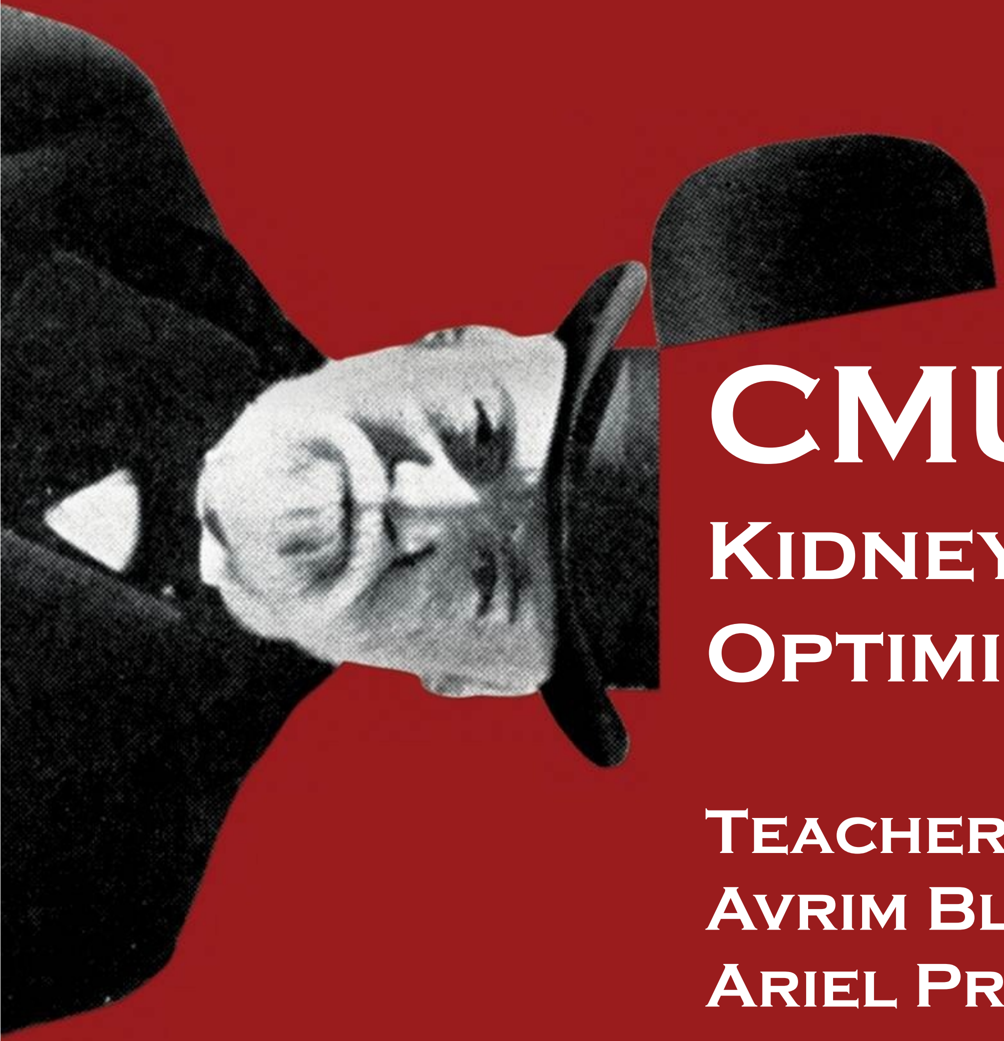


CMU 15-896

Kidney Exchange: Optimization

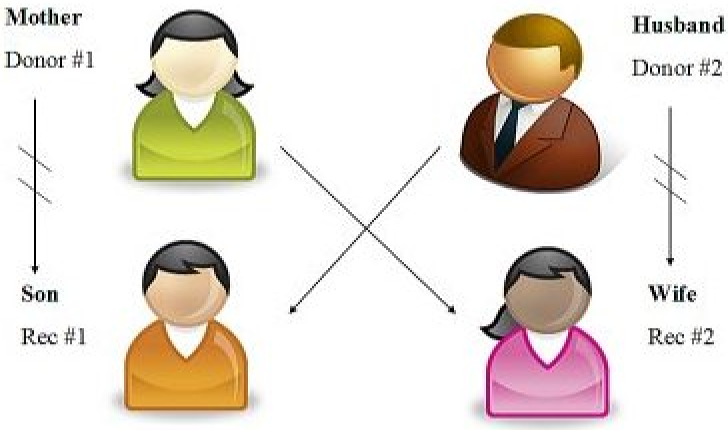
Teachers:
Avrim Blum
Ariel Procaccia (this time)

Kidney transplants

- Kidney failure can be fatal
- Options: dialysis, kidney transplant
- In 2010:
  - 4,654 people died waiting for a kidney transplant.
  - 34,418 people were added to the national waiting list
  - 10,600 people left the list by receiving a deceased donor kidney
  - The waiting list had 89,808 people, and the median waiting time is between 2-5 years, depending on blood type

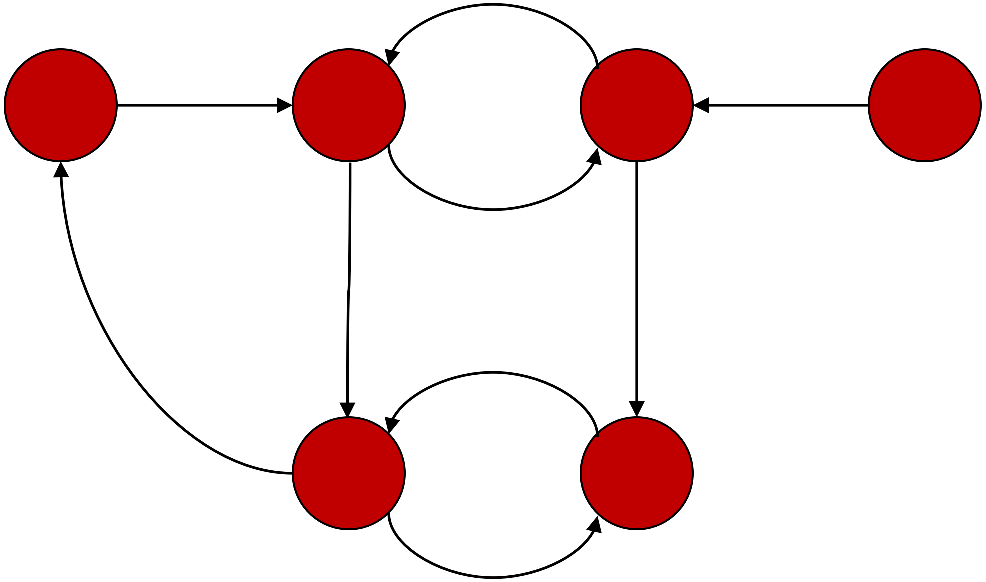
Kidney exchange

- Best option: live donor
- In 2010 there were 5467 live donations in the US
- Most patients are incompatible with potential donors
- Kidney exchange = patients swap incompatible donors to obtain a compatible donor



