Graduate AI
Lecture 20: Social Choice I

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Social choice theory

• A mathematical theory that deals with aggregation of individual preferences
• Origins in ancient Greece
• Formal foundations: 18th Century (Condorcet and Borda)
• 19th Century: Charles Dodgson
• 20th Century: Nobel prizes to Kenneth Arrow and Amartya Sen
Computational social choice

• Two-way interaction with AI
• $\text{AI} \Rightarrow \text{social choice}$
  o Algorithms and computational complexity
  o Machine learning in social choice
  o Knowledge representation
  o Markov decision processes
Computational social choice

• Social choice ⇒ AI
  o Multiagent systems: reducing communication
  o Human computation: aggregating peoples’ opinions
The voting model

• Set of voters $N = \{1, \ldots, n\}$
• Set of alternatives $A, |A| = m$
• Each voter has a ranking over the candidates
• $x >_i y$ means that voter $i$ prefers $x$ to $y$
• Preference profile = collection of all voters’ rankings

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Voting rules

• *Voting rule* = function from preference profiles to alternatives that specifies the winner of the election

• Plurality
  
  o Each voter awards one point to top alternative
  
  o Alternative with most points wins
  
  o Used in almost all political elections
More voting rules

• Borda count
  o Each voter awards $m-k$ points to alternative ranked $k$’th
  o Alternative with most points wins
  o Proposed in the 18th Century by the chevalier de Borda
  o Used in the national assembly of Slovenia
  o Similar to rule used in the Eurovision song contest

Lordi, Eurovision 2006 winners
More voting rules

- **Veto**
  - Each voter vetoes his least preferred alternative
  - Alternative with least vetoes wins

- **Positional scoring rules**
  - Defined by a vector \((s_1, \ldots, s_m)\)
  - Each voter gives \(s_k\) points to \(k\)’th position
  - Plurality: \((1,0,\ldots,0)\); Borda: \((m-1,m-2,\ldots,0)\), Veto: \((1,\ldots,1,0)\)
More voting rules

• a beats b in a *pairwise election* if the majority of voters prefer a to b

• Plurality with runoff
  o First round: two alternatives with highest plurality scores survive
  o Second round: pairwise election between these two alternatives
More voting rules

• Single Transferable vote (STV)
  o m-1 rounds
  o In each round, alternative with least plurality votes is eliminated
  o Alternative left standing is the winner
  o Used in Ireland, Malta, Australia, and New Zealand (and Cambridge, MA)
STV: example

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Marquis de Condorcet

- 18th Century French Mathematician, philosopher, political scientist
- One of the leaders of the French revolution
- After the revolution became a fugitive
- His cover was blown and he died mysteriously in prison
Condorcet winner

- Condorcet winner = alternative that beats every other alternative in pairwise election
- Condorcet paradox = Condorcet winner may not exist
- Condorcet criterion = elect a Condorcet winner if one exists
- Does plurality satisfy criterion? Borda?
More voting rules

• Copeland
  o Alternative’s score is \#alternatives it beats in pairwise elections
  o Why does Copeland satisfy the Condorcet criterion?

• Maximin
  o Score of x is \( \min_y |\{i \in N : x >_i y\}| \)
  o Why does Maximin satisfy the Condorcet criterion?
Awesome example

• Plurality: a
• Borda: b
• Condorcet winner: c
• STV: d
• Plurality with runoff: e

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Manipulation

- Using Borda count
- Top profile: b wins
- Bottom profile: a wins
- By changing his vote, voter 3 achieves a better outcome!
**Strategyproofness**

- A voting rule is *strategyproof (SP)* if a voter can never benefit from lying about his preferences:
  \[ \forall <, \forall i \in \mathbb{N}, \forall <'_i, f(<) \geq_i f(<'_i, <_{-i}) \]

- If there are two candidates then plurality is SP
Gibbard-Satterthwaite

- A voting rule is *dictatorial* if there is a voter who always gets his most preferred alternative
- A voting rule is *onto* if any alternative can win
- **Theorem (Gibbard-Satterthwaite):** If \( m \geq 3 \) then any voting rule that is SP and onto is dictatorial
- In other words, any voting rule that is onto and nondictatorial is manipulable
Proof of G-S Theorem

• We prove the following statement on the board

• If \( m \geq 3 \) and \( n = 2 \) then any voting rule that is SP and onto is dictatorial

• The proof also appears in: L.-G. Svensson. The proof of the Gibbard-Satterthwaite Theorem revisited, Theorem 1 (available from course website)
Lemmas

• A voting rule satisfies **monotonicity** if:
  \[ f(<) = a, \forall i \in N, x \in A, [x \leq a \Rightarrow x \leq' a] \]
  implies that \( f(<') = a \)

• **Lemma:** Any SP voting rule is monotonic

• A voting rule satisfies **Pareto optimality (PO)** if:
  \( \forall i \in N, x >_i y \Rightarrow f(<) \neq y \)

• **Lemma:** Any SP and onto voting rule is PO