Auctions as Games: Equilibria and Efficiency

Near-Optimal Mechanisms

Éva Tardos, Cornell

Games and Quality of Solutions

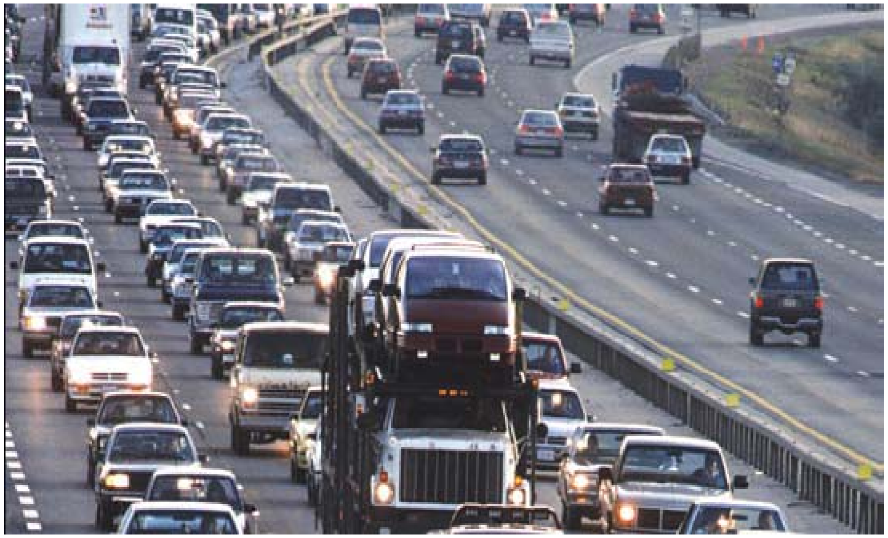
Tragedy of the Commons

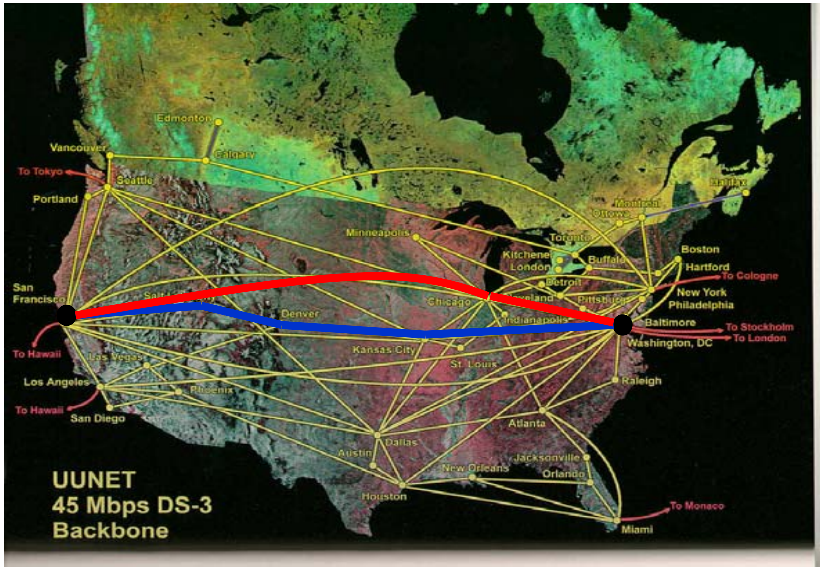
- Rational selfish action can lead to outcome bad for everyone

Model:

- Value for each cow decreasing function of # of cows
- Too many cows: no value left

Good Example: Routing Game



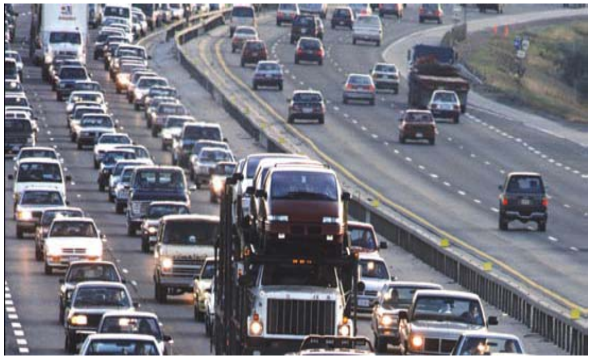


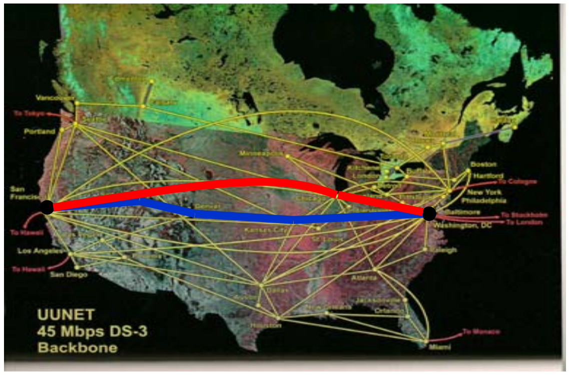
- Traffic subject to congestion delays
- cars and packets follow shortest path

Congestion game =cost (delay) depends only on congestion on edges

Simple vs Optimal

- Simple practical mechanism, that lead to good outcome.
- optimal outcome is not practical





Simple vs Optimal

- Simple practical mechanism, that lead to good outcome.
- optimal outcome is not practical

Also true in many other applications:

- Need distributed protocol that routers can implement
- Models a distributed process

e.g. Bandwidth Sharing, Load Balancing,

Games with good Price of Anarchy

- Routing:
- Cars or packets though the Internet
- Bandwidth Sharing:
- routers share limited bandwidth between processes
- Facility Location:
- Decide where to host certain Web applications
- Load Balancing
- Balancing load on servers (e.g. Web servers)
- Network Design:
- Independent service providers building the Internet

Today Auction "Games"

Basic Auction: single item Vickrey Auction

$2 $5 $7 $3 $4

Pays $5

Player utility $v_i - p_i$ — item value -price paid

Vickrey Auction
(second price)

- Truthful
- Efficient
- Simple

Extension VCG (truthful and efficient),
but not so simple

Vickrey, Clarke, Groves

Combinatorial Auctions

Buyers have values for any subset S: $v_i(S)$

user utility $v_i(S) - p_i$ — value –price paid

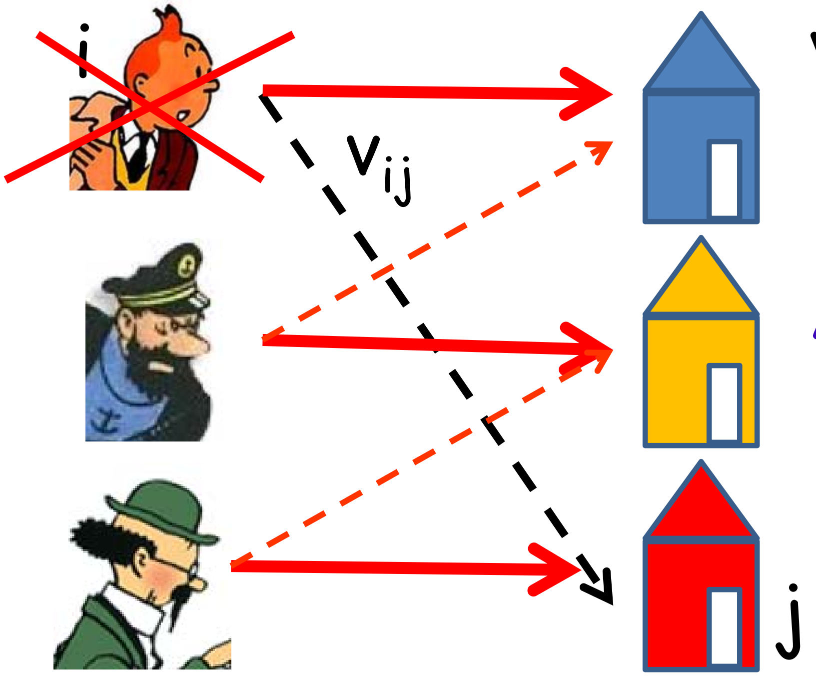
- Efficient assignment: $\max \sum_i v_i(S^*_i)$ over partitions S^*_i
- Payment: welfare loss of others

$$p_i = \max \sum_{j\neq i} v_j(S_j) - \sum_{j\neq i} v_j(S^*_j)$$

Truthful!

Truthful Auction

Special case: unit demand bidders:



v_{ij} = buyer i's value for house j

$$v_i(S) = \max_{j \in S} v_{ij}$$

Assignment: max value matching $\boldsymbol{M^* = \{(i, j_i^*)\}}$

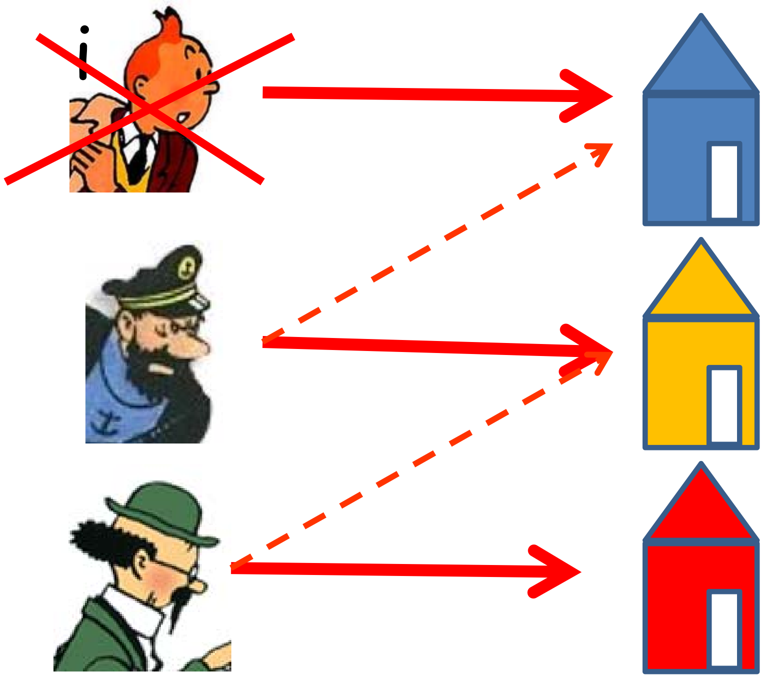
$$\max_{M^*} \sum\nolimits_i v_{ij_i^*}$$