More generally…

- Directed graph $G = (V, E)$
- Each $v \in V$ is a donor-patient pair
- Edge $(u, v) \in E$ if donor of u is compatible with patient of v
- Exchanges along cycles

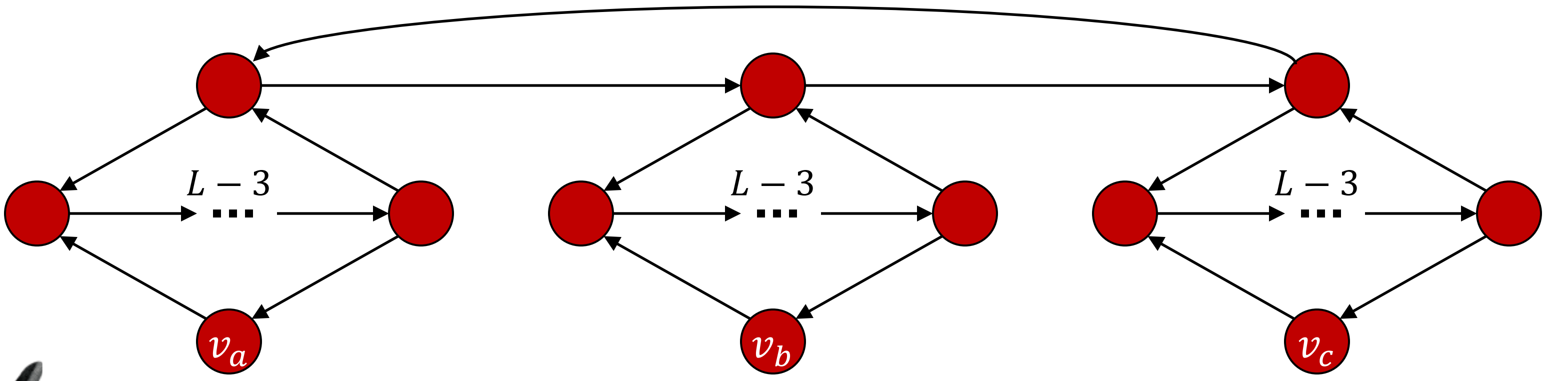


Cycle cover

- Maximum cover by cycles
- If cycle length is unrestricted, problem is in P [homework 4 q3]
- Cycle cap is a medical necessity
- **Theorem [Abraham et al. 2007]:** Given G, $L \geq 3$, computing a max cycle cover with cycles of length $\leq L$ is NP-hard
- Trivial for $L = 2$

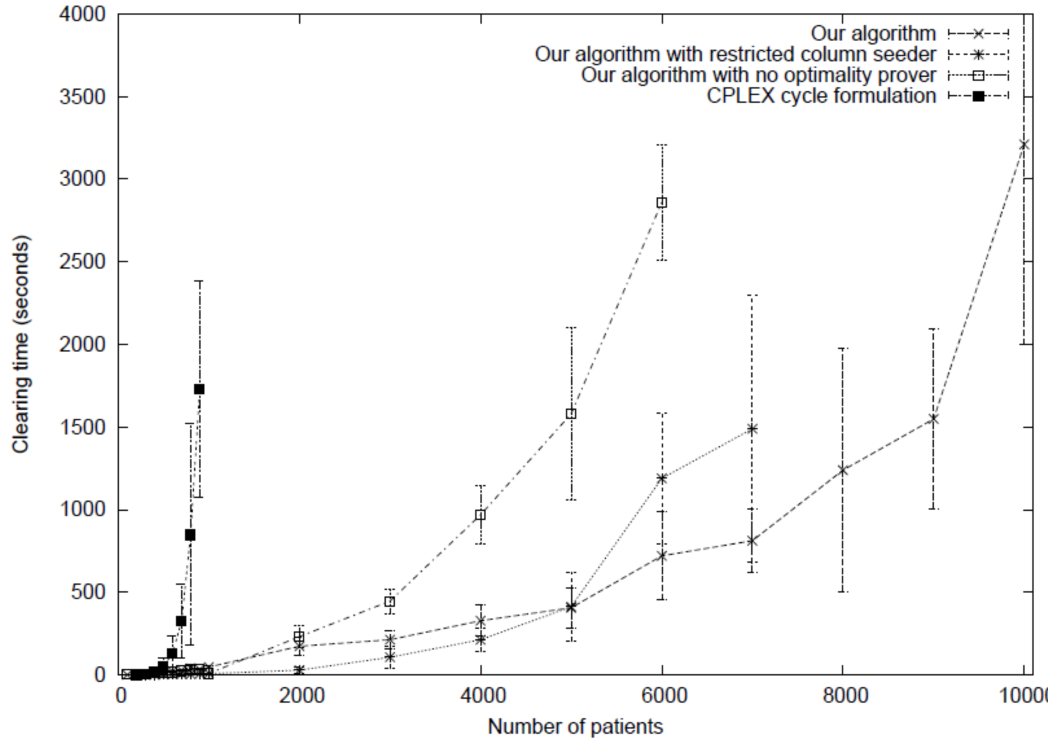
Proof by illustration

- Reduction from 3D-Matching: Given disjoint sets A, B, C of size q and triples $T \subseteq A \times B \times C$, is there a disjoint $M \subseteq T$ of size q?
- For each $x \in A \cup B \cup C$ construct v_x
- For each triple (a, b, c) construct gadget below
- 3D matching $\Leftrightarrow$ perfect cycle cover



