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Measuring the Difficulty of Assembly Tasks from Tool Access Information*

Antonio Díaz-Calderón D. Navin-Chandra Pradeep K. Khosla
Engineering Design Research Center
and The Robotics Institute,
Carnegie Mellon University,
Pittsburgh, PA 15213.

Abstract

Focusing on accessibility issues, this paper presents a qualitative measure of assembly task difficulty based on concepts from information theory. It is proposed that assembly task difficulty is directly proportional to the logarithm of the assembly task efficiency, this efficiency being a function of the average information content of the assembly task. Preliminary evaluation results are presented where the performance of this metric is tested. The results suggest that the new measure does a good job in capturing assembly difficulty as a function of assembly geometry.

1 Introduction

Given the geometric configuration, the spatial relations of the components of the assembly, and the required tools, we present a measure of assembly task difficulty. This measure is based on the average information content of an assembly task†, giving an indicator its difficulty. The goal of providing such a measure is to extend the capabilities of existing CAD systems to that of assisting the designer in evaluating the design to detect possible problems that may only appear late in the design cycle, thus reducing design cycle time.

A number of factors influence the difficulty in executing an assembly task. These include picking, moving, and positioning parts and tools. We focus on difficulty arising from spatial constraints, for example, when using a screwdriver. Measuring the difficulty of a task will provide an indicator of how difficult it is to assemble the product. By comparing different designs for the same product, the measure can be used to design for easy assembly. We focus on three issues that affect the execution of a task:

tool access measures the space available to bring the tool to the target point.

tool operability measures the difficulty involved in operating the tool under less than ideal conditions as required by the tool.

hand clearance measures the difficulty involved in using the tool at a given orientation considering obstacles in the vicinity.

1.1 Proposed approach

The approach we propose in this paper is based on the fact that every task involves certain information content that can be measured by analyzing the geometry of the assembly. This idea is shown using the example in Fig. 1. The peg insertion task in Fig. 1a does not present any difficulty so long as the ratio of hole size to peg size is large. However, in the other two cases (Fig. 1b and Fig. 1c) one has to make more precise movements to properly match the peg and the hole. The axial asymmetry of the peg in Fig. 1c permits assembly in exactly one relative position.

This shows that as the geometry of both peg and hole gets more complex, the task becomes more difficult. One reason is that the information content of the task increases as the complexity of the geometry of the assembly increases. In other words, the agent executing the task has to consider more information to complete the task, (i.e, the agent needs to know the relationship between the shape of the peg and the shape of the hole and their relative dimensions Fig. 1c).

1.2 Related work

Much work has been done in design for assembly. Boothroyd [1] developed a method based on empirical data drawn from a set of experiments that provide

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†Henceforth referred to merely as *task*.

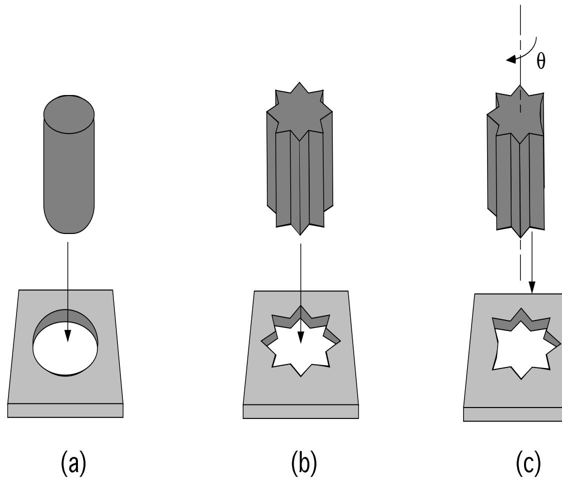

Figure 1: Example of peg-and-hole insertion for different shapes. The complexity of inserting the peg into the hole increases as the geometry of the peg and the hole gets more complex.

a relationship between design features and assembly times. The information provided by this method can be used to compare designs and/or to suggest modifications to existing designs. Although comprehensive, problems may arise when the assembly does not completely fit the data given. This can lead to inaccurate predictions on assembly times. Fitts [2] proposes the use of the *index of difficulty* to measure the performance of the human motor system in controlling movements that require high degree of dexterity. Sturges [3] use this concept to quantify human manual dexterity and extends his analysis to robotic manipulation. Sanderson [4] defines a probability distribution on the location and orientation of a mechanical part to measure handling complexity which he defined to be the entropy of the distribution. Accessibility issues were first described in [5, 6]. However they focused on deriving assembly plans and provided no systematic way of measuring difficulty. In [5] Sedas proposes the use of access cones which he called *direction cones* to determine access directions to a point in the assembly. Although limited to 2D, his method can be extended to 3D to handle more general assemblies (we will discuss this extension in Section 2.) The issue of accessibility has also been addressed by [7, 8, 9] but with the objective of finding feasible plans, not for design.

