GRADUATE AI
Lecture 23: Game theory II
April 16, 2012

Teachers:
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A curious game

Iterated elimination $\Rightarrow$ Unique NE at (up,left)
**Commitment is good**

- Suppose the game is played as follows:
  - Row player commits to playing a row
  - Column player observes the commitment and chooses column
- Row player can commit to playing down!
Commitment to mixed strategy

- By committing to a mixed strategy, row player can guarantee a reward of 2.5
- Called a Stackelberg (mixed) strategy

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0,0</td>
<td>2,1</td>
</tr>
<tr>
<td>1</td>
<td>1,1</td>
<td>3,0</td>
</tr>
</tbody>
</table>

P = \frac{1}{2} = 0.5 \quad Q = \frac{1}{2} = 0.5
Computing Stackelberg

- **Theorem** [Conitzer and Sandholm, EC 2006]: In 2-player normal form games, an optimal Stackelberg strategy can be found in poly time.

- **Theorem** [ditto]: the problem is NP-hard when the number of players is $\geq 3$. 
Tractability for 2 players

• For each pure follower strategy $t$, we compute via the LP below a strategy for the leader such that
  o Playing $t$ is a best response for the follower
  o Under this constraint, the leader strategy is optimal
• Choose $t^*$ that maximizes leader value

\[
\begin{align*}
\text{maximize} & \quad \sum_{s \in S} p_s u_l(s, t) \\
\text{subject to} & \quad \text{for all } t' \in T, \sum_{s \in S} p_s u_f(s, t) \geq \sum_{s \in S} p_s u_f(s, t') \\
& \quad \sum_{s \in S} p_s = 1
\end{align*}
\]
Application: Security

- Airport security: deployed at LAX
- Federal Air Marshals
- Coast Guard

Idea:
  - Defender commits to mixed strategy
  - Attacker observes and best responds
The Element of Surprise

To help combat the terrorism threat, officials at Los Angeles International Airport are introducing a bold new idea into their arsenal: random security checkpoints. Can game theory help keep us safe?

WEB EXCLUSIVE

By Andrew Murr
Newsweek
Updated: 1:00 p.m. PT Sept 28, 2007

Sept. 28, 2007 - Security officials at Los Angeles International Airport now have a new weapon in their fight against terrorism: complete, baffling randomness. Anxious to thwart future terror attacks in the early stages while plotters are casing the airport, LAX security patrols have begun using a new software program called ARMOR, NEWSWEEK has learned, to make the placement of security checkpoints completely unpredictable. Now all airport security officials have to do is press a button labeled "Randomize," and they can throw a sort of digital cloak of invisibility over where they place the cops' antiterror checkpoints on any given day.
SECURITY GAMES

- Model due to [Kiekintveld et al., AAMAS 2009]
- Set of targets $T$
- Set of security resources $\Omega$ available to the defender (leader)
- Set of schedules $S \subseteq 2^T$
- Resource $\omega$ can be assigned to one of the schedules in $A(\omega) \subseteq S$
- Attacker chooses one target to attack
- Utilities depend on target and whether it is defended
Solving security games

- Consider the case of $S=T$, i.e., resources are assigned to individual targets, i.e., schedules have size 1
- Nevertheless, number of leader strategies is exponential
- **Theorem** [Korzhyk et al., AAAI 2010]: Optimal leader strategy can be computed in poly time
A compact LP

- LP formulation similar to previous one
- Advantage: logarithmic in #leader strategies
- Disadvantage: do probabilities correspond to strategy?

\[
\begin{align*}
\text{maximize } & \quad U_d(t^*, c) \\
\text{subject to } & \quad \forall \omega \in \Omega, \forall t \in A(\omega) : 0 \leq c_{\omega,t} \leq 1 \\
& \quad \forall t \in T : c_t = \sum_{\omega \in \Omega : t \in A(\omega)} c_{\omega,t} \leq 1 \\
& \quad \forall \omega \in \Omega : \sum_{t \in A(\omega)} c_{\omega,t} \leq 1 \\
& \quad \forall t \in T : U_a(t, c) \leq U_a(t^*, c)
\end{align*}
\]
Fixing the probabilities

\[
\begin{array}{c}
\omega_1 \\
\omega_2
\end{array}
\begin{array}{c}
.7 \\
.2 \\
.3 \\
.7
\end{array}

\begin{array}{c}
t_1 \\
t_2 \\
t_3
\end{array}

\begin{array}{ccc}
t_1 & t_2 & t_3 \\
\omega_1 & .7 & .2 & .1 \\
\omega_2 & 0 & .3 & .7
\end{array}

\begin{array}{ccc}
t_1 & t_2 & t_3 \\
\omega_1 & 0 & 0 & 1 \\
\omega_2 & 0 & 1 & 0
\end{array}

\begin{array}{ccc}
t_1 & t_2 & t_3 \\
\omega_1 & 0 & 1 & 0 \\
\omega_2 & 0 & 0 & 1
\end{array}

\begin{array}{ccc}
t_1 & t_2 & t_3 \\
\omega_1 & 1 & 0 & 0 \\
\omega_2 & 0 & 1 & 0
\end{array}

\begin{array}{ccc}
t_1 & t_2 & t_3 \\
\omega_1 & 1 & 0 & 0 \\
\omega_2 & 0 & 0 & 1
\end{array}
Fixing the probabilities

- The probabilities $c_{\omega,t}$ satisfy theorem’s conditions.
- By 3, each matrix consists of $\{0,1\}$ entries.
- Interpretation by 4: $\omega$ assigned to $t$ iff corresponding entry is 1.
- By 1, we get a mixed strategy.
- By 2, gives right probs.