Cycle covers in practice

- In practice optimal cycle covers are computed on a weekly basis at CMU

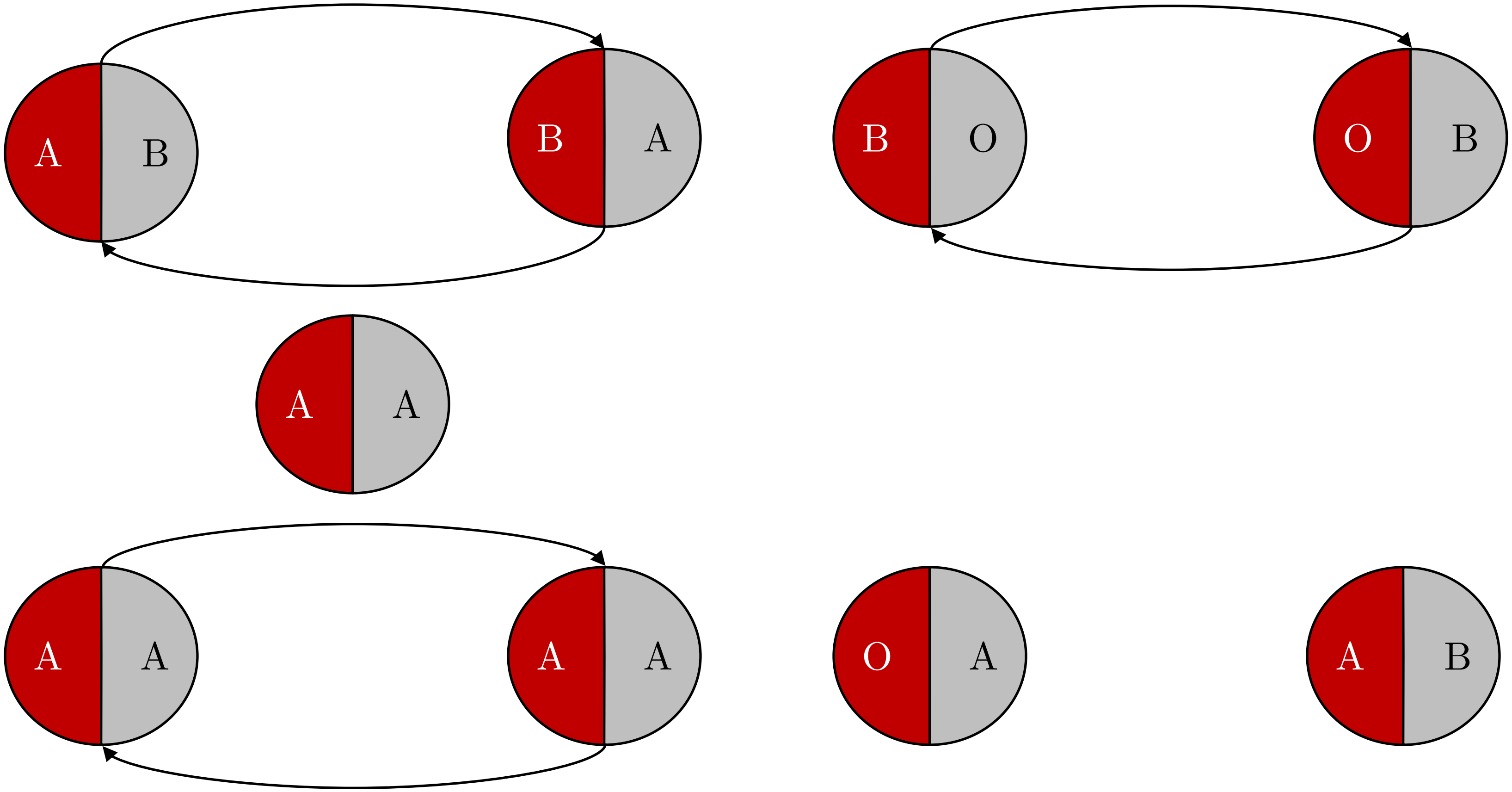


[Abraham et al., 2007]