The rest of the paper is organized as follows: section 2 states the formulation of the problem, section 3 defines the function used to measure the task difficulty based on the geometric information provided by the assembly model, section 4 presents an example of the application of this measure, and we conclude in section 5.

2 Geometric Tools

An assembly operation is defined to be the establishment of all contacts between two subassemblies placing them in their final relative position. It is composed of a set of assembly tasks such that when all tasks in the set are successful, the assembly operation is successfully completed. An assembly task can be divided up into smaller parts which we call *primitive operations* or p-operations. A p-operation is defined to be the smallest operation that composes a task. Each p-operation will be related to an efficiency value (called q-value[‡]) which measures the goodness of the feature related with the p-operation. An example of a p-operation is the operation *rotate* peg which rotates the peg in Fig. 1c by θ degrees. In this example, the feature of interest is the amount of turning needed to align the peg, which can be measured by the cosine of the angle θ (i.e., $q - value = \cos(\theta)$). This choice of q-value shows that the p-operation executed on the pegs shown in Fig. 1a and Fig. 1b is highly efficient, while the efficiency of the p-operation executed on the peg shown in Fig. 1c depends on the amount of turning needed to align it.

The initial set of p-operations that we consider is the following:

Tool access Measures the region of space—called *access space*—from which a part can be reached by a tool. This depends only on the spatial configuration of the neighboring parts.

Tool orientation Measures the difference between the actual orientation of the tool to the orientation required for its operation under ideal conditions.

Tool clearance Measures the smallest distance from the tool to the assembly.

The rest of this section is devoted to the development of methods to compute the q-values for the initial set of p-operations using geometric models of parts and tools.

2.1 Access maps

The computation of q-values for each p-operation is based on the information provided by *access maps*. Access maps are maximal sets of directions that represent regions of space from which a point $\mathbf{p}$ in the assembly can be reached.

[‡] $0 < q \leq 1$

Let $\mathbf{f}$ be a face of a subassembly $S_i \in \mathcal{S}$[§], $\mathbf{P}$ be a tangent plane to $\mathbf{f}$, and $\mathbf{p}$ be the target point on a face $\mathbf{f}$ with tangent plane $\mathbf{P}$ (Fig. 2). The set of local access directions to $\mathbf{p}$ consists of all vectors $\mathbf{v} \in \Re^3$ such that $||\mathbf{v}|| = 1$ and $\mathbf{v}^{\mathrm{T}}\mathbf{m} \geq 0$, where $\mathbf{m}$ is the unit normal vector to $\mathbf{P}$. Since we are interested in only those directions from which the tool can be used, some of the directions included in this set must be discarded.

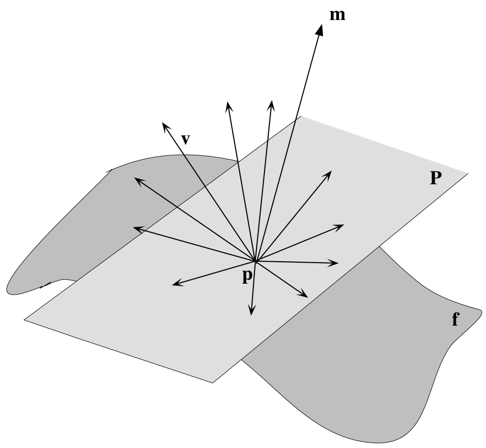

Figure 2: Local access directions.

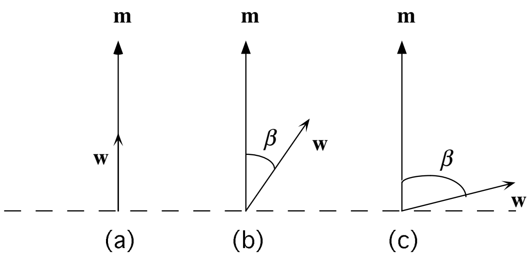

Figure 3: Classification of tools.

