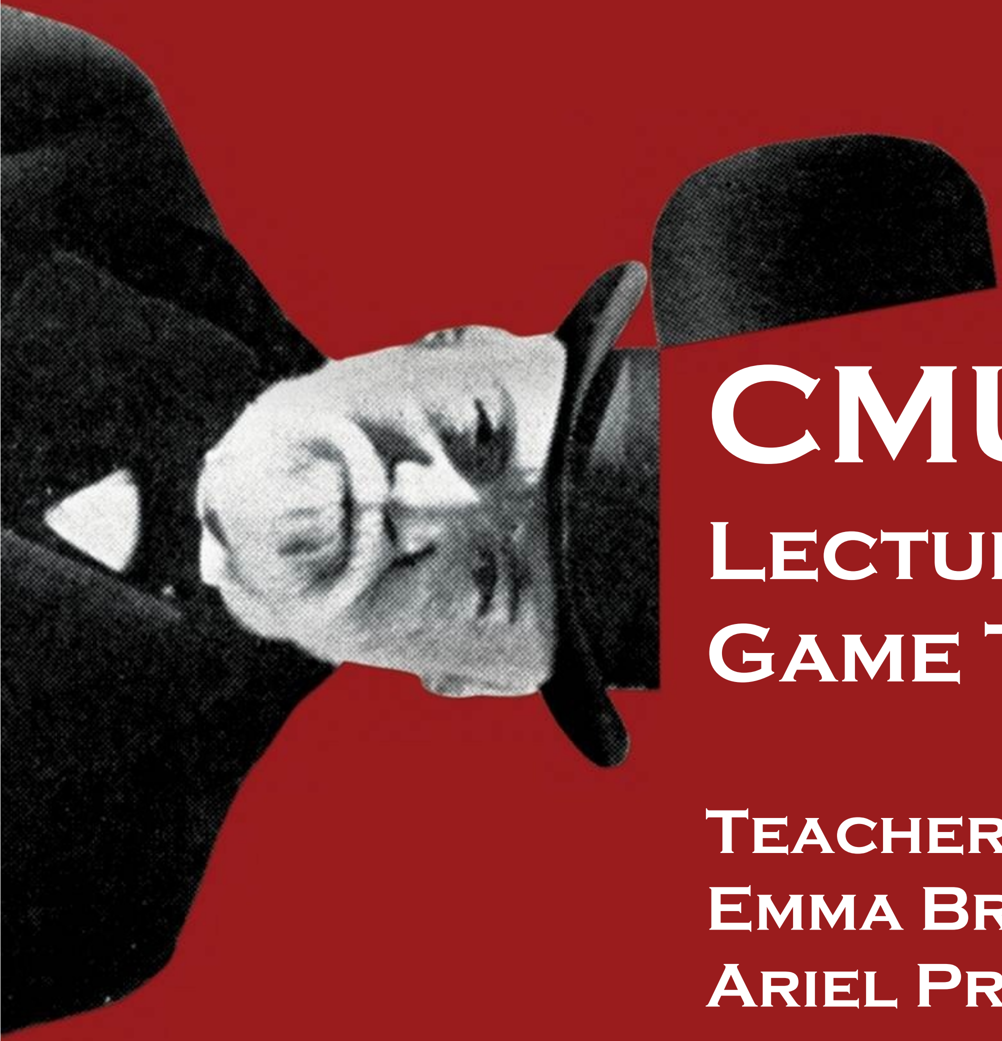

CMU 15-381

Lecture 17: Game Theory 1

Teachers:
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Ariel Procaccia (this time)

The prisoner's dilemma

- Two men are charged with a crime
- They are told that:
  - If one rats out and the other does not, the rat will be freed, other jailed for nine years
  - If both rat out, both will be jailed for six years
- They also know that if neither rats out, both will be jailed for one year

The prisoner's dilemma

	Cooperate	Defect
Cooperate	-1,-1	-9,0
Defect	0,-9	-6,-6

Click: what would you do?

Understanding the dilemma

- Defection is a dominant strategy
- But the players can do much better by cooperating
- Related to the tragedy of the commons

In real life

- Presidential elections
  - Cooperate = positive ads
  - Defect = negative ads
- Nuclear arms race
  - Cooperate = destroy arsenal
  - Defect = build arsenal
- Climate change
  - Cooperate = curb CO_2 emissions
  - Defect = do not curb

The professor's dilemma

Professor \ Class	Listen	Sleep
Make effort	10^6,10^6	-10,0
Slack off	0,-10	0,0

Click: dominant strategies?

Nash equilibrium

- Each player's strategy is a best response to strategies of others
- Formally, a Nash equilibrium is a vector of strategies $s=(s_1,\ldots,s_n)$ such that $\forall i \in N,\ \forall s'_i \in S_i,\ u_i(s) \geq u_i(s'_i, s_{-i})$
- https://www.youtube.com/watch?v=CemLiSI5ox8
- Click: what are the Nash equilibria of the professor's dilemma?

Rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Click: Nash equilibrium?