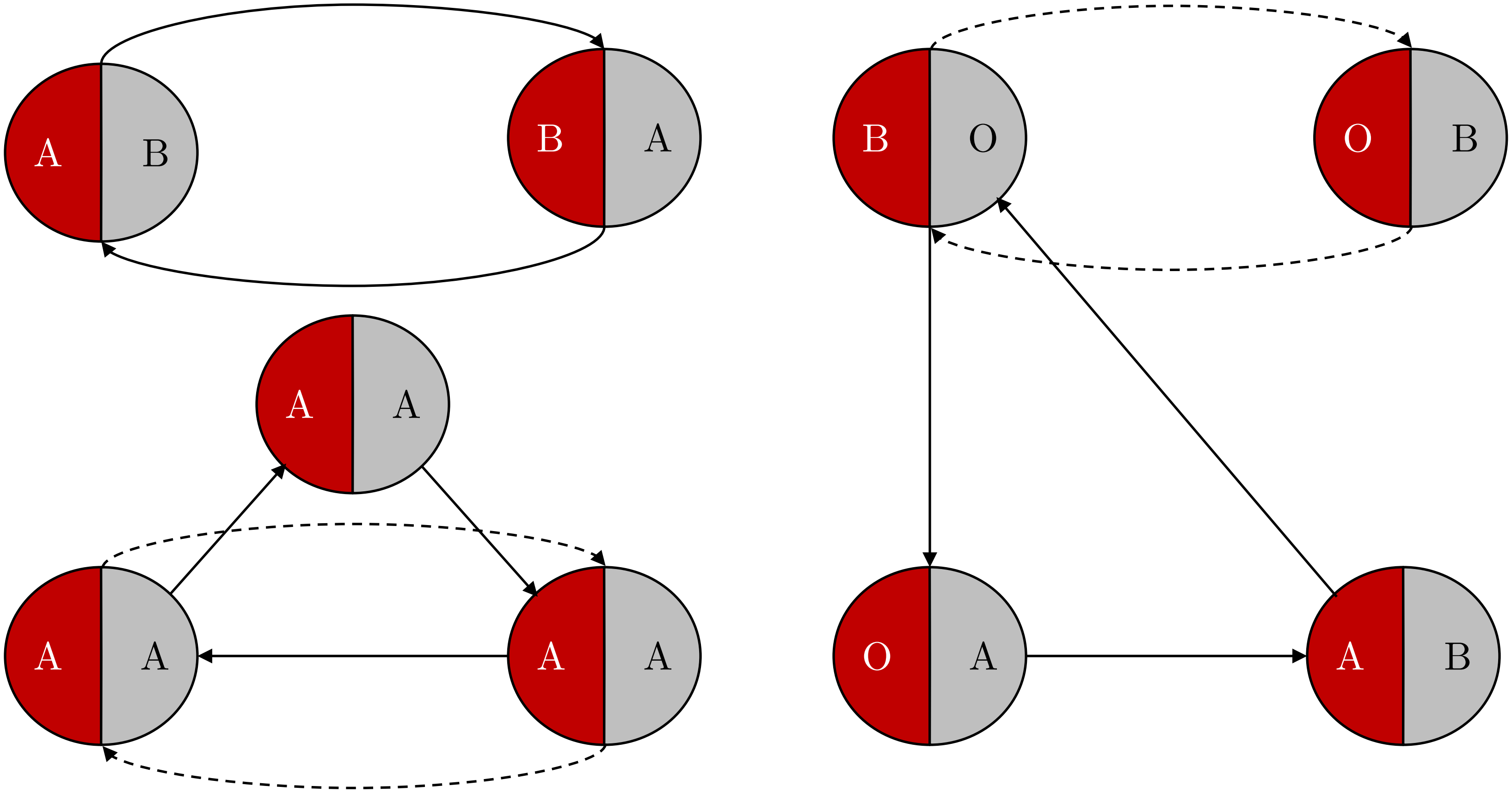
Are long cycles needed?

- Model of [Roth, Sonmez, and Unver 2007]
- Four blood types: O, A, B, AB
- Donor is compatible with patient if latter has "more letters" (O is empty set)
  - Example: A can donate to A or AB, but not to B or O
- **Assumption:** There are no tissue-type incompatibilities between pairs

3-CYCLES CAN HELP

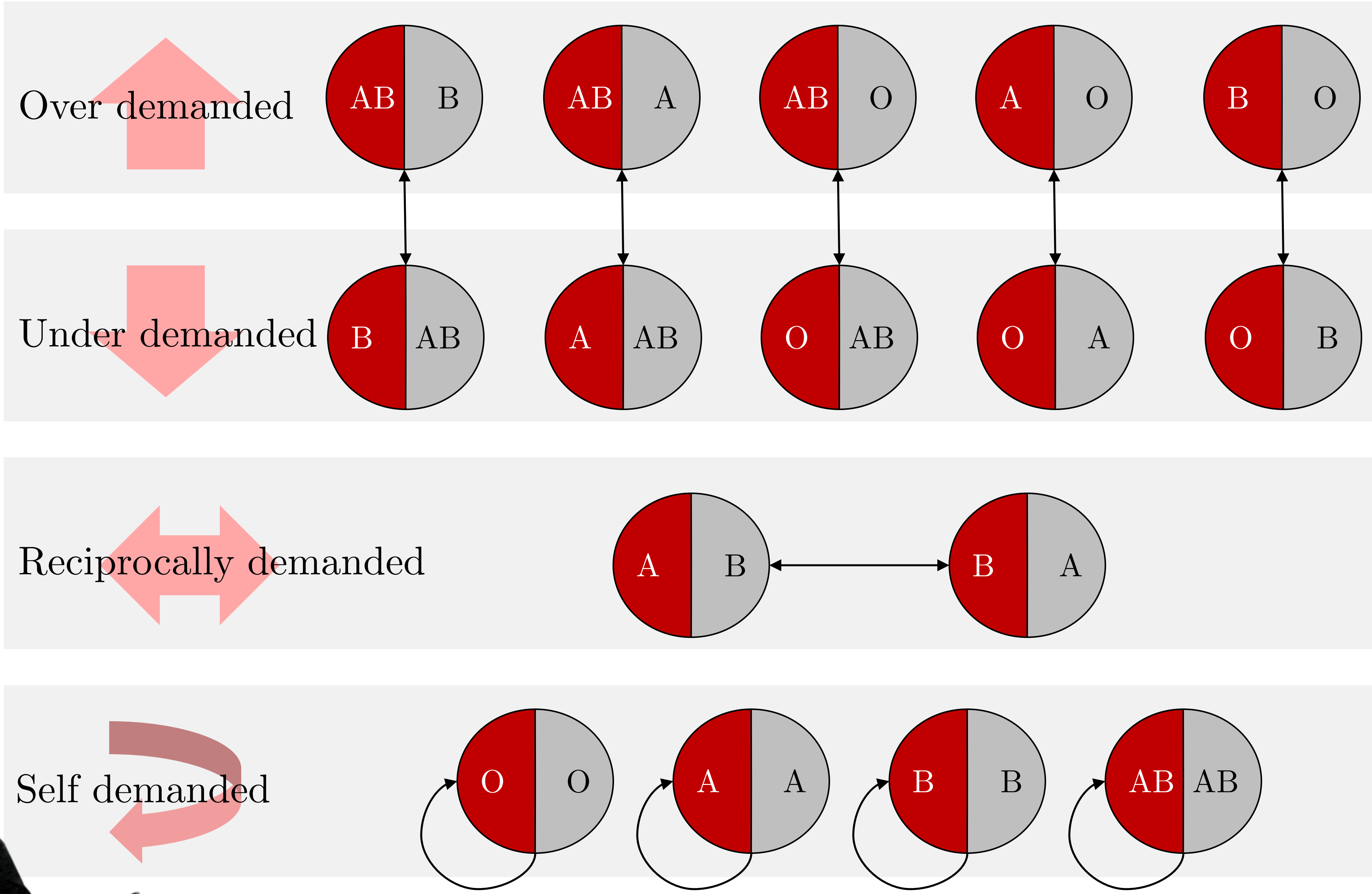


3-cycles can help



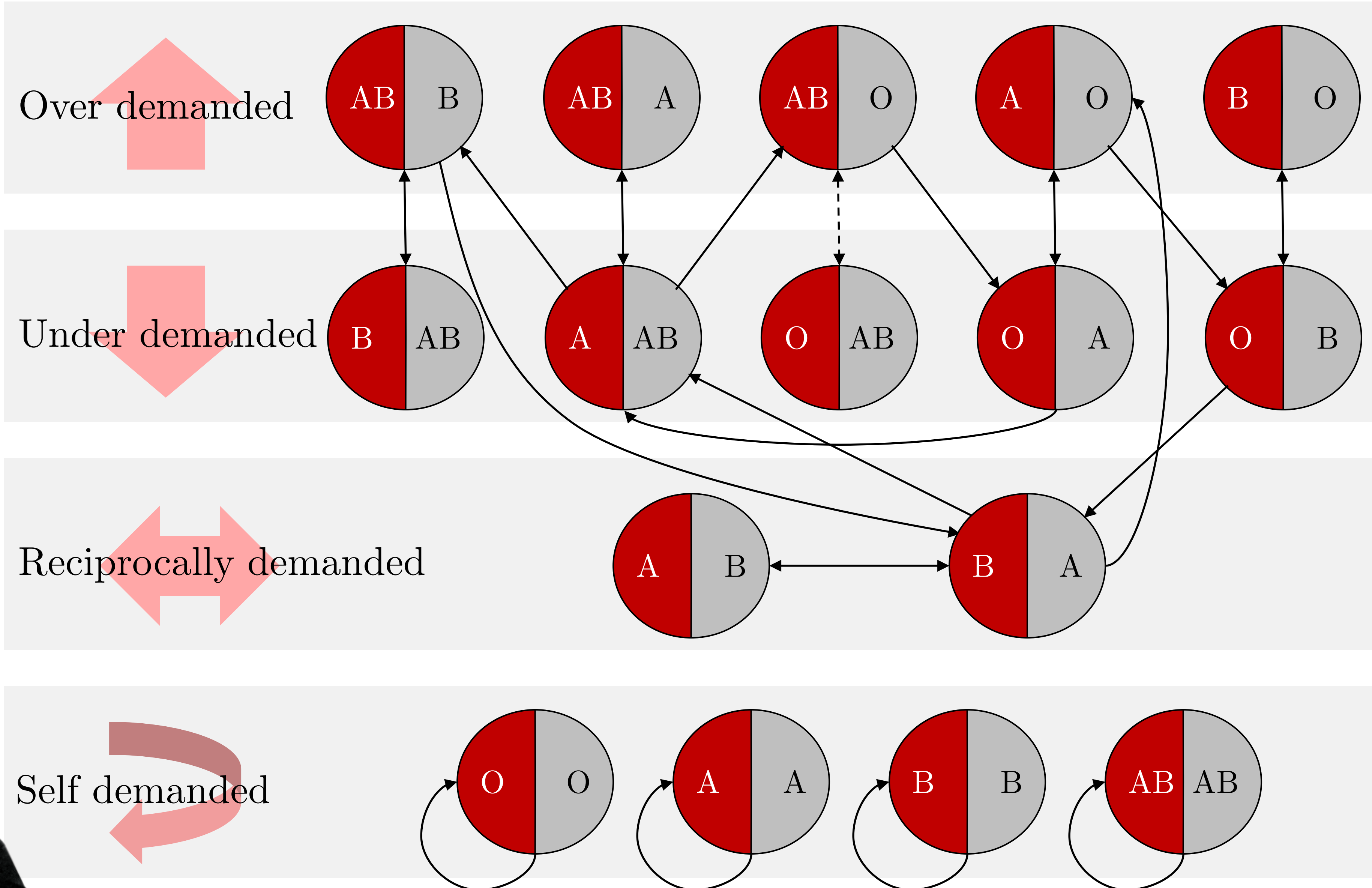
Classification of pairs

- We classify donor-patient pairs into four types:
  - Self-Demanded: X-X
  - Reciprocally demanded: A-B and B-A
  - Over-demanded: X-Y that are blood-type compatible
  - Under-demanded: X-Y that are blood-type incompatible
- **Assumption:** There is an endless supply of under-demanded pairs
- Next two slides show optimal allocations for 2-cycles and 3-cycles


Over demanded
AB B
AB A
AB O
A O
B O
Under demanded
B AB
A AB
O AB
O A
O B
Reciprocally demanded
A B
B A
Self demanded
O O
A A
B B
AB AB

3-cycles can help, revisited

- Why do 3-cycles help?
  1. Odd number of pairs in a self-demanded set
  2. Each AB-O pair can form a 3-cycle with O-A, A-AB or O-B, B-AB
  3. Remaining A-B or B-A pairs can be matched in 3-cycles, e.g., (A-B, B-O,O-A)
- Assume that we draw each pair from product dist. over blood types; each type has constant probability
- Vote: Which item gives $\Omega(n)$ extra matches?


Over demanded
AB B
AB A
AB O
A O
B O
Under demanded
B AB
A AB
O AB
O A
O B
Reciprocally demanded
A B
B A
Self demanded
O O
A A
B B
AB AB