- price = welfare loss of others

$$p_i = \max_{M=\{(k,j_k)\}} \sum\nolimits_{k \neq i} v_{kj_k} - \sum\nolimits_{k \neq i} v_{kj_k^*}$$

Truthful Auction

Special case: unit demand bidders:



Assignment: max value matching

$$\max_{M^*} \sum\nolimits_i v_{i j_i^*}$$

price = welfare loss of others

$$p_i = \max_{M=\{(k,j_k)\}} \sum\nolimits_{k\neq i} v_{k j_k} - \sum\nolimits_{k\neq i} v_{k j_k^*}$$

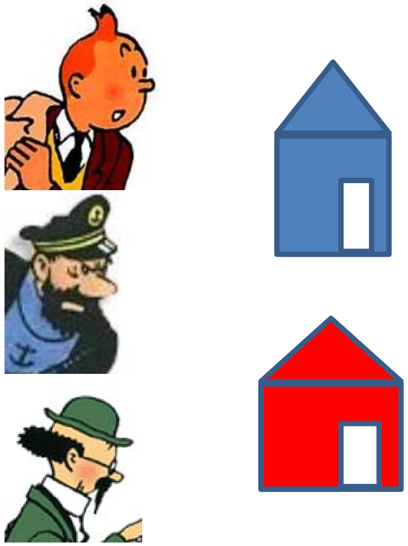
- Requires computation and coordination
- pricing unintuitive

Auctions as Games

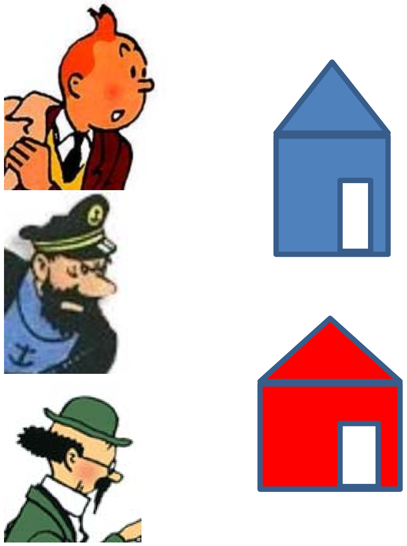
simpler auction game are better in many settings.

- analyze simple auctions
- understand which auctions well and which work less well

First idea: simultaneous second price



Auctions as Games



- Simultaneous second price?
  Christodoulou, Kovacs, Schapira ICALP'08
  Bhawalkar, Roughgarden SODA'10
- Greedy Algorithm as an Auction Game
  Lucier, Borodin, SODA'10
- AuAuctions (GSP)
  Paes-Leme, T FOCS'10, Lucier, Paes-Leme + CKKK EC'11
- First price?
  Hassidim, Kaplan, Mansour, Nisan EC'11
- Sequential auction?
  Paes Leme, Syrgkanis, T SODA'12, EC'12

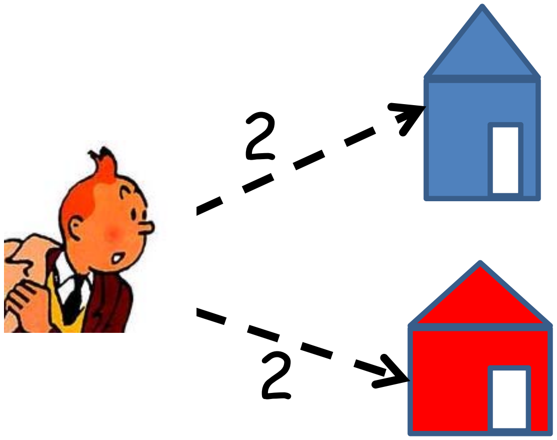
Question: how good outcome to expect?

Simultaneous Second Price unit demand bidders

- Is simultaneous second price truthful

No!

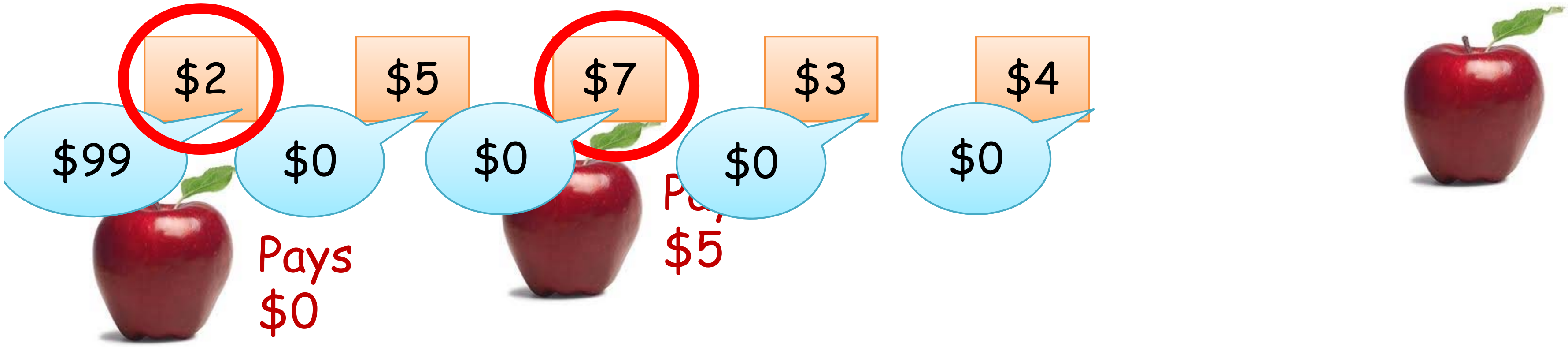
limited bidding language



How about Nash equilibria?

Nash equilibria of bidding games

Vickrey Auction - Truthful, efficient, simple (second price)



but has many bad Nash equilibria

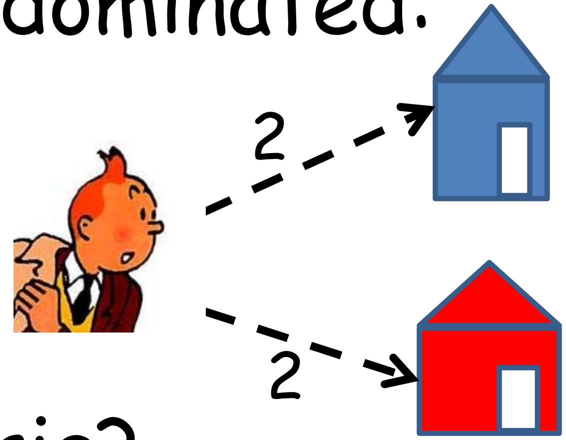
Assume bid ≤ value (higher bid is dominated)

Theorem: all Nash equilibria efficient: highest value winning

Simultaneous Second Price unit demand bidders

Bidding above the item value is dominated:

Assume $b_{ij} \leq v_{ij}$ all i&j.



Question:

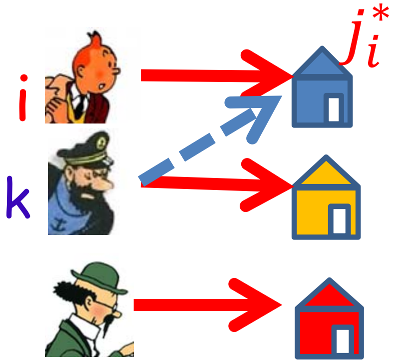
How good are Nash equilibria?

Price of Anarchy

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]

Total value v(N)=$\sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least ½ of optimum OPT= $\max_{M^*} \sum_i v_{ij_i^*}$ (assuming $b_{ij} \le v_{ij}$ ∀ i&j).

Proof Consider the optimum M^*.
If i won j_i^* he has the same value as in OPT
Else, some other player k won j_i^*
Current solution is Nash: i cannot improve his utility by changing his bid



Price of Anarchy

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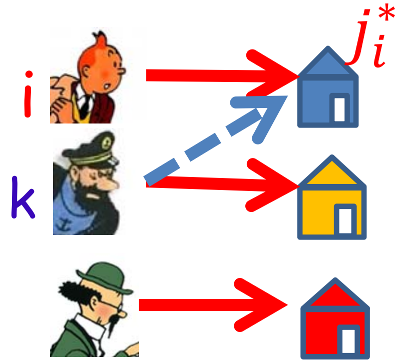
Proof (cont.) player k won $j_k = j_i^*$



player i could bid $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0 \ \forall j \neq j_i^*$

- If he wins he gets value $v_{ij_i^*}$ - $b_{kj_i^*}$

- Else $v_{ij_i^*} \leq \ b_{kj_i^*}$

In either case

$$v_{ij_i} \geq v_{ij_j^*} - b_{kj_i^*} \ \geq v_{ij_i^*} - v_{kj_k} \quad \text{(assuming } b_{ij} \leq v_{ij}\text{)}$$

Sum over all players: Nash ≥ OPT - Nash

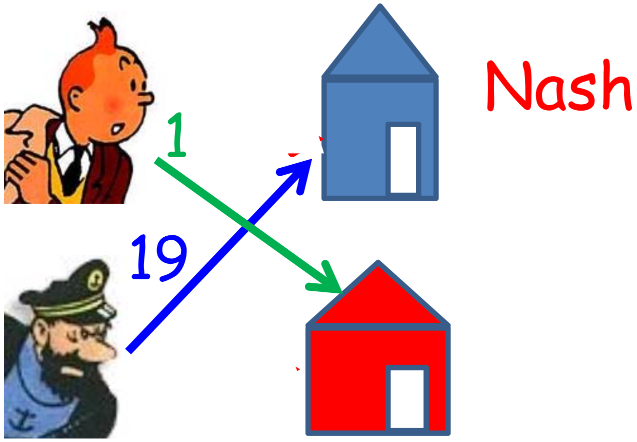
Unit Demand Bidders: example

Nash value 19+1=20

Bids 0, 1, 19, 0

OPT value 20+20=40

Inequalities




$1 \geq 20-19$ winner of his item has high value at Nash


$19 \geq 20-1$ he has high value at Nash

Both “charging” to the same high value at OPT

Our questions

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]

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- ✓ Quality of Nash Equilibria

- What if stable solution is not found?

