

# 15-251: Great Theoretical Ideas In Computer Science

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## Recitation 9

### Conditions permitting

- (a) Mary flips a fair coin. If it lands heads, she rolls a 3-sided die; otherwise she rolls a 4-sided die. What is the probability that she rolls at least a 3?
- (b) Martha is making three pancakes, each of which burns with probability 0.5. Given that at least one pancake burns, what is the probability that they all burn?

### T totals

- (a) The law of total probability states that any three events  $A, B, T$  where  $A, B$  partition the sample space ( $A \cap B = \emptyset$  and  $A \cup B = S$ ),

$$Pr[T] = Pr[T|A] \cdot Pr[A] + Pr[T|B] \cdot Pr[B]$$

Prove this fact and note that it extends to partitions of any size.

- (b) Alice flips a fair coin  $n + 1$  times and Bob flips a fair coin  $n$  times. What is the probability that Alice's coin comes up heads strictly more than Bob's does?

### Dog gone

You lose sight of your dog running in the park one day. When you chase after, you come to a fork. Each branch of the fork goes on infinitely far, but your dog chose one of the branches, walked  $d$  meters down it, and stopped. Describe a good algorithm to find your dog and analyze its competitive ratio. (Cost is the total distance you walk.)

### Blunt truths

Your friend Norville is showing off his fair 7-sided die and wants to play a game where you keep rolling the die until you roll a 7. He claims that the game is more likely to end after an odd number of rolls “because 7 is, like, odd, man.” Is Norville right?

### Online dating

There are  $n$  people with whom you can go on dates, and you would like to propose marriage to the best one. Of course, once you've proposed to somebody, it would be rude to keep going on dates with other people, but you can only propose to the last person you went on a date with. So after each date, you must immediately decide whether or not to propose to that person. (We assume that you can compare people and that no two people are exactly equally good.)

Your strategy is as follows: first, you go on dates with  $t$  people and reject them. Then, continue dating until you meet somebody who is better than all of the first  $t$ , and propose to that person. (If nobody meets this criterion, remain forever alone.)

For what value of  $t$  do you maximize the probability that you propose to the best candidate?