A RANDOM GRAPH MODEL

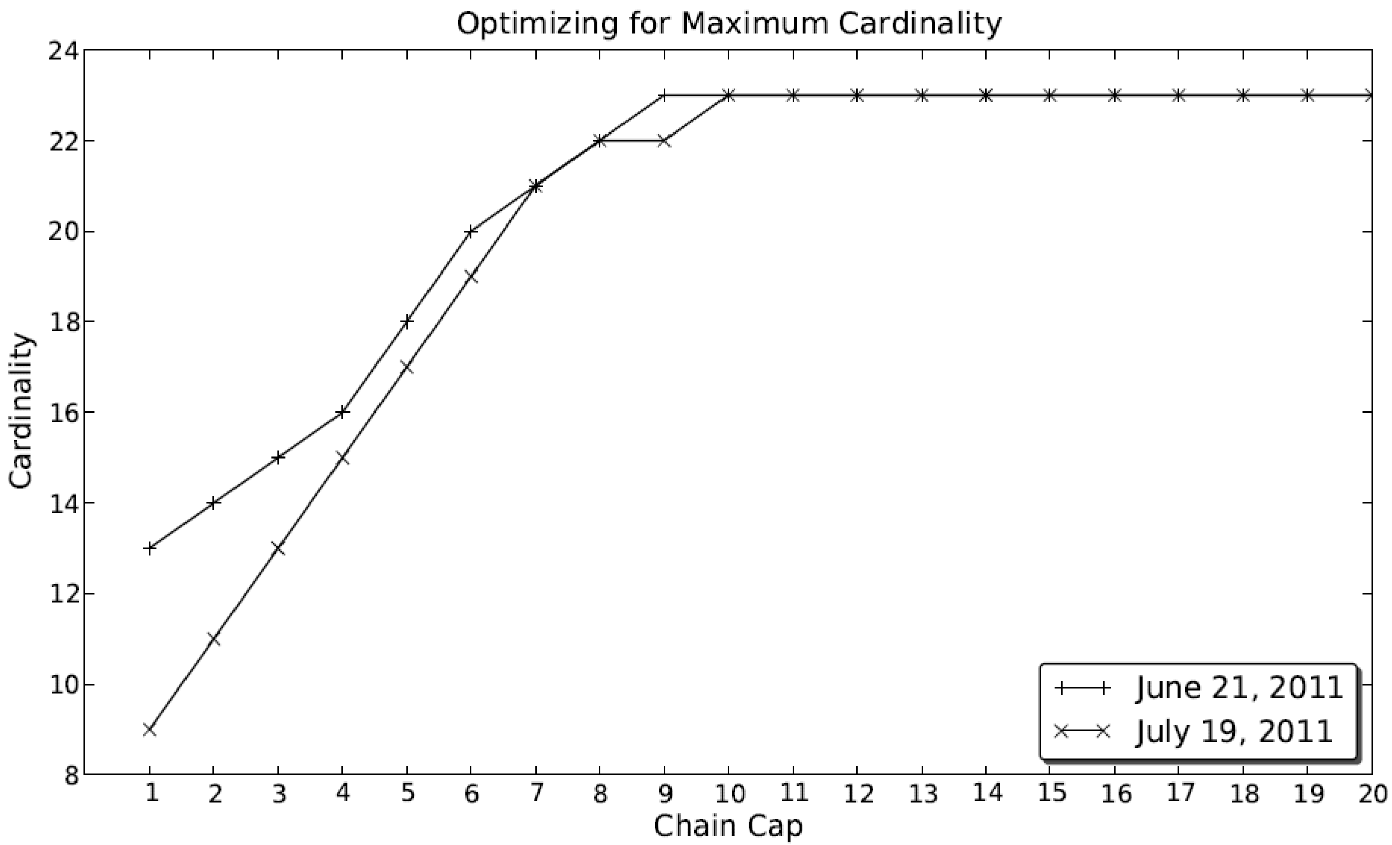
- Each blood type X has probability μ_X
- Draw blood types for patient and donor
- Blood-type compatible donor and patient are tissue-type incompatible with probability $\gamma > 0$
- If donor-patient pair is internally compatible, remove them
- Otherwise, randomly generate edges to blood-type compatible pairs
- **Theorem [Ashlagi and Roth 2011]:** In large random graphs, w.h.p. $\exists$opt allocation with cycles of length ≤ 3

Introducing: Chains

- Altruistic donors can initiate a chain
- Long chains can have a huge impact on #matches

Chain of length 60, from NYT

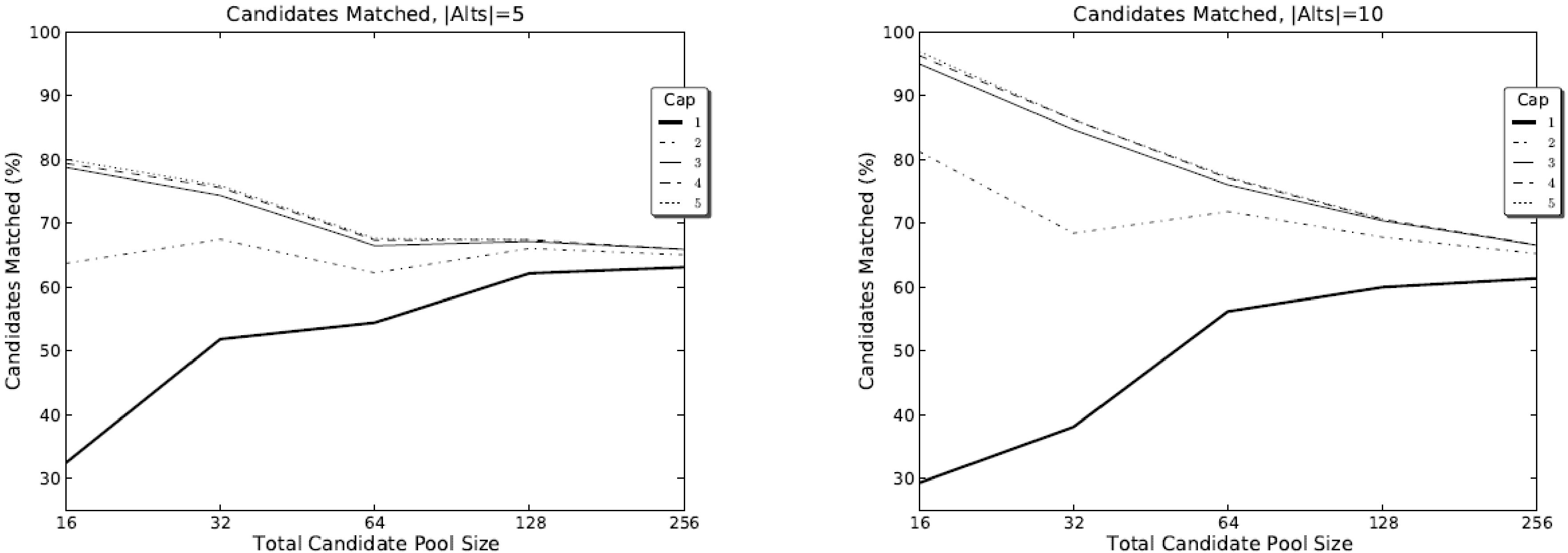
Chains in real exchanges



[Dickerson et al., 2012a]