To see how this affects the selection of access directions, consider the schematic representation of three different tools (where the tool is represented by the line $\mathbf{w}$) (Fig. 3). From the figure, we see that the limits on the orientation of the tools will determine the selection of access directions. For instance if we are using a tool having operation limits as the one depicted in (Fig. 3a), the direction vector is restricted to be the vector $\mathbf{m}$, whereas for tools with operation limits of those depicted in (Fig. 3b), the direction vectors are those for which $\beta \leq \pi/4$ with respect to $\mathbf{m}$.

[§] An assembly $\mathcal{S}$ is composed of subassemblies $S_1, S_2, \ldots, S_n$, with each S_i being composed of one or more parts.

We define the set of local access directions (LAD) as

$$\mathsf{LAD} \stackrel{\mathrm{def}}{=} \left\{\mathbf{v} \in \Re^3 \mid ||\mathbf{v}|| = 1, \cos(\beta) \leq \mathbf{v}^{\mathrm{T}}\mathbf{m} \leq 1\right\} \quad (1)$$

Where the angle β determines the operational limits for the tool. Geometrically, LAD defines a cone with its apex at point $\mathbf{p}$ and $\mathbf{m}$ as its symmetry axis. Unfortunately not all vectors in LAD can be used to reach the point $\mathbf{p}$ if there are obstacles on the way. The problem now is to find a subset of directions of LAD such that there exists no obstructions with any $S_i \in \mathcal{S}$. We call this directions *global access directions*. The direction vectors in this subset are subject to the constraint

$$\mathbf{r}(\alpha) \bigcap_{i=1}^{n} S_i = \emptyset \quad (2)$$

where $\mathbf{r}(\alpha) = \mathbf{p} + \alpha\mathbf{v}$, $\mathbf{v} \in \mathsf{LAD}$. The set of global access directions (GAD) is defined as

$$\mathsf{GAD} \stackrel{\mathrm{def}}{=} \left\{\mathbf{v} \in \mathsf{LAD} \mid \mathbf{r}(\alpha) \bigcap_{i=1}^{n} S_i = \emptyset\right\} \quad (3)$$

The intersection of LAD and GAD with the unit sphere $\mathbf{S}$ defines a set of points σ on the surface of the sphere which we call *local access map* and *global access map* respectively (Fig. 4). Note that since $\mathsf{GAD} \subseteq \mathsf{LAD}$ implies $\mathrm{GAM} \subseteq \mathrm{LAM}$. In the special case where $\mathrm{GAM} = \mathrm{LAM}$ then the access to point $\mathbf{p}$ is totally unrestricted.

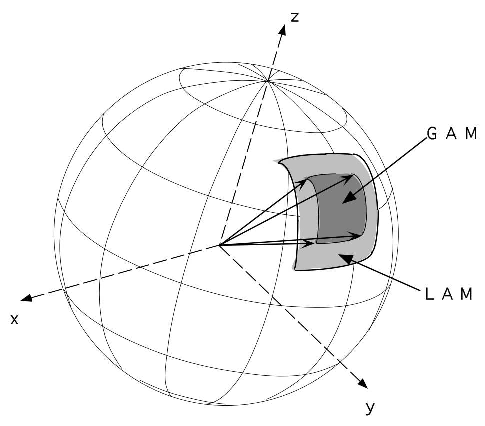

Figure 4: Local and Global access maps on the unit sphere: darker area represents global access directions.

Given that we now have a characterization of the available access space, we can proceed to define the q-values in terms of this maps.

Tool access The q-value for this p-operation measures the amount of free space available to access a particular point $\mathbf{p}$ in the assembly. This is defined as

$$\mathrm{AS} = \frac{\mathrm{area(GAM)}}{\mathrm{area(LAM)}} \tag{4}$$

Tool orientation We want to find the best direction vector such that

$$\begin{aligned} &\mathbf{v} \in \mathsf{GAD}, \\ &O(\mathbf{v}, T) \cap \mathcal{S} = \emptyset \\ &\max_{\mathbf{v}\in\mathrm{GAD}} \left(\mathbf{v}^{\mathrm{T}}\mathbf{m}\right) \end{aligned} \tag{5}$$

where $O(\mathbf{v}, T)$ represents the tool T after applying the orientation specified by $\mathbf{v}$. The constraint $\max(\mathbf{v}^{\mathrm{T}}\mathbf{m})$ ensures that the direction vector will be selected such that the tool is oriented closest to $\mathbf{m}$. Then the number

$$\mathrm{TO} = \mathbf{v}^{\mathrm{T}}\mathbf{m} \tag{6}$$

is the q-value for this p-operation and gives a measure of how far the tool is from its optimal orientation.

Tool clearance The q-value for tool clearance measures the amount of space between the tool and the assembly $\mathcal{S}$ when the tool has been positioned at point $\mathbf{p}_0$ and oriented as specified by Eq. 5. Let $\mathcal{F}$ be a finite set of faces of $\mathcal{S}$, such that for each face $\mathbf{f}_i \in \mathcal{F}$, we have $\mathbf{n}_i^{\mathrm{T}}\mathbf{q} - \mathbf{p}_0 \geq 0$ where $\mathbf{p}_0$ and $\mathbf{q}$ are points of $\mathcal{S}$. Let $\mathcal{G}$ be a finite set of faces of the tool such that $\mathcal{G}$ contains only those faces that are involved in grasping the tool. Then the minimum distance d from a face $\mathbf{g} \in \mathcal{G}$ to a face $\mathbf{f} \in \mathcal{F}$ (Fig. 5) is given by

$$\begin{aligned} d &= \min_{\substack{\mathbf{f}\in\mathcal{F},\\ \mathbf{g}\in\mathcal{G}}}(\|\mathbf{r}(\alpha)\|), \\ \alpha &= f(\mathbf{w}, \mathbf{p}) \end{aligned} \tag{7}$$

where $\mathbf{r}(\alpha) = \mathbf{p} + \alpha\mathbf{w}$, is the shortest line from a point $\mathbf{p}$ in $\mathbf{g}_i$ in the direction of the face normal $\mathbf{w}$ to $\mathbf{f}_i$ and $\alpha = f(\mathbf{w}, \mathbf{p})$ is the scale factor such that the line $\mathbf{r}(\alpha)$ intersects with $\mathbf{f}_i$.

A dimension commonly used in the establishment of hand clearance is the hand thickness at metacarpal [10]. This dimension specifies the minimum space required between the operator's hand and the device he/she is working on. We have called this dimension D and its permissible values as given in [10] are the following:

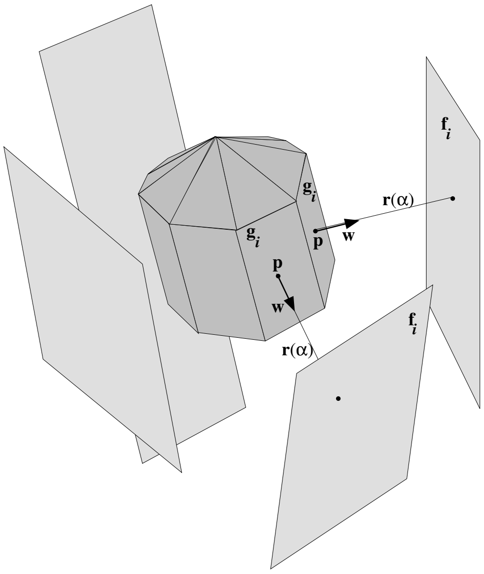

Figure 5: Finding the closest face $\mathrm{f}_i \in \mathcal{S}$ to the tool.

	5th	50th	95th
Males	1.1	1.2	1.3
Females	0.8	1.0	1.1

Table 1: Hand thickness for adults on different percentiles [10]. All dimensions are in inches.

Then the q-value TC is defined as follows

$$\mathrm{TC} = \begin{cases} \frac{d}{\mathrm{D}}, & \text{for } d < \mathrm{D} \\ 1, & \text{for } d \geq \mathrm{D} \end{cases} \tag{8}$$

3 A definition of assembly task difficulty

In this section we develop a measure of assembly task difficulty based on the geometric information extracted from the model.