  Is such a bound possible outside of Nash outcome?
- What if other player's values are not known

  Is such a bound possible for a Bayesian game?
- Other games?

  Do bounds like this apply other kind of game?

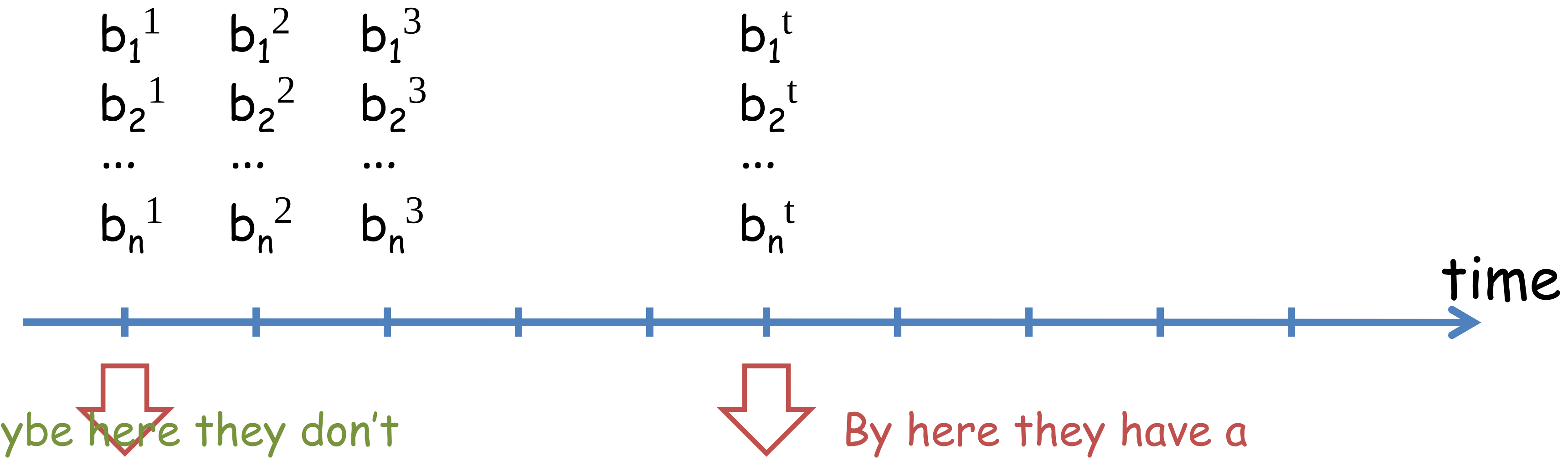
Selfish Outcome (2)?

Is Nash the natural selfish outcome?

How do users coordinate on a Nash equilibrium, e.g., which do the choose?

- Does natural behavior lead no Nash?
- Which Nash?
- Finding Nash is hard in many games…
- What is natural behavior?
  - Best response?
  - Noisy Best response (e.g. logit dynamic)
  - learning?
  - Copying others?

Auctions and No-Regret Dynamics



b_1^1 b_1^2 b_1^3 b_1^t

b_2^1 b_2^2 b_2^3 b_2^t

… … … …

b_n^1 b_n^2 b_n^3 b_n^t

Maybe here they don't know how to bid, who are the other advertisers,…

Run Auction on
$(b_1^1, b_2^1, \ldots, b_n^1)$

By here they have a better idea…

Run Auction on
$(b_1^t, b_2^t, \ldots, b_n^t)$

Vanishingly small regret for any fixed strat x:

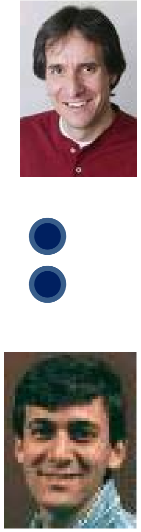
$$\sum_t u_i(b_i^t, b_{-i}^t) \geq \sum_t u_i(x, b_{-i}^t) - o(T)$$

Learning:

see Avrim Blum starting Wednesday

Iterated play where users update play based on experience

Traditional Setting: stock market

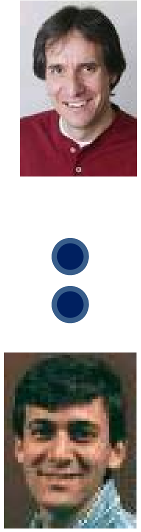


m experts N options

Goal: can we do as well as the best expert?

Regret = average utility of single best strategy with hindsight - long term average utility.

No Regret Learning



Goal: can we do as well as the best expert?

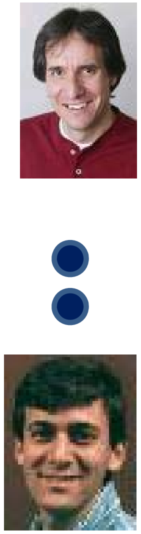
-as the single stock in hindsight?

Idea: if there is a real expert, we should find out who it is after a while.

No regret: too hard (would need to know expert at the start)

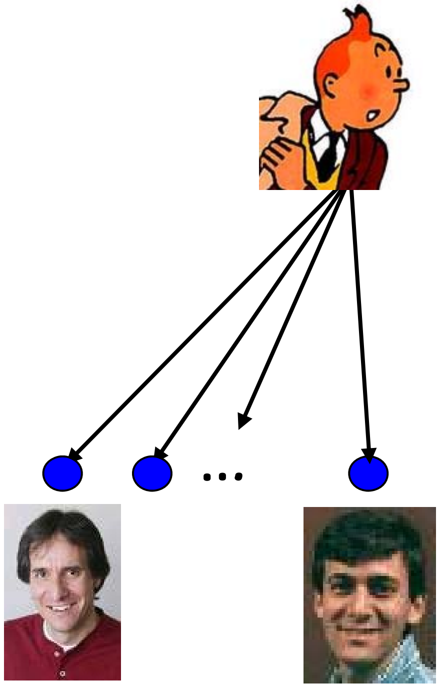
Goal: small regret compared to range of cost/benefit

Learning in Games



Goal: can we do (almost) as well as the best expert?

Games?



Focus on a single player:

experts = strategies to play

Goal: learn to play the best strategy with hindsight

Best depends on others

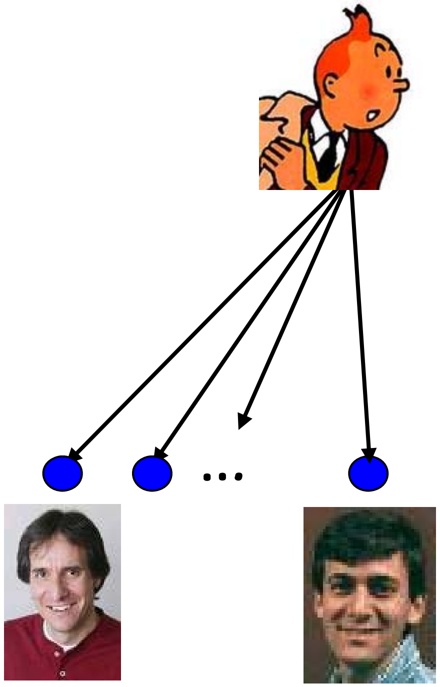
Learning in Games

Focus on a single player:

experts = strategies to play

Goal: learn to play the best strategy with hindsight

Best depends on others did



Example: matching pennies

With q=(½ ,½), best value with hindsight is 0.

Regret if our value < 0

½	½
-1 1	1 -1
1 -1	-1 1

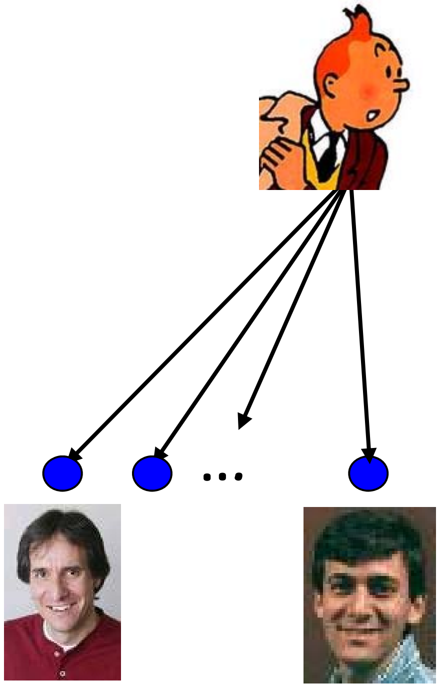
Learning in Games

Focus on a single player:

experts = strategies to play

Goal: learn to play the best strategy with hindsight

Best depends on others did



Example: matching pennies

With q=(**¾ ,¼),** best value with hindsight is ½ **(**by playing top). Regret if our value < ½

¾	¼
-1 1	1 -1
1 -1	-1 1

Learning and Games

see Avrim Blum starting Wednesday

- Regret = average utility of single best strategy with hindsight - long term average utility.

Nash = strategy for each player so that players have no regret

Hart & Mas-Colell: general games → Long term average play is (coarse) correlated equilibrium

Simple strategies guarantee vanishing regret.

(Coarse) correlated equilibrium

Coarse correlated equilibrium: probability distribution of outcomes such that for all players

expected utility ≥ exp. utility of any fixed strategy

Correlated eq. & players independent = Nash

Learning:

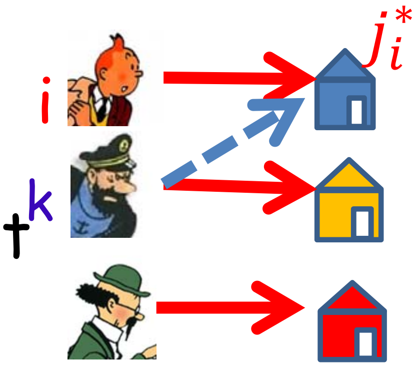
Players update independently, but correlate on shared history

Quality of learning outcome

Theorem Unit demand bidders, the total value v(N)=$\sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least $\frac{1}{2}$ of optimum OPT= $\max_{M^*} \sum_i v_{ij_i^*}$ (assuming $b_{ij} \leq v_{ij}$ ∀ i&j).

How about outcome of no-regret learning (coarse correlated equilibria)?

Same bound applies!



Idea: proof was based on "player i has no regret about one strategy"

bid $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0 \ \forall j \neq j_i^*$

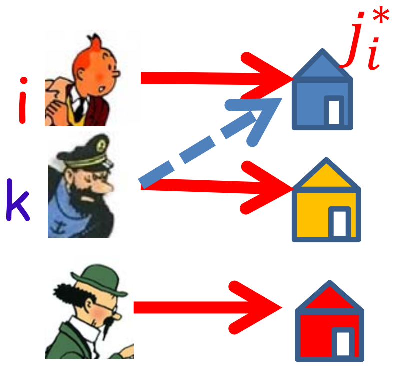
outcome of no-regret learning: no regret about any strategy!

Quality of learning outcome

Theorem Unit demand bidders, the total value E[v(N)]=$E[\sum_i v_{ij_i}]$ expected value at an outcome distribution D= $\{(i, j_i)\}$ with no regret is ≥½ of OPT= $\max_{M^*} \sum_i v_{ij_i^*}$ (assuming $b_{ij} \le v_{ij}$ ∀ i&j& all bids).

Proof: player i has no regret about one strategy
bid $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0 \ \forall j \neq j_i^*$

Price of j_i^* is a bid by an other player ≤ value
v_{ij_i} = value for player i
b_j = bid winning item j ≤v(j)= value for winner



$$E(v_{ij_i}) \ge v_{ij_i^*} - E(b_{j_i^*}) \ge v_{ij_i^*} - E(v(j_i^*))$$

Sum over all player $E_D(SW) \ge OPT - E_D(SW)$

Our questions

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]

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- ✓ Quality of Nash Equilibria
- ✓ What if stable solution is not found?

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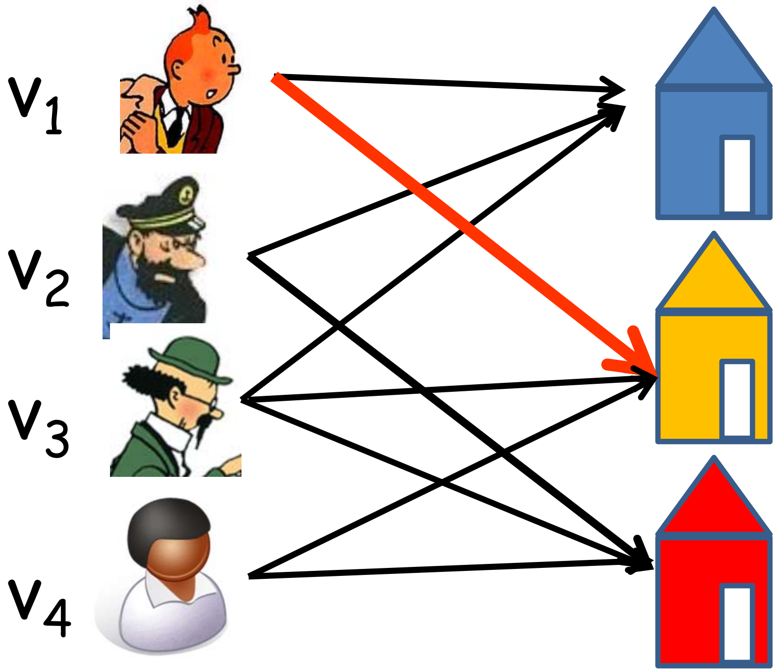
  Do bounds like this apply other kind of game?

Bayesian Auction games

Valuations v drawn from distribution $\mathcal{F}$

For simplicity assume for now

- single value v_i for items of interest
- $(v_1, \ldots, v_n) \in \mathcal{F}$ drawn from a joint distribution



- OPT i_j^* random
- Depends on information i doesn't have!

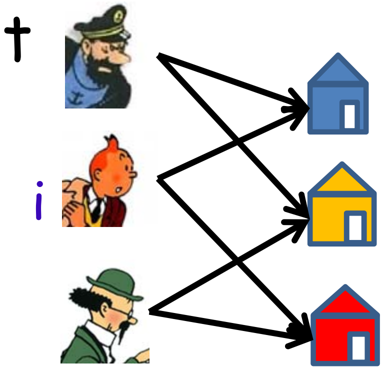
Bayesian Price of Anarchy

Theorem Unit demand bidders, the total value v(N)=$\sum_i v_{ij_i}$ at a Nash equilibrium $N = \{(i, j_i)\}$ is at least ½ of optimum OPT= $\max_{M^*} \sum_i v_{ij_i^*}$ (assuming $b_{ij} \leq v_{ij}$ ∀ i&j).

How about outcome of Bayesian game?

proof was based on "player i has no regret about one strategy"

bid $b_{ij_i^*} = v_{ij_i^*}$ and $b_{ij} = 0\ \forall j \neq j_i^*$

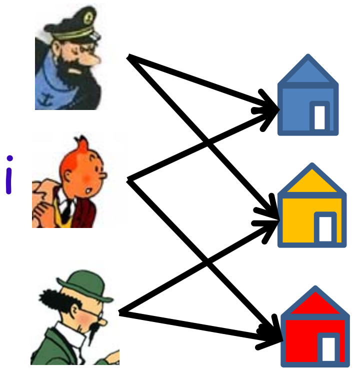


- Optimal item j_i^* depends on others
- Player can have no regret about any fixed item j, but not about j_i^*

Bayesian Price of Anarchy

Theorem Unit demand single parameter bidders, total expected value E(v(N))=$\mathrm{E}(\sum_{i\in N} v_i)$ at an equilibrium distr. $N = \{(i,j)\}$ (assuming $b_i \leq v_i \forall \mathrm{i}$) is at least $\frac{1}{4}$ of the OPT=$E(\max_{M} \sum_{i\in \mathrm{M}} v_i)$ assuming auction guarantees max one assigned item

proof "player i has no regret about bidding $\frac{1}{2}v_i$"



- If player wins: price ≤ $b_i \leq \frac{1}{2}v_i$ hence utility at least $\frac{1}{2}v_i$
- If he looses, all his items of interest, went to players with bid (and hence value) at least $\frac{1}{2}v_i$.

In either case

$$\frac{1}{2}v_{ij_i^*} \geq v_{ij_i} + b_{j_i^*} \geq v_{ij_i} + v(j_i^*)$$

Sum over player, and take expectation over v∈$\mathcal{F}$

$\frac{1}{2}$OPT≥ E(v(N)+ E(v(N))

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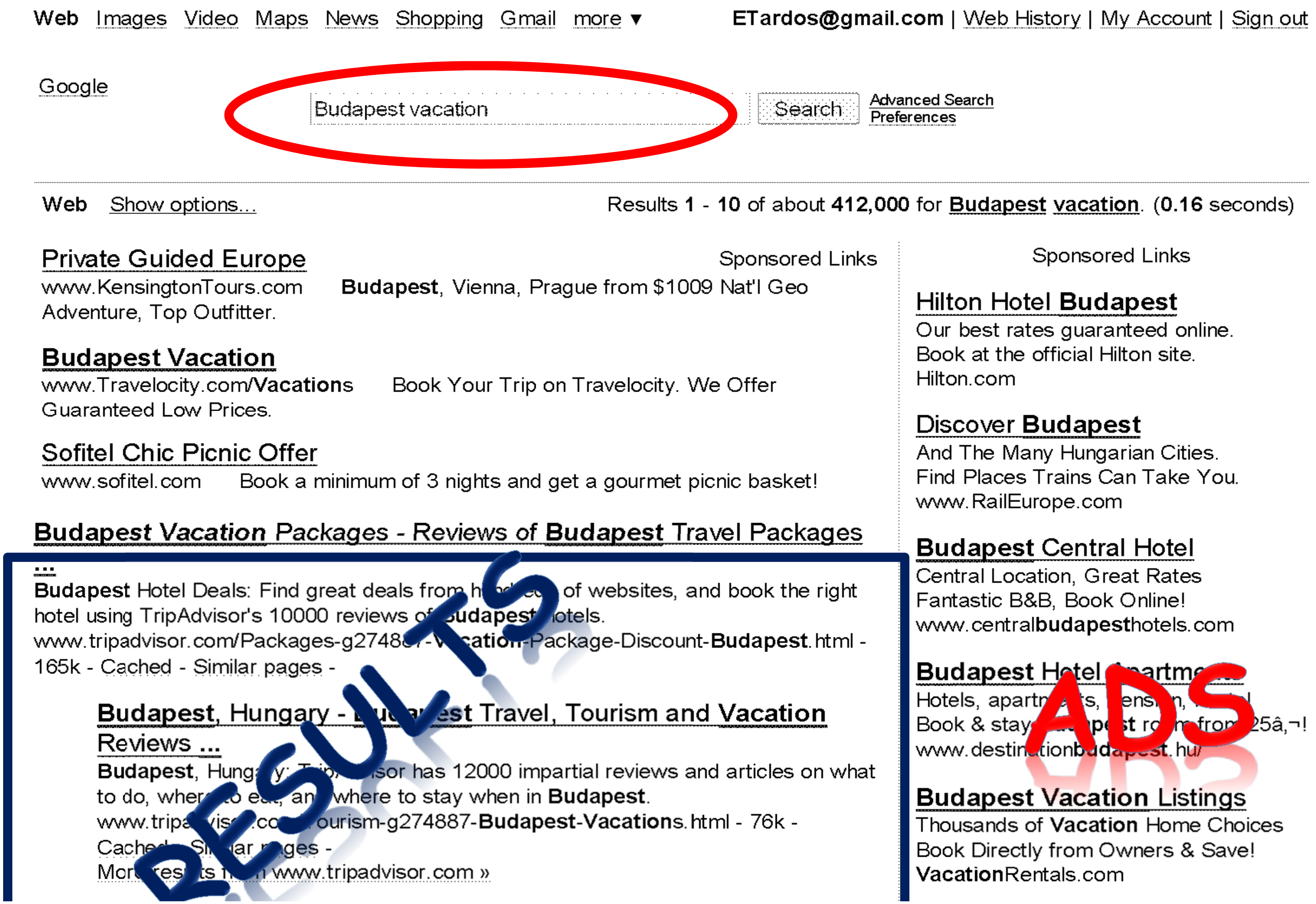
AdAuction



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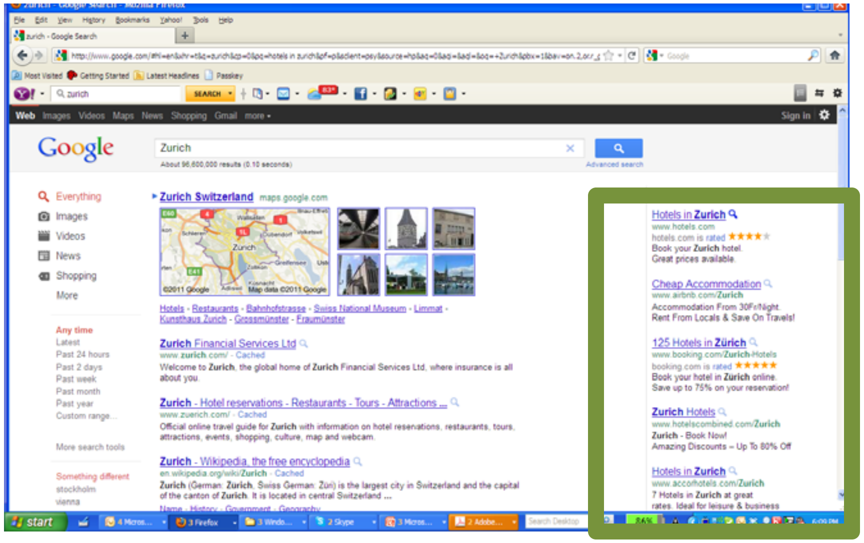

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7 Hotels in Zurich at great
rates. Ideal for leisure & business

Online auctions:

- Display ads
- Search Ads

Powerful ad:

customized by information about user

Search term, History of user, Time of the day, Geographic Data, Cookies, Budget

- Millions of ads each minute, and all different!

- Needs a simple and intuitive scheme

Model of Sponsored Search

Ordered slots, higher is better

Advertisers:

Hilton, RailEurope, CentralBudapestHotels, DestinationBudapest, RacationRentals.com, Travelzoo.com, TravelYahhoo.com, BudgetPlace.com

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Budapest Hotel Apartments
Hotels, apartments, pension, hostel
Book & stay **Budapest** room from 25â,¬!
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Budapest Vacation Listings
Thousands of **Vacation** Home Choices
Book Directly from Owners & Save!
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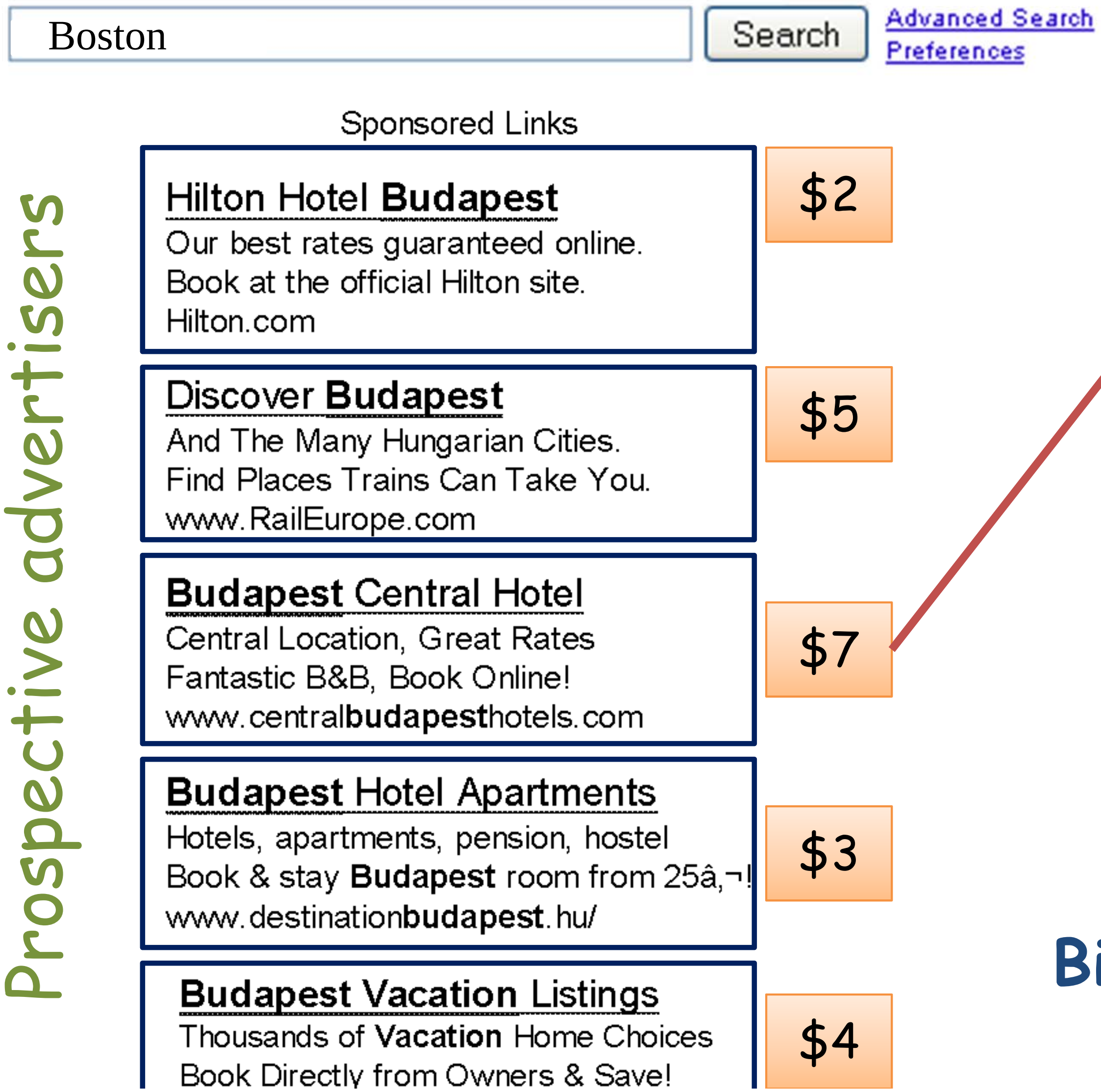
Vacations to Budapest
Browse the Best Travel Deals to
Budapest & more. Use Travelzoo!

Selling one Ad Slot

Prospective advertisers



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Budapest Vacation Listings
Thousands of **Vacation** Home Choices
Book Directly from Owners & Save!

α=click rate

Pays $5

Vickrey Auction

- Truthful
- Efficient
- Simple
- …

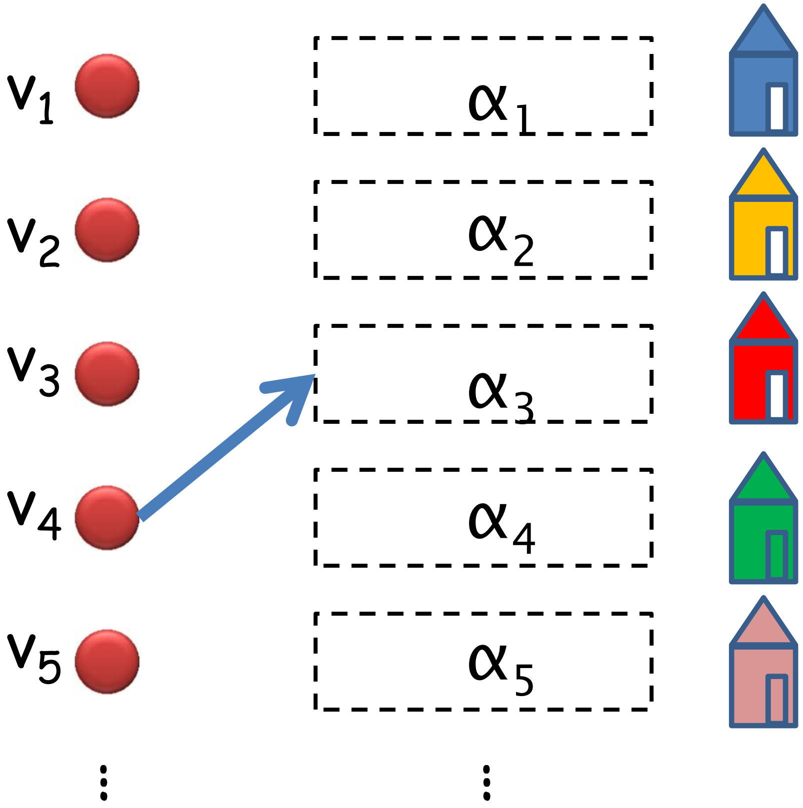
Bids on click value

Keyword Auction=Matching Problem

Version 1

- n ads and n slots
- Each advertiser has a value v_k per click
- Each slot has click through rate α_j
- Value of slot j for k

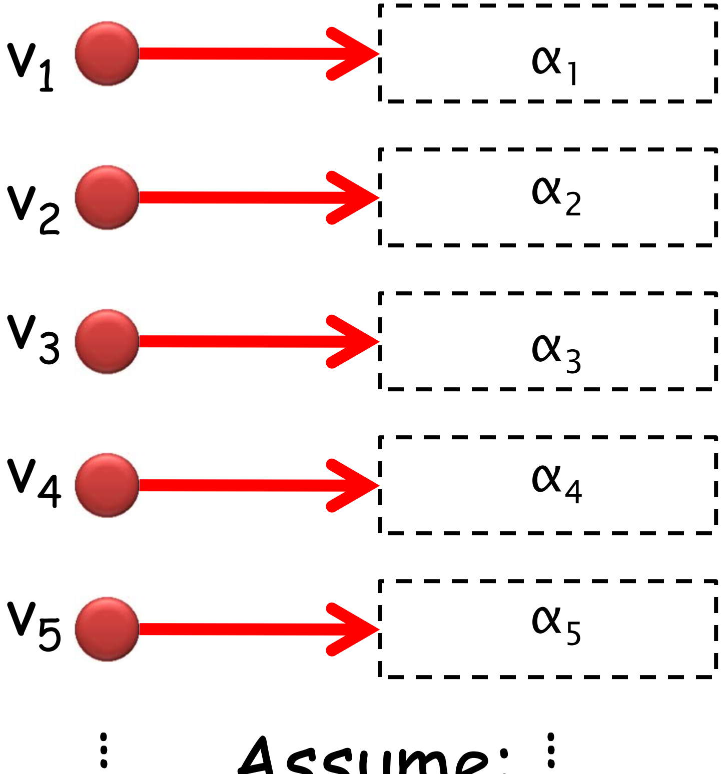
$$v_{kj} = v_k \alpha_j$$



$$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$$

Maximizing welfare (matching)

- n advertisers and n slots
- Each advertiser has a value v_i
- Click through rate is a_j
- max $\sum_j \alpha_j v_j$ =total value



Assume:

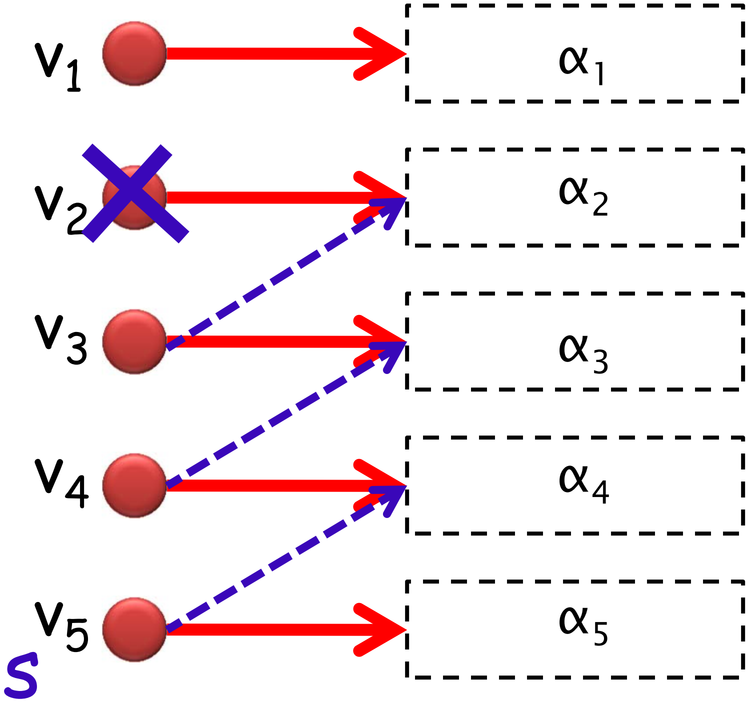
$v_1 \geq v_2 \geq \ldots \geq v_n$

$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$

VCG for AdAuctions

- n advertisers and n slots

Assignment: max total value $\sum_i \alpha_i v_i$



Price paid

p_i= welfare loss of others

$$p_i = \sum_{j>i} (\alpha_{j-1} - \alpha_j) v_j$$

⋮ Assume: ⋮

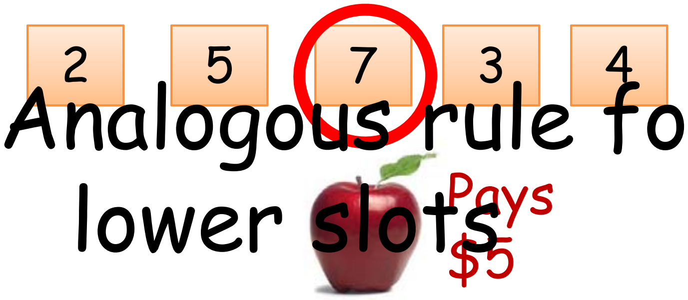
$$v_1 \geq v_2 \geq \ldots \geq v_n$$

$$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$$

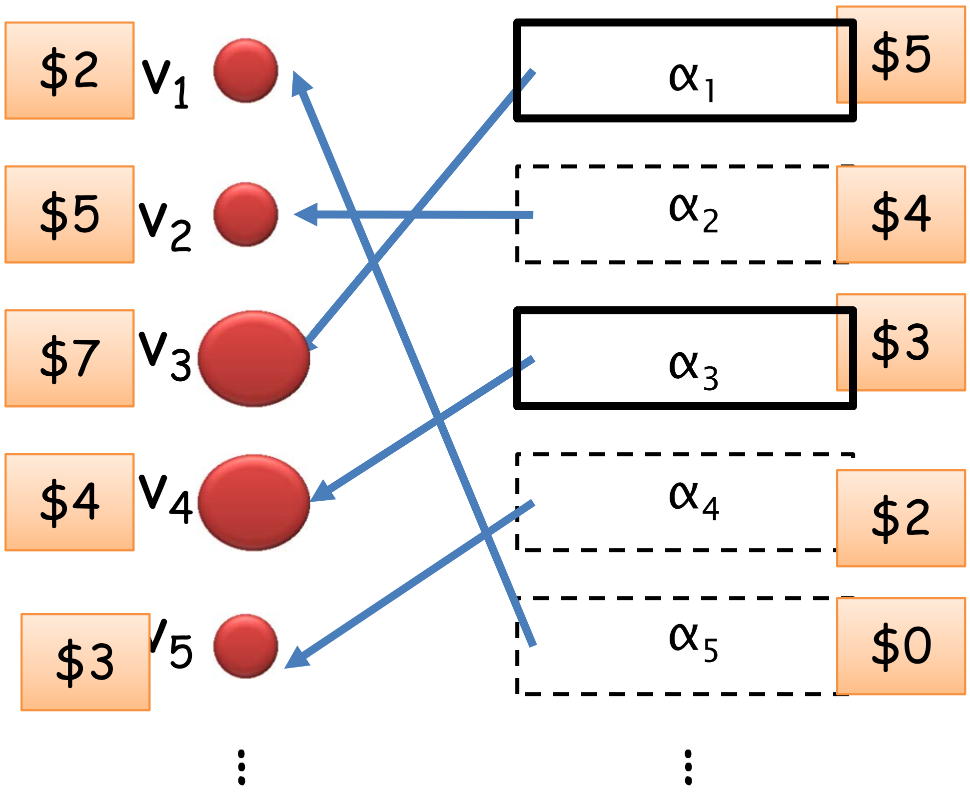
Generalized Second Price (GSP)

- Users bid per click
- Sort by bid
- Charge next lower bid for each click

Recall:



Analogous rule for lower slots



Sort by

$b_{\pi(1)} \geq b_{\pi(2)} \geq \ldots \geq b_{\pi(n)}$

Is GSP truthful?

Is bidding $b_k = v_k$ Nash equilibrium for the bidders?

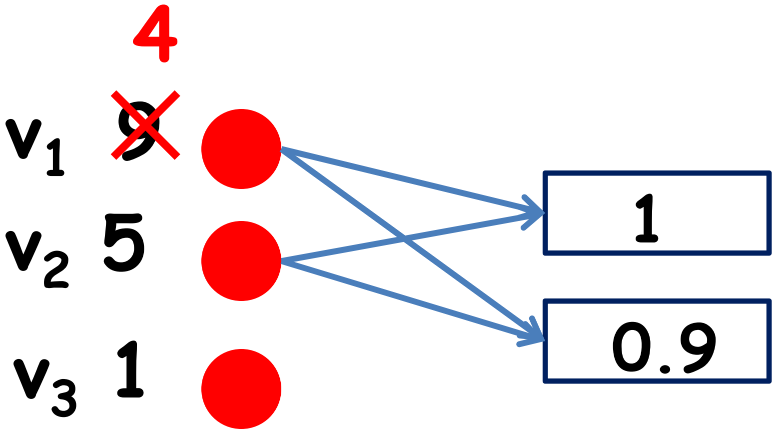
Example:

Bidder 1's value if telling the truth

$(9-5) \cdot 1 = 4$

If bidding $b_1 < 5$

$(9-1) \cdot 0.9 = 7.2$



Sort by bid value

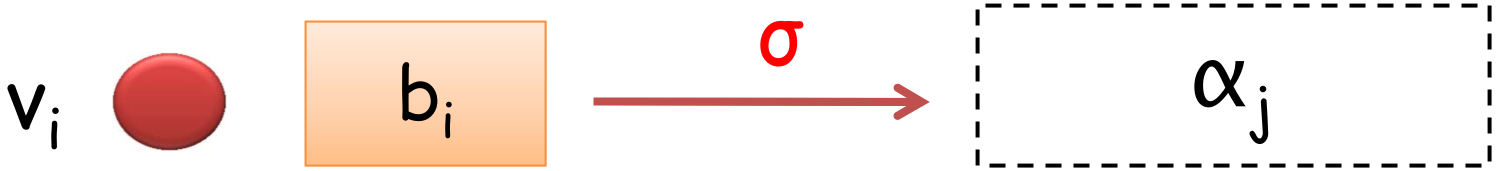
$b_1 > b_2 > b_3 > b_4 > \ldots$

Charge next price $p = b_{k+1}$

Value to bidder k

$(v_k - b_{k+1}) \cdot \alpha_k$

Measuring efficiency

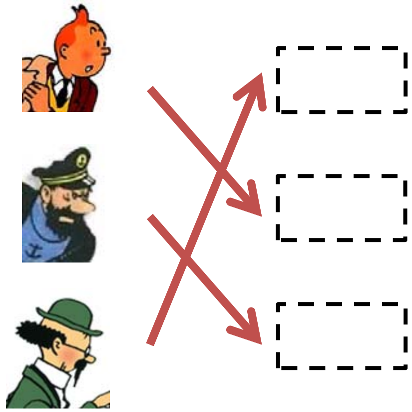


v_i b_i $\xrightarrow{\sigma}$ α_j

$j = \sigma(i)$

Social welfare

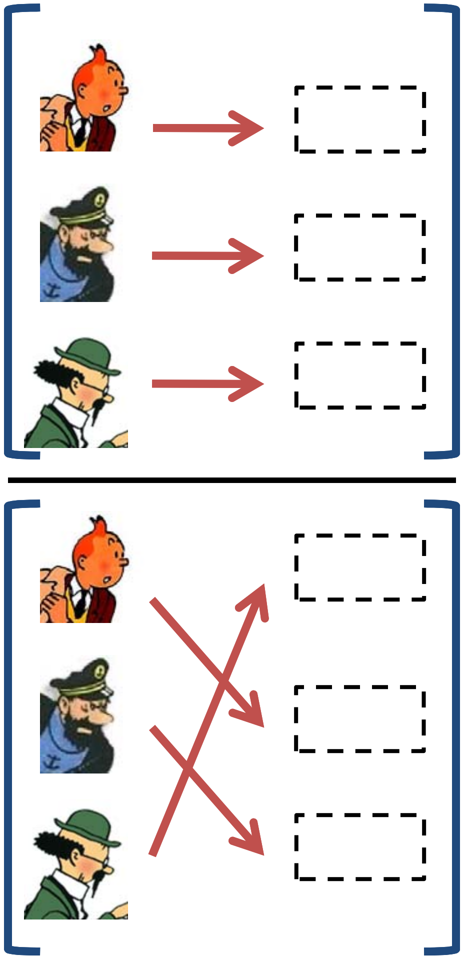
$= \text{click} \cdot \text{value} = \sum_i v_i \alpha_{\sigma(i)}$



Measuring inefficiency

Price of Anarchy $=\max_{Nash}\dfrac{\max SW}{SW(Nash)}$

Price of Stability $=\min_{Nash}\dfrac{\max SW}{SW(Nash)}$



Equilibrium selection?

Full Information: ∃ Good equilibria

Theorem [Edelman, Ostrovsky, Schwarz'07 & Varian'06] Envy free equilibria maximize social welfare, and envy free ∃. (Price of stability 1)

Theorem [Paes Leme, T, FOCS'10] Price of Anarchy bounded by 1.618.

[Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, EC'11] improved to 1.282

True in the full information model only

Today: a game with uncertainty

Two forms of uncertainty:

- participants
  Bayesian game
- *quality factors*

Bayesian setting (no efficient Nash)
[Gomes, Sweeney 09]

Keyword Auction with quality factors

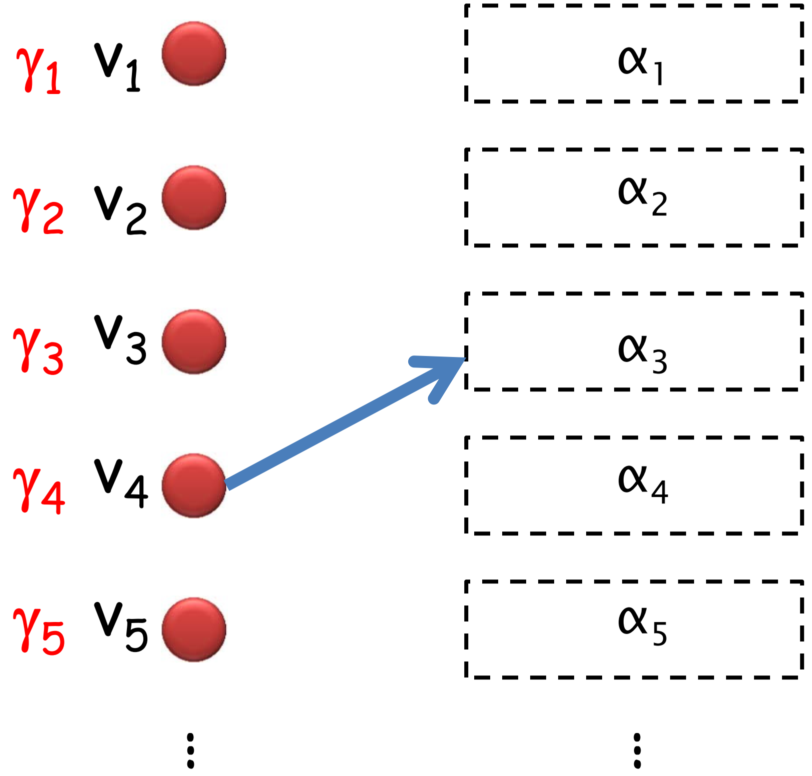
Version 2

- n ads and n slots
- Each advertiser has a value v_k per click
- Each slot has click through rate α_j
- "ad-quality" a click through rate γ_k

- Click through rate of slot j for k

  $\gamma_k \alpha_j$

  separable model



- Value of slot j for k

  $\boxed{\gamma_k v_k}\alpha_j$ Effective value

Generalized Second Price (GSP)

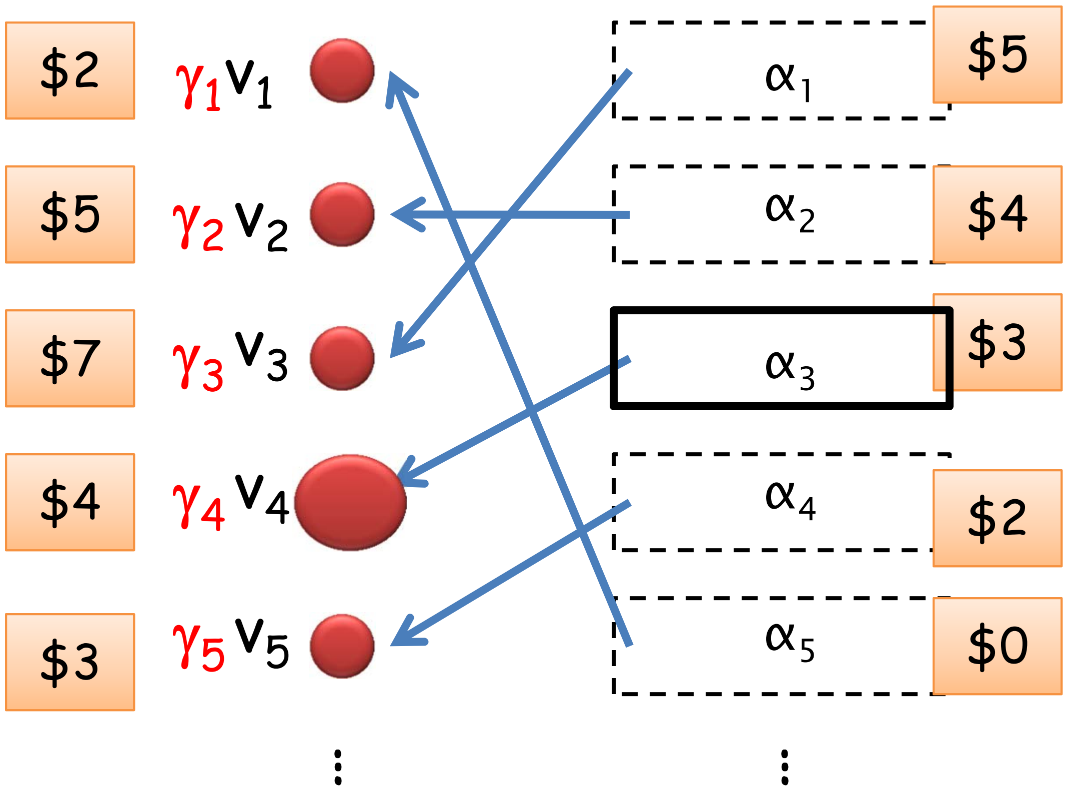
- Users bid per click
- Sort by $\text{bid}_{*}\gamma$
- Charge critical price for each click

Value of player k in slot j:

$$k = \pi(j)$$

$$u_k = \alpha_j \gamma_k (v_k - p_k)$$

$$\boxed{\gamma_k\, p_k = \gamma_{\pi(j+1)}\, b_{\pi(j+1)}}$$



Sort by $\gamma_k b_k$

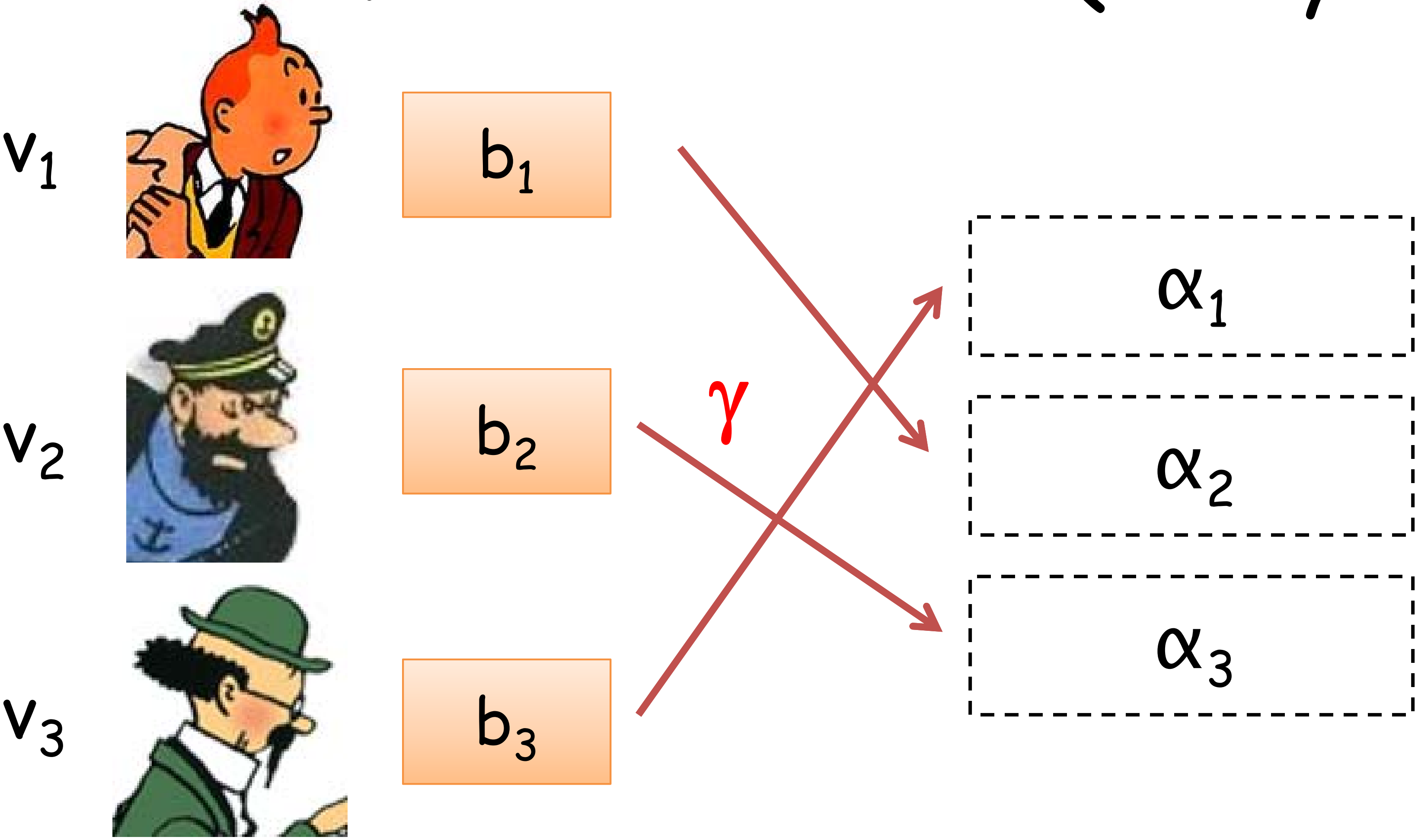
Uncertainty about Ad Quality

$b_k \Rightarrow b_k \gamma_k$

Computer via machine learning from

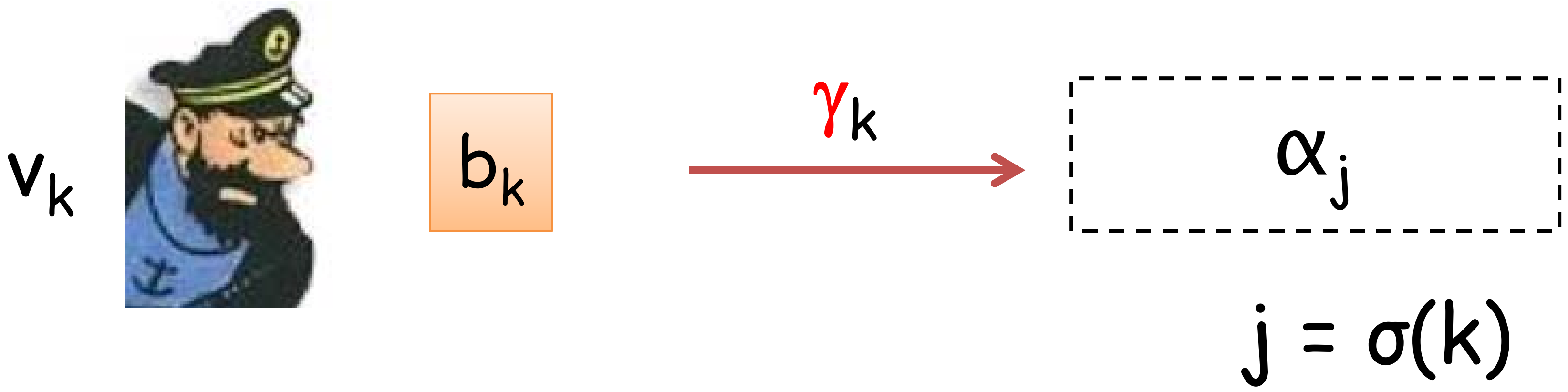
- History of user
- Time of the day
- Geographic Data
- Cookies
- Budget, …

Model of Uncertain Ad Quality



- valuations fixed (full information) or Bayesian.
- But Ad Quality uncertain, only distribution known (possibly correlated)

Model with Ad Quality Uncertainty



v_k b_k γ_k α_j

$j = \sigma(k)$

Nash equilibrium:

$$E[u_k(b_k,b_{-k})] \ge E[u_k(b'_k,b_{-k})]$$

Expectation over participants and quality factors γ

Simple proof PoA for welfare

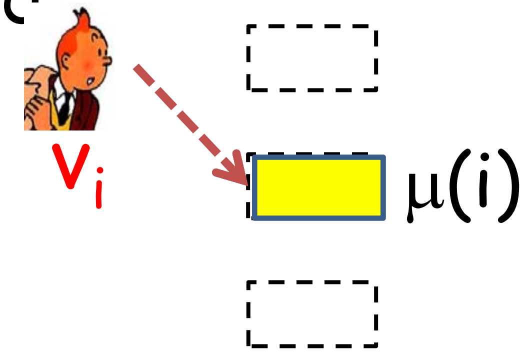
Theorem: [Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, Lucier, Paes Leme, T] Even if values are arbitrarily correlated, the PoA is bounded by 4

Assume
γ_i=1 all i

Proof sketch for bound of 4 full info:

- Focus on person i with slot in Opt $\mu(i)$
- Deviate to $\frac{1}{2}v_i$ whenever your value is v_i
- Either you get slot $\mu(i)$ or better and

$$u_i(\tfrac{1}{2}v_i, b_{-i}) \geq \tfrac{1}{2}\alpha_{\mu(i)}v_i$$



Simple proof PoA for welfare

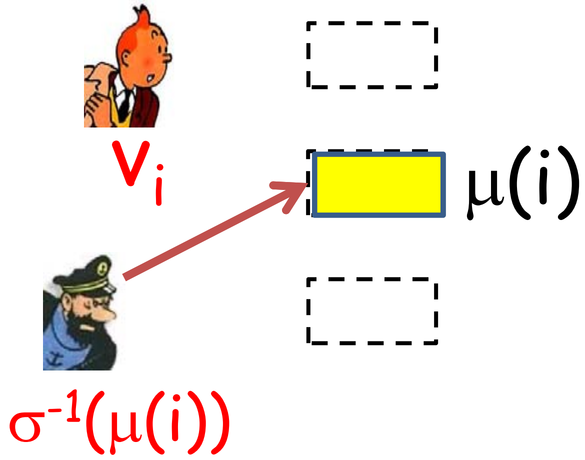
Proof sketch for bound of 4 full info:

- Deviate to $\frac{1}{2}v_i$ whenever your value is v_i
- either get slot $\leq\mu(i)$ and $u_i(\frac{1}{2}v_i,b_{-i})\geq \frac{1}{2}\alpha_{\mu(i)} v_i$
- Or the player in that slot has value $\geq \frac{1}{2}v_i$

$$\alpha_{\mu(i)}\, v_{\sigma^{-1}(\mu(i))} \geq \alpha_{\mu(i)}\frac{1}{2}\, v_i$$

Add two options

$$u_i(\frac{1}{2}v_i,b_{-i}) +\alpha_{\mu(i)}\, v_{\sigma^{-1}(\mu(i))} \geq \frac{1}{2}\alpha_{\mu(i)}\, v_i$$



Simple proof PoA for welfare

Theorem: Even if values are arbitrarily correlated, the PoA is bounded by 4

Proof sketch for bound of 4 Bayesian :

- Deviate to $\frac{1}{2}v_i$ whenever your value is v_i

$$u_i(\tfrac{1}{2}v_i, b_{-i}) + \alpha_{\mu(i)}\, v_{\sigma^{-1}(\mu(i))} \geq \tfrac{1}{2}\alpha_{\mu(i)}\, v_i$$

- true for every realization of the random vars
- sum all players, take expectations, use Nash

$$\Sigma_i\, E(u_i(v)) + \Sigma_j\, E(\alpha_{\sigma(j)} v_j) \geq \tfrac{1}{2}\Sigma_i\, E(\alpha_{\mu(i)} v_i)$$

NASH + NASH $\geq \frac{1}{2}$ OPT

Efficiency of Outcome

Proof idea: deviate to $\frac{1}{2}v_i$ when your value is v_i

This is a "no-regret" style bound:
don't regret not playing $\frac{1}{2}v_i$

$\Rightarrow$ Bound applies to learning outcomes

If proof uses only "no-regret"-bound then extends to learning outcomes.

If regret only used for $\frac{1}{2}$ v_i (depends on v_i only), extends to Bayesian game with correlated types.

Simple Auction Games

What we have seen so far

- item bidding games simple item bidding
- Generalized Second Price
- Very simple valuations: unit demand or even single parameter

Simple proof technique bounding outcome quality (Nash, Bayesian Nash, learning outcomes)

References and Better results

- [Christodoulou, Kovacs, Schapira ICALP'08] Price of anarchy of 2 assuming conservative bidding, and fractionally subadditive valuations, independent types
- [Bhawalkar, Roughgarden SODA'10] subaddivite valuations
- [Syrgkanis, T] Improved bound of 3 for unit-demand single value version with correlated types

- [Caragiannis, Kaklamanis, Kanellopoulos, Kyropoulou, Lucier, Paes-Leme,T] Improved bound of 2.93 for GSP with uncertainty either Bayesian model or quality factor uncertainty.