Chains in large exchanges

- **Theorem [Ashlagi et al. 2012, Dickerson et al., 2012a]:** In large random graphs, w.h.p. ∃opt allocation with cycles of length ≤ 3 and chains of length ≤ 3

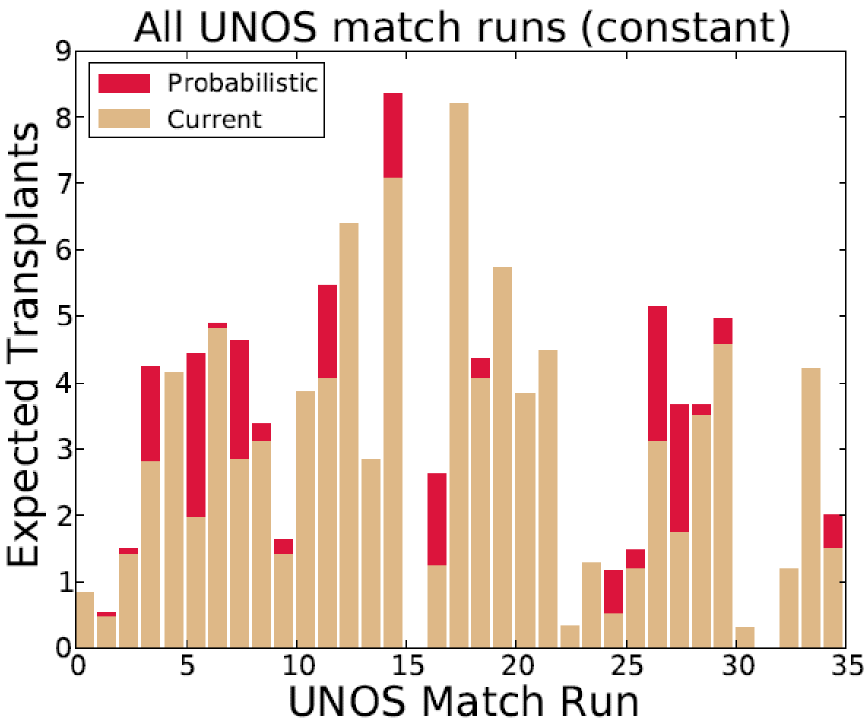


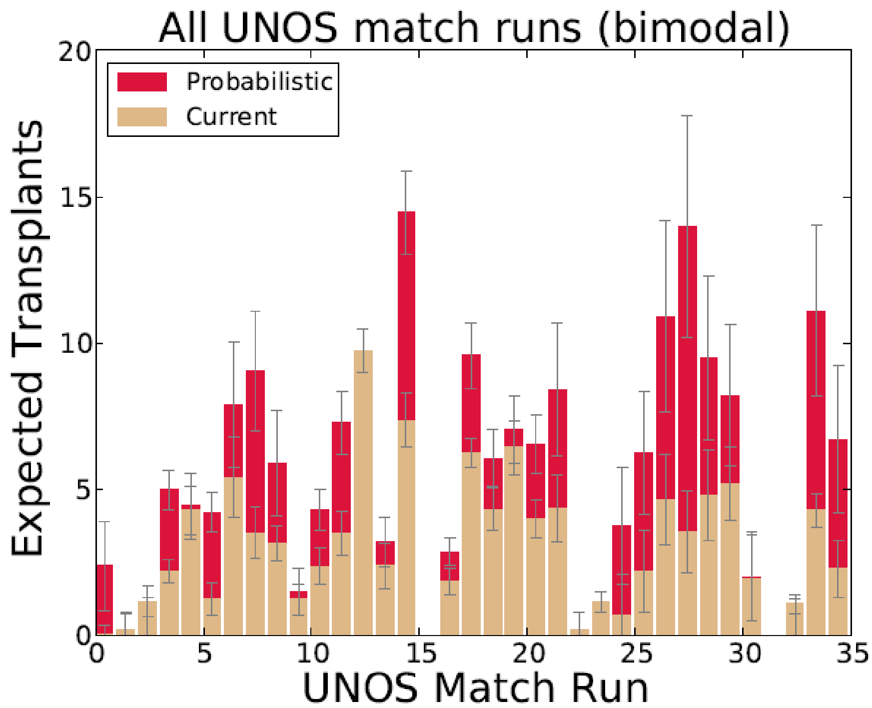
[Dickerson et al., 2012a]

