1. Term, formula, hybrid program, or none of the above?
For each of the following, determine if the expression is a \( dC \) term, a well-formed \( dC \) formula, a well-formed hybrid program, or none of the above (ie. it is not well-formed). In the case that the expression is none of the above, give a short explanation.

(a) \(? (3 > 2)\)
(b) \([x := 1; \{ x'' = x \} (-1 \leq x \wedge x \leq 1) \leftrightarrow [y := \cos z] (-1 \leq y \wedge y \leq 1)\]
(c) \(x = 42\)
(d) \(x := y \cup x := z\)
(e) \((x > y)^*\)
(f) \(42\)
(g) \(L > 0 \wedge U > 0 \rightarrow [(x := *; ?(L < x \wedge x < U))^*](x = y)\)
(h) \(x\)
(i) \(y := \pi\)
(j) \((x > y \wedge (y > z \cup y > w))\)
(k) \(x := 0 \rightarrow [{\{x' = 1, y' = 1 \& y \leq 10}\} x \leq 10\]
(l) \((x := x + 1)^*\)
(m) \(\forall x \exists y \ (x < y)\)
(n) \([x := 42]\)
(o) \(x := y^2\)
(p) \(y > 1 \wedge z > 1 \rightarrow [x := z^y](x > z)\)
(q) \([x = y; y = z]? (x \geq z)\)

2. Practice writing hybrid programs.

(a) The if-then-else construct is not actually in the grammar of hybrid programs; it’s just syntactic sugar. Write an equivalent hybrid program which does not depend on if-then-else:

\[
\text{if } (x^2 \geq 5 \wedge x \leq 0) \ x := x - 1 \ \text{else} \ x := x + 1
\]

(b) Non-deterministic Choice: Write a hybrid program that assigns either \( a \) or \( b \) to the variable \( x \).
3. Safety and Contracts

(a) A safety property is something that a cyber-physical system should always maintain. Describe a cyber-physical system and then name three safety properties that it should never violate.

(b) Suppose you want to prove that a property $\phi$ is a safety property of hybrid program $\alpha$, i.e. that $\phi$ holds under all possible runs of hybrid program $\alpha$. Write the $\mathcal{DL}$ formula that expresses this.

(c) Now, suppose you want to prove that, given initial conditions $\psi$, another property $\phi$ is a safety property of hybrid program $\alpha$. Write the $\mathcal{DL}$ formula that expresses this.

(d) Consider hybrid program $\alpha$:

$$\alpha \equiv x := w; ((y := 3; z := 20) \cup (z := 6; y := w + 7)); \{x' = 10\}; ?(y \leq 30)$$

For each variable in $\alpha$, list the set of all values they can reach at the end of a run of $\alpha$.

(e) Come up with at least two properties that hold at the end of all runs of $\alpha$ (example: $y \leq 30$ is a property that the program always satisfies, but now you can’t use upper bounds on $y$). Write down a $\mathcal{DL}$ formula which is true iff hybrid program $\alpha$ always satisfies those properties.

4. Non-Deterministic Evolution

$$\beta \equiv x := x_0; v := v_0; t := 0; \{x' = v, v' = a, t' = 1 \& v \geq 0\}; ?(v = 0)$$

(a) Assume that $a < 0 \land v_0 \geq 0$. At the end of a run of hybrid program $\beta$, what is the value of $t$ as a function of $x_0$, $v_0$, and $a$?

(b) Suppose we remove the guard $? (v = 0)$ at the end of hybrid program $\beta$. Again assuming that $a < 0 \land v_0 \geq 0$, what are the possible values of $t$ at the end of any run of this modified version of $\beta$?

(c) Suppose we assume instead that $a < 0 \land v_0 \leq 0$ ($v_0$ is less than or equal to zero). What are the possible values of $t$ at the end of any run of $\beta$?