Let Ω be the ordered set of p-operations pertaining to a given task. We can partition the task Ω in a finite number of p-operations (i.e., p-operation$_k$) whose probabilities p_k are known such that $\bigcup_{k=1}^{m} p\text{-operation}_k = \Omega$ and $\sum_{k=1}^{m} p_k = 1$. The problem of interest is to associate a measure of difficulty, $D(p_1, p_2, \ldots, p_m)$ with the task Ω. To begin with, let us define the probability values as $p_k = \frac{q_k}{Q}$ where $Q = \sum_{i=1}^{m} q_i$. The probability p_k of the p-operation$_k$ will be given by the normalized q-values for the entire task Ω. To associate a metric of information to the task Ω, we borrow the concept of *average* *information content* from *Information Theory* [11]:

$$H(\Omega) = -\sum_{i=1}^{m} p_i \log_2 p_i \qquad (9)$$

The quantity $-\log_2(p_k)$ is called *the information content* of the p-operation$_k$

$$I(p\text{-operation}_k) = -\log_2 \frac{q_k}{Q} \qquad (10)$$

The information content of the *k-th* p-operation will give a measure for the difficulty of executing it. Large values of $I(p\text{-operation}_k)$ mean the p-operation$_k$ has a high degree of difficulty, whereas small values of $I(p\text{-operation}_k)$ mean the p-operation$_k$ has a low degree of difficulty. Eq. 9 provides a measure of the easiness of the task which we call *Index of Easiness*. Large values of $H(\Omega)$ indicate low difficulty in executing the task. Eq. 9, takes on its maximum value when all $p_1 = p_2 = \cdots = p_m = 1/m$

$$H(\Omega) = -\sum_{i=1}^{m} p_i \log_2 p_i = \log_2 m \qquad (11)$$

this means that a task Ω having different q-values is not a very good choice. It is a good choice to have a task having similar q-values since this would lead to large values of $H(\Omega)$. Similar q-values can only be achieved when we have "ideal conditions" for each p-operation. Therefore the function $H(\Omega)$ takes on its maximum value when all the p-operations have q-values equal to 1, and at this point, $\max(H(\Omega)) = \log_2 m$, where m is the number of p-operations. Using this fact, we can find the efficiency of the task which is given by

$$E(\Omega) = \frac{H(\Omega)}{\log_2 m} \qquad (12)$$

To associate a measure of difficulty with the task, we need to find a functional relation such that $D(p_1, p_2, \ldots, p_m) = \phi(E(p_1, p_2, \ldots, p_m))$. We propose to use a logarithmic relation (i.e., $\phi = \ln$) since that relation has shown to have some correlation with the empirical data [1]. The difficulty measure for task Ω is defined to be

$$D(\Omega) \stackrel{\text{def}}{=} -\ln E(\Omega) \qquad (13)$$

If the geometry of the assembly changes in such a way that the efficiency decreases, the difficulty value for the assembly task increases. Note that the range of $D(p_1, p_2, \ldots, p_m)$ is $D \in [0, \infty)$, and $D(\Omega) \to \infty$ as the accessibility to the point approaches to zero.

Assembly	*Task Efficiency*	*Task Difficulty* $D = -\ln(E)$
Fig. 6a	0.791366	0.233995
Fig. 6b	0.786230	0.240506
Fig. 7a	0.750019	0.287657
Fig. 7b	0.881538	0.126087
Carburetor	0.762062	0.271727

Table 2: Difficulty measures for assembly examples.

4 Experimental results

In this section we present results obtained with the measure described in this paper. We tested it on 5 assemblies. In all the examples, the tool (screwdriver) is shown in the orientation selected when computing the q-value for tool orientation. The geometry of the assembly shown in Fig. 6a, limits the access space due to the two walls surrounding the fastener. This is reflected in a difficulty measure equal to 0.233995. Similarly, the cylindrical obstacle in the assembly of Fig. 6b contributes to an even smaller access space being reflected in a higher difficulty value. An extra constraint is added to the assembly of Fig. 7a for which the cylindrical obstacle has been grown up to affect the operability of the tool. This is reflected by a larger difficulty measure of $D = 0.287657$. Finally, the difficulty measure for the assembly of Fig. 7b is the smallest due to the larger access space provided by its geometry. A more complex example is shown in Fig. 8. Here the task is to separate the carburetor from the gas tank. To achieve this task four screws must be removed and as we can see, the air filter is obstructing the access to all of them. We have measured the difficulty to remove one of the screws and found a feasible orientation for the tool. Although the tool is oriented in a feasible orientation, this orientation is far from the optimal tool orientation. This orientation and the fact that the access space is very limited, give the task a high difficulty measure ($D = 0.271727$). Table 2 presents a summary of the different difficulty measures obtained for this assemblies.