Introducing: Crossmatches

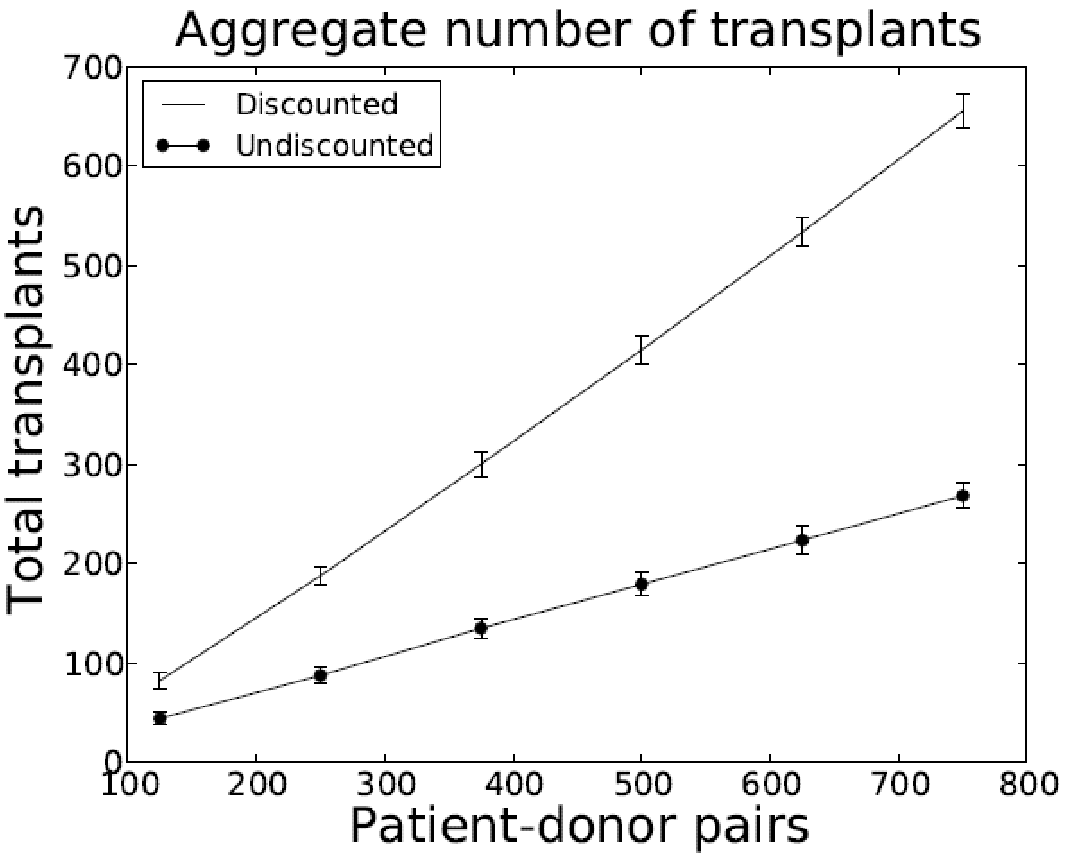
- Mixing cells and serum to determine whether patient will reject the kidney
- Adds another level of uncertainty: assume that crossmatch is negative (match possible) with some probability
- Optimization should now favor short cycles and short chains

Results from real data





Results from simulations



Introducing: dynamics

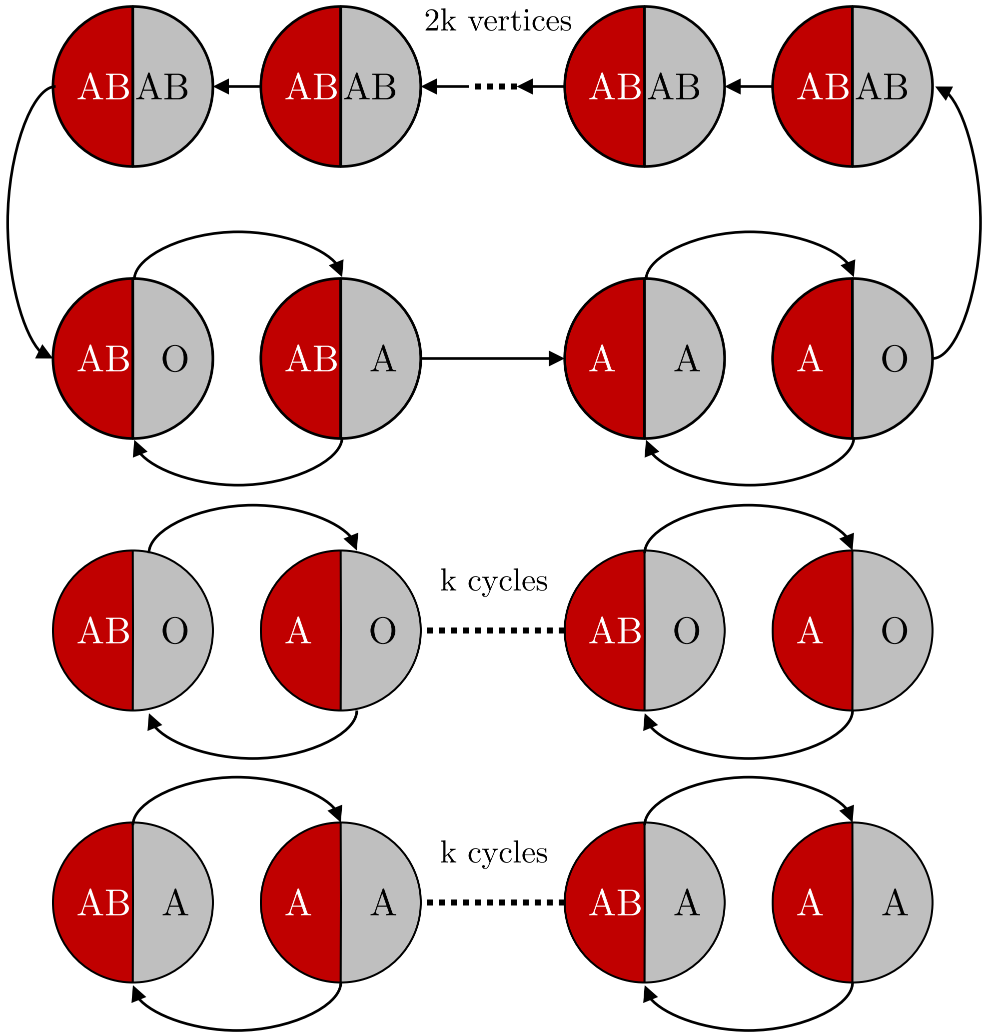
- Every month new pairs enter the pool, and some pairs leave
- Matching myopically may not be optimal; should we save an AB-O pair for later?
- How can we look into the future?

VERTEX POTENTIALS

- Assign a potential to each donor-patient pair and each altruistic donor according to blood type [Dickerson et al., 2012b]
- In each round, maximize cardinality of matching minus total potential removed
- Optimize potentials using local search

Vertex potentials are bad?

- Opt matches 6k+4
- Match pulsing cycles ⇒ total at most 4k+4
- Do not match pulsing cycles ⇒
  - $P_{AB\text{-}O}+P_{AB\text{-}A} > 2$
  - $P_{A\text{-}A}+P_{A\text{-}O} > 2$
- Either
  - $P_{AB\text{-}O}+P_{A\text{-}O} > 2$
  - $P_{A\text{-}A}+P_{AB\text{-}A} > 2$
- Do not match k cycles in first stage
- Match 4k+4 overall



Vertex potentials are good