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Load sensitive topology control: Towards minimum energy consumption in dense ad hoc sensor networks ☆

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Abstract

Sensor networks are usually composed of tiny and resource constraint devices, which make energy conservation a vital concern of their design and deployment. Reducing energy consumption has been addressed through different aspects till now. Topology Control (TC) is a well-known approach which tries to determine transmission ranges of nodes to optimize their energy utilization while keeping some network properties like connectivity. However, in current TC schemes, the transmission range of each node is mostly accounted as the exclusive estimator for its energy consumption while ignoring the amount of data it sends or relays. In this paper, we deliberately reformulate the problem of topology control, regarding both network load and transmission range parameters. Our approach is particularly formulated for dense sensor networks with one or more base stations. The problem is considered in three different environmental conditions and then, proper mathematical relations are presented to find the optimum solutions. Finally, we show the advantages of our proposal through experiments.

Keywords: Wireless sensor networks; Energy conservation; Topology control; Transmission range; Traffic load

1. Introduction

Ad hoc sensor networks, which are composed of tiny and resource constrained wireless devices, have been widely deployed for monitoring and controlling applications in various physical environments. Communication among such devices is typically established through wireless channels in absence of any fixed predetermined infrastructure. Usually, sensors are powered by limited batteries, whose

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charging is hardly possible due to difficulties imposed by the operating environment. Therefore, energy conservation is one of the main challenging problems for ad hoc and especially sensor networks. The limited power supply, each sensor contains initially, dissipates gradually through computation and communication tasks. As the required power for transmission/reception is usually an order of magnitude higher than what needed for computation, most proposals have focused on degrading this essential power [1].

Thus far, different techniques have been suggested to address the energy conservation problem, