

# Correction to Spectral Mixture (SM) Kernel Derivation for Multidimensional Inputs

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## Abstract

This note corrects a typo in the spectral mixture kernel in [Wilson and Adams \(2013\)](#) for the case of multidimensional inputs.

Spectral densities  $S(s)$  and stationarity kernels  $k(x, x') = k(x - x') = k(\tau)$  are Fourier duals:

$$k(\tau) = \int_{\mathbb{R}^P} S(s) e^{2\pi i s^\top \tau} ds. \quad (1)$$

[Wilson and Adams \(2013\)](#) derive the *spectral mixture* kernel by modelling  $S(s)$  as a symmetrized scale-location mixture of Gaussians:

$$S(s) = \sum_q w_q [\mathcal{N}(s; \mu_q, \Sigma_q) + \mathcal{N}(-s; \mu_q, \Sigma_q)]. \quad (2)$$

Substituting Eq. (2) into Eq. (1) we find

$$k_{\text{spectral mixture}}(x - x') = \sum_q w_q \frac{|\Sigma_q|^{\frac{1}{2}}}{(2\pi)^{\frac{P}{2}}} \exp\left(-\frac{1}{2} \left\| \Sigma_q^{\frac{1}{2}}(x - x') \right\|^2\right) \cos \langle x - x', 2\pi \mu_q \rangle \quad (3)$$

In [Wilson and Adams \(2013\)](#), for multidimensional inputs, and a diagonal covariance  $\Sigma_q = \text{diag}(v_q^{(1)}, \dots, v_q^{(P)})$ , the spectral mixture kernel is **incorrectly** written as

$$k(\tau) = \sum_{q=1}^Q w_q \prod_{p=1}^P \exp\{-2\pi^2 \tau_p^2 v_q^{(p)}\} \cos(2\pi \tau_p \mu_q^{(p)}) \quad (4)$$

Eq. (4) is incorrect. The cosine should be outside of the product. **The correct form for a diagonal covariance scale-location Gaussian mixture is:**

$$k_{\text{SM}}(\tau) = \sum_{q=1}^Q w_q \cos(2\pi\tau^\top \mu_q) \prod_{p=1}^P \exp\{-2\pi^2 \tau_p^2 v_q^{(p)}\} \quad (5)$$

The corrected version can be found in Wilson (2014) and Yang et al. (2015). **Note that the typo in Eq. (4) does not make any difference for  $P = 1$  dimensional inputs.** The correct multivariate version of the the spectral mixture kernel is implemented as `covSM.m` in the GPML toolbox (Rasmussen and Nickisch, 2010).

## References

- Rasmussen, C. E. and Nickisch, H. (2010). Gaussian processes for machine learning (GPML) toolbox. *Journal of Machine Learning Research (JMLR)*, 11:3011–3015.
- Wilson, A. G. (2014). *Covariance kernels for fast automatic pattern discovery and extrapolation with Gaussian processes*. PhD thesis, University of Cambridge.
- Wilson, A. G. and Adams, R. P. (2013). Gaussian process kernels for pattern discovery and extrapolation. *International Conference on Machine Learning (ICML)*.
- Yang, Z., Smola, A. J., Song, L., and Wilson, A. G. (2015). A la carte - learning fast kernels. *Artificial Intelligence and Statistics*.