**Theorem 1 (Birkhoff-von Neumann (Birkhoff 1946)).** Consider an $m \times n$ matrix $M$ with real numbers $a_{ij} \in [0, 1]$, such that for each $1 \leq i \leq m$, $\sum_{j=1}^{n} a_{ij} \leq 1$, and for each $1 \leq j \leq n$, $\sum_{i=1}^{m} a_{ij} \leq 1$. Then, there exist matrices $M^1, M^2, \ldots, M^q$, and weights $w^1, w^2, \ldots, w^q \in (0, 1]$, such that:

1. $\sum_{k=1}^{q} w^k = 1$;
2. $\sum_{k=1}^{q} w^k M^k = M$;
3. for each $1 \leq k \leq q$, the elements of $M^k$ are $a_{ij}^k \in \{0, 1\}$;
4. for each $1 \leq k \leq q$, we have: for each $1 \leq i \leq m$, $\sum_{j=1}^{n} a_{ij}^k \leq 1$, and for each $1 \leq j \leq n$, $\sum_{i=1}^{m} a_{ij}^k \leq 1$.

Moreover, $q$ is $O((m + n)^2)$, and the $M^k$ and $w^k$ can be found in $O((m + n)^{4.5})$ time using Dulmage-Halperin algorithm (Dulmage and Halperin 1955; Chang, Chen, and Huang 2001).
**Generalizing?**

- Schedules of size 2
- Air Marshals domain has such schedules: outgoing + incoming flight (bipartite graph)
- Previous approach fails
- **Theorem** [Korzhyk et al., AAAI 2010]: (even bipartite) problem is NP-hard
Mechanism design!

• A subfield of game theory that focuses on designing the rules of the game to achieve desirable properties
• We will only cover a tiny fraction of the very basics of auction theory
Ad auctions

Google search results for "auctions"

- Quibids: Discount Auctions | Quibids.com
- Police Auctions | policeauctions.com
- Overstock Auction Prices | DealDash.com
- M Davis Group | mdaingroup.com
- Munauction | munauction.com
- Harry Davis & Co | harrydavis.com
- Aspire Auctions, Inc | aspireauctions.com
- Internet Auction Group | internetauctiongroup.com
- True Blue Auctions | trueblueauctions.com
- Auction My Card LLC

Map for auctions

- Record™ Auctions | recordauctions.com
- Liquidation.com Auctions | liquidation.com
- Best Auction Deals Online | bdo.com
- Penny Auction - Free Bids | pennybidday.com
- Tired of eBay? | yardstacl.com
- Government Auctions | governmentauctions.org

Carnegie Mellon University
English auctions

• Most well-known type of auctions
  o Ascending
  o Open cry
  o First price

• Dominant strategy: successively bid slightly more than current highest bid until price reaches valuation

• Susceptible to:
  o Winner’s curse: why doesn’t anyone else want the good at the final price?
  o Shills: work for auctioneer and drive prices up
Other boring auctions

• Dutch
  o Auctioneer starts at high price
  o Auctioneer lowers price until a bidder makes a bid at current price

• First-price sealed-bid auction
  o Bidders submit sealed bids
  o Good is allocated to highest bidder
  o Winner pays price of highest bid

• Bids generally do not match valuation!
Vickrey auction

• Bidders submit sealed bids
• Good is allocated to highest bidder
• Winner pays price of *second highest* bid!!
• Amazing observation: bidding true valuation is a dominant strategy!!
Truthfulness: bidding high

- Three cases based on highest other bid (blue dot)
- Higher than bid: lose before and after
- Lower than valuation: win before and after, pay same
- Between bid and valuation: lose before, win after but overpay
Truthfulness: bidding low

• Three cases based on highest other bid (blue dot)
• Higher than valuation: lose before and after
• Lower than bid: win before and after, pay the same
• Between valuation and bid: win before with profit, lose after
SEQUENTIAL AUCTIONS ARE BAD

• A computer and screen are sold in two Vickrey auctions
• Each is worthless alone but together their value to you is $500
• What should bid in the first auction?
  o Say you bid $200 and lose to a $300 bid; the screen may sell for $50
  o Say you bid $200 and win; the screen may sell for $500
Combinatorial auctions

- Bidders submit bids for *subsets* of goods
- Example:
  - ({A, C, D}, 7)
  - ({B, E}, 7)
  - ({C}, 3)
  - ({A, B, C, E}, 9)
  - ({D}, 4)
  - ({A, B, C}, 5)
  - ({B, D}, 5)
- What is the optimal solution?
Winner determination

• Allocate to maximize social welfare
• Consider the special case of *single minded bidders*: each bidder $i$ values a subset $S_i$ of items at $v_i$ and any subset that does not contain $S_i$ at 0

• **Theorem (folk)**: optimal winner determination is NP-complete, even with single minded bidders
**NP-HARDNESS + PIC**

- **INDEPENDENT SET (IS):** given a graph, is there a set of vertices of size $k$ such that no two are connected?

- Given an instance of IS:
  - The set of items is $E$
  - Player for each vertex
  - Desired bundle is adjacent edges, value is 1

- A set of winners $W$ satisfies $S_i \cap S_j$ for every $i \neq j \in W$ iff the vertices in $W$ are an independent set
Final remarks

• Vickrey auction can be generalized to yield a truthful mechanism (VCG) for combinatorial auctions

• Requires optimally solving the winner determination problem

• Resorting to approximation is no longer truthful

• Tons of research on practical algorithms for solving CAs, and on approximation algorithms that are truthful