigma(i)} \]
Measuring inefficiency

Price of Anarchy $= \max_{\text{Nash}} \frac{\max SW}{SW(\text{Nash})}$

Price of Stability $= \min_{\text{Nash}} \frac{\max SW}{SW(\text{Nash})}$

Equilibrium selection?
Theorem [Edelman, Ostrovsky, Schwarz’07 & Varian’06] Envy free equilibria maximize social welfare, and envy free 3.

(Price of stability 1)


[Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, EC’11] improved to 1.282

True in the full information model only

Full Information: ∃ Good equilibria
Today: a game with uncertainty

Two forms of uncertainty:
- participants
- quality factors

Bayesian setting (no efficient Nash)

[Gomes, Sweeney 09]
Keyword Auction with quality factors

Version 2

- n ads and n slots
- Each advertiser has a value $v_k$ per click
- Each slot has click through rate $\alpha_j$
- "ad-quality" a click through rate $\gamma_k$
- Click through rate of slot j for k $\gamma_k \alpha_j$

separable model

\[
\gamma_1 v_1 \quad \frac{\alpha_1}{\ldots} \\
\gamma_2 v_2 \quad \frac{\alpha_2}{\ldots} \\
\gamma_3 v_3 \quad \frac{\alpha_3}{\ldots} \\
\gamma_4 v_4 \quad \frac{\alpha_4}{\ldots} \\
\gamma_5 v_5 \quad \frac{\alpha_5}{\ldots} \\
\vdots \\
\gamma_k v_k \alpha_j \quad \text{Effective value}
\]
Generalized Second Price (GSP)

- Users bid per click
- Sort by bid $\cdot \gamma$
- Charge critical price for each click

Value of player $k$ in slot $j$:
$$k = \pi(j)$$

$$u_k = \alpha_j \gamma_k (v_k - p_k)$$

$$\gamma_k p_k = \gamma \pi(j+1) b_{\pi(j+1)}$$

Sort by $\gamma_k b_k$
Uncertainty about Ad Quality

Google

```
pizza
About 1,280,000,000 results (0.14 seconds)
```

Computer via machine learning from

- History of user
- Time of the day
- Geographic Data
- Cookies
- Budget, ...

\[ b_k \rightarrow b_k \gamma_k \]
- valuations fixed (full information) or Bayesian.
- But Ad Quality uncertain, only distribution known (possibly correlated)
Model with Ad Quality Uncertainty

\[ v_k \]  \quad b_k \quad \gamma_k \quad \alpha_j \quad j = \sigma(k) \]

Nash equilibrium:
\[ E[u_k(b_k, b_{-k})] \geq E[u_k(b'_k, b_{-k})] \]

Expectation over participants and quality factors \( \gamma \)
Theorem: [Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, Lucier, Paes Leme, T] Even if values are arbitrarily correlated, the PoA is bounded by 4.

Proof sketch for bound of 4 full info:
- Focus on person $i$ with slot in Opt $\mu(i)$
- Deviate to $\frac{1}{2}v_i$ whenever your value is $v_i$
- Either you get slot $\mu(i)$ or better and

\[ u_i(\frac{1}{2}v_i, b_{-i}) \geq \frac{1}{2} \alpha_{\mu(i)} v_i \]

Assume $\gamma_i = 1$ all $i$
Simple proof PoA for welfare

Proof sketch for bound of 4 full info:
- Deviate to $\frac{1}{2}v_i$ whenever your value is $v_i$
- either get slot $\leq \mu(i)$ and $u_i(\frac{1}{2}v_i, b_{-i}) \geq \frac{1}{2}\alpha_{\mu(i)} v_i$
- Or the player in that slot has value $\geq \frac{1}{2}v_i$

$$\alpha_{\mu(i)} v_{\sigma^{-1}(\mu(i))} \geq \alpha_{\mu(i)} \frac{1}{2} v_i$$

Add two options

$$u_i(\frac{1}{2}v_i, b_{-i}) + \alpha_{\mu(i)} v_{\sigma^{-1}(\mu(i))} \geq \frac{1}{2}\alpha_{\mu(i)} v_i$$
**Simple proof PoA for welfare**

**Theorem**: Even if values are arbitrarily correlated, the PoA is bounded by 4

**Proof sketch** for bound of 4 Bayesian:
- Deviate to $\frac{1}{2}v_i$ whenever your value is $v_i$

$$u_i\left(\frac{1}{2}v_i, b_{-i}\right) + \alpha_{\mu(i)} v_{\sigma^{-1}(\mu(i))} \geq \frac{1}{2} \alpha_{\mu(i)} v_i$$

- true for every realization of the random vars
- sum all players, take expectations, use Nash

$$\Sigma_i E(u_i(v)) + \Sigma_j E(\alpha_{\sigma(j)} v_j) \geq \frac{1}{2} \Sigma_i E(\alpha_{\mu(i)} v_i)$$

$$\text{NASH} + \text{NASH} \geq \frac{1}{2} \text{OPT}$$
Efficiency of Outcome

Proof idea: deviate to $\frac{1}{2}v_i$ when your value is $v_i$.  
This is a “no-regret” style bound:  
don’t regret not playing $\frac{1}{2}v_i$  
$\Rightarrow$ Bound applies to learning outcomes

If proof uses only “no-regret”-bound then extends to learning outcomes.

If regret only used for $\frac{1}{2}v_i$ (depends on $v_i$ only), extends to Bayesian game with correlated types.
Simple Auction Games

What we have seen so far

• item bidding games simple item bidding
• Generalized Second Price
• Very simple valuations: unit demand or even single parameter

Simple proof technique bounding outcome quality (Nash, Bayesian Nash, learning outcomes)
References and Better results

- [Christodoulou, Kovacs, Schapira ICALP’08] Price of anarchy of 2 assuming conservative bidding, and fractionally subadditive valuations, independent types
- [Bhawalkar, Roughgarden SODA’10] subadditive valuations
- [Syrgkanis, T] Improved bound of 3 for unit-demand single value version with correlated types

- [Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, Lucier, Paes-Leme, T] Improved bound of 2.93 for GSP with uncertainty either Bayesian model or quality factor uncertainty.