To verify that our method is giving meaningful values we took an example from the literature [1] and adapted it to fit our model. The experimental set up is shown in (Fig. 9). Bootrhoyd has measured the time it takes to make the initial engagement for machine screws Fig. 10. Our experiment assumes the use of standard screw and assumes the screw is already in final position. We measure the difficulty involved in engaging the tool to the screw. As we can see in (Fig. 11), the difficulty is high when the screw is close to the obstacle and it decreases as the screw moves far

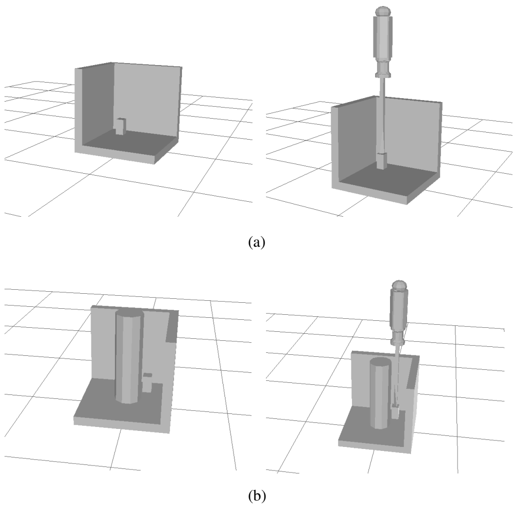

Figure 6: Difficulty values on test models: (a) $D = 0.233995$**, (b)** $D = 0.240506$

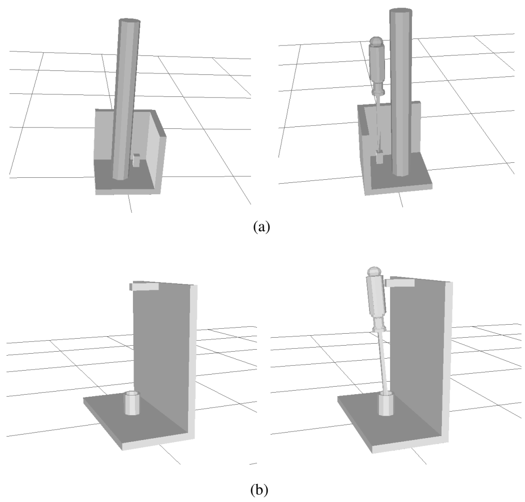

Figure 7: Difficulty values on test models: (a) $D = 0.287657$**, (b)** $D = 0.126087$

away from the obstacle. We can see some correlation between the curve given in Fig. 10 and Fig. 11. This suggests that the measures our method is giving indeed capture the geometric constraints that affect assembly tasks and map them to a difficulty index that numerically represents those geometric constraints.

5 Conclusions

We have presented a measure of the difficulty of assembly tasks focusing on the difficulty arising from spatial constraints. The average information content of a set of p-operations which is used to measure the difficulty of the related task is a good estimate since it captures the geometry of the assembly and translates it into information that exhibits some correlation with empirical observations of the executions of the task.

While this measure promises to be a good estimator, work needs to be done to find a good correlation to data drawn from observation to prove its effectiveness, to extend its applicability to a wider range of tools. Also, research must be done to discover new ways of measuring different attributes that affect the assembly process of a product.

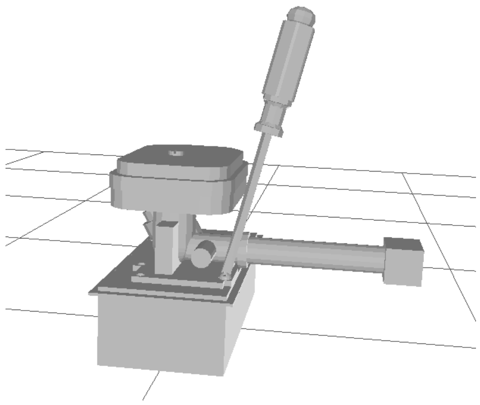

Figure 8: Carburetor assembly. $D = 0.271727$

References

[1] G. Boothroyd, *Design for assembly handbook.* Department of Mechanical Engineering University of Massachusetts, Amherst, MA, 1983.