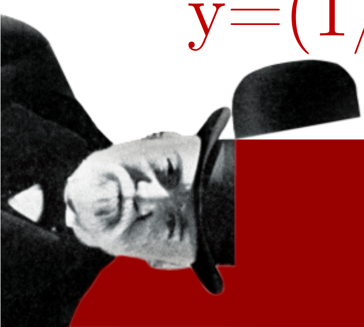
Mixed strategies

- A mixed strategy is a probability distribution over pure strategies
- The mixed strategy of player 1 is $x=(x_1,\ldots,x_n)$, $x_i = \Pr[1 \text{ plays strategy } s_i]$
- Similarly, player 2 plays $y=(y_1,\ldots,y_n)$
- The utility of player i is $u_i(x,y) = \Sigma_{j,k} [x_j \cdot y_k \cdot u_i(s_j,s_k)]$

Exercise: Mixed NE

- $x=(1/2, 1/2, 0)$,
  $y=(0, 1/2, 1/2)$,
  what is $u_1(x,y)$?
- $x=(1/3, 1/3, 1/3)$,
  $y=(1/3, 1/3, 1/3)$,
  what is $u_1(x,y)$?
- $x=(1/2, 1/2, 0)$,
  $y=(1/2, 1/2, 0)$, is NE?
- $x=(1/3, 1/3, 1/3)$,
  $y=(1/3, 1/3, 1/3)$, is NE?

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0



Nash's Theorem

- **Theorem [Nash, 1950]:** if everything is finite then there exists at least one (possibly mixed) Nash equilibrium
- However, how does one *compute* a Nash equilibrium?
- Computing a Nash equilibrium is computationally hard [Daskalakis et al., STOC 2006]

Application: Interdomain routing

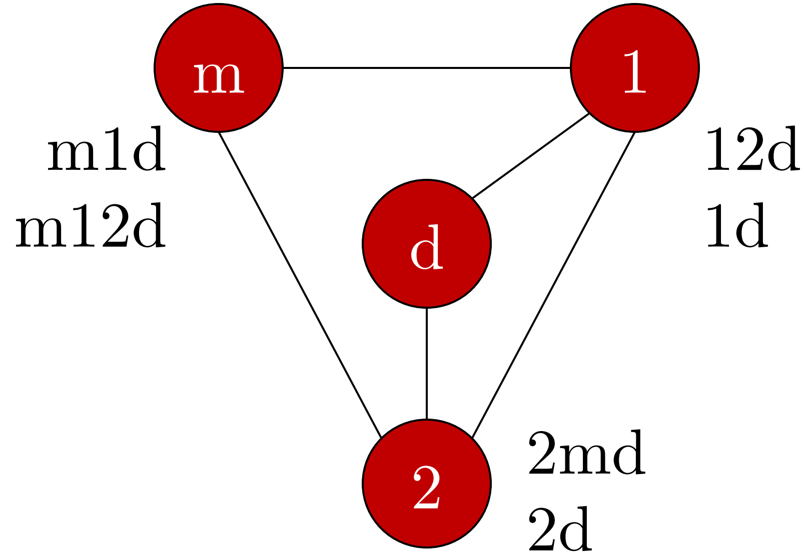
- Internet composed of smaller networks called *autonomous systems* (AS)
- Owned by competing entities (Microsoft, AT&T, etc.)
- Interdomain routing = establishing routes between ASes
- Standard protocol: BGP

Application: Interdomain routing

- Graph with n source nodes (players) and a destination node
- Each player has preferences over routes to the destination
- Under BGP ASes continuously:
    - Receive updates about routes of neighbors
    - Choose a neighbor to send traffic to
    - Announce new route to neighboring nodes

Application: Interdomain routing

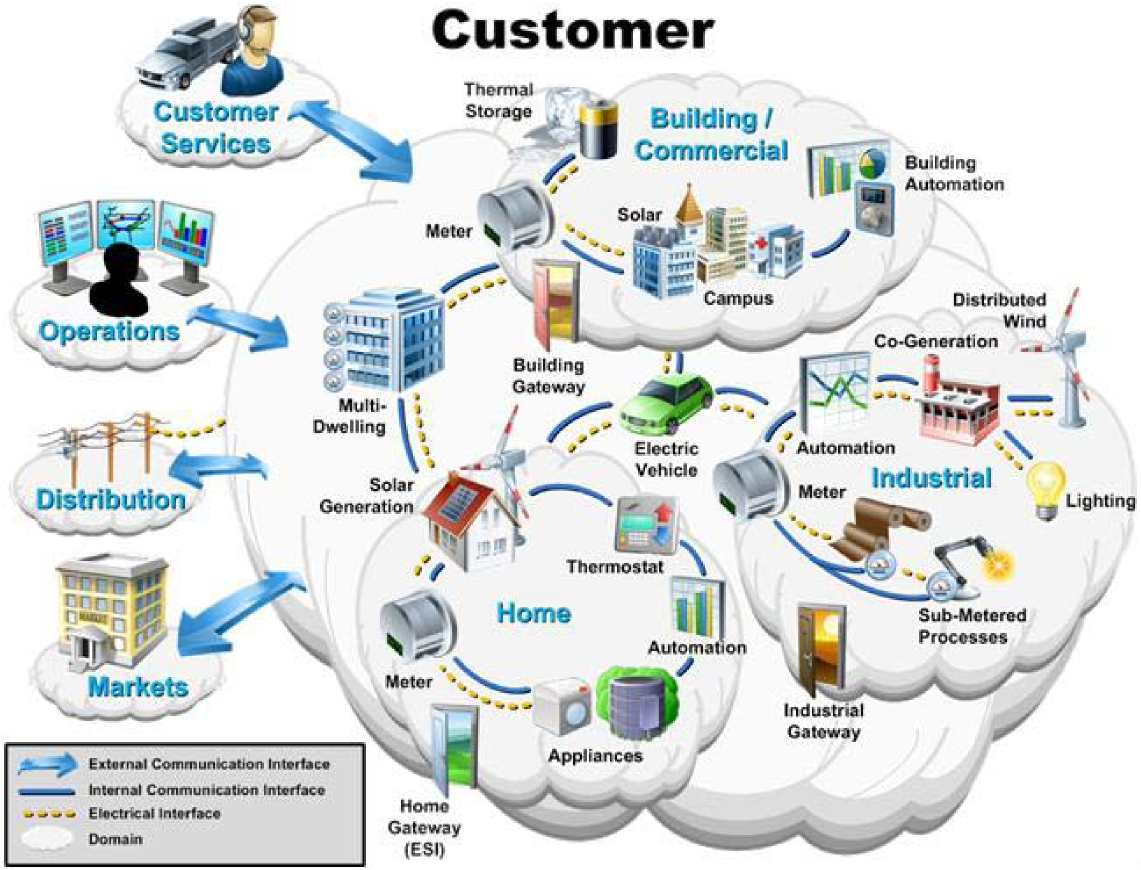
- **Theorem [Levin et al, STOC 2008]:** Following BGP is not an (ex-post) NE
- BGP converges to the NE (12d,2d,m12d)
- But… if m repeatedly announces to 2 the route md
- 2 would go with 2md
- 1 would go with 1d
- m gets m1d!



Application: Interdomain routing

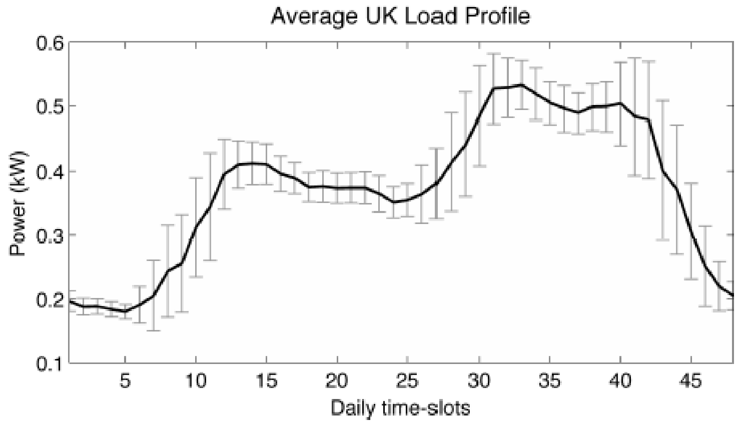
- Route verification = players can verify that neighbors' declared paths actually exist
- **Theorem [Levin et al., STOC 2008]:** Assuming route verification (+mild technical condition), following BGP is an (ex-post) Nash equilibrium!
- Provides partial explanation for why interdomain routing functions so well!

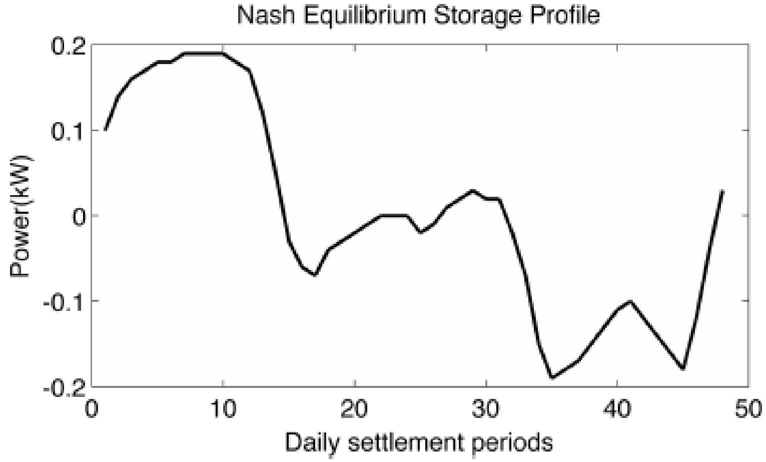
Application: Smart grid



Application: Smart grid

- Energy storage devices advocated for saving energy in future smart grid
- Bad if all are charged at the same time
- Solution: agent-based management system that allows storage devices to converge to equilibrium [Vytelingum et al., AAMAS 2010]





Application: Smart grid

- Strategy of an agent: how much to charge in each half hour of the day
- The behavior of electricity suppliers is specified by a supply curve
- Equilibrium can be analytically computed
- Simulations show that in eq., savings of 13% on electricity bill in UK