[2] P. M. Fitts, "The information capacity of the human motor system in controlling the amplitude of movement," *Journal of Experimental Psychology*, vol. 47, pp. 381–391, June 1954.

[3] R. H. Sturges, "A quantification of manual dexterity:the design for an assembly calculator," *Robotics & Computer-Integrated Manufacturing*, vol. 6, no. 3, pp. 237–252, 1989.

[4] A. C. Sanderson, "Parts entropy methods for robotic assembly system design," in *Proceedings of IEEE International Conference on Robotics*, (Atlanta, GA), pp. 600–608, IEEE, March 1984.

[5] S. W. Sedas, "Disassembly expert," Master's thesis, Department of Electrical and Computer Engineering, Carnegie Mellon University, December 1986.

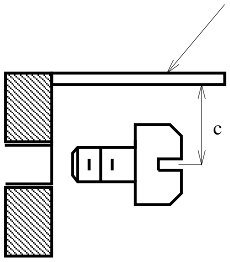

Figure 9: Example used to measure task difficulty as a function of restricted accessibility [1].

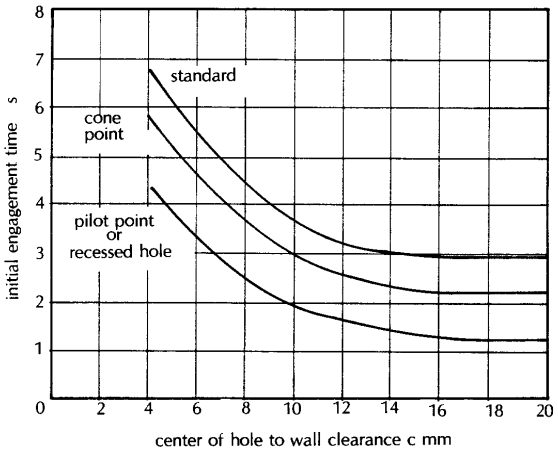

Figure 10: Results presented by Boothroyd [1].

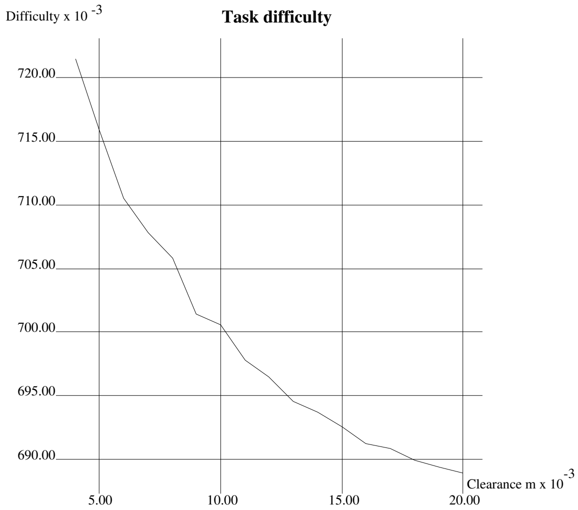

Figure 11: Results given by or method.

[6] R. S. Mattikalli and P. K. Khosla, "Determining the assembly sequence from a 3d model," Tech. Rep. EDRC 24-16-89, Engineering Design Research Center, Carnegie Mellon University, Pittsburgh, PA, 1989.

[7] S. Lee and Shin. Y. G., "Assembly planning based on geometric reasoning," *Computers & Graphics*, vol. 14, no. 2, pp. 237–250, 1990.

[8] J. M. Miller and R. L. Hoffman, "Automatic assembly planning with fasteners," in *Proceedings of IEEE International Conference on robotics and Automation*, (Scottsdale, AZ), pp. 69–74, IEEE, May 1989.

[9] M. Santochi and G. Dini, "Computer-aided planning of assembly operations: The selection of assembly sequences," *Robotics & Computer-Integrated Manufacturing*, vol. 9, no. 6, pp. 439–446, 1992.

[10] W. E. Woodson, *Human factors design handbook: Information and guidelines for the design of systems, facilities, equipment, and products for human use.* New York: McGraw-Hill, 1981.

[11] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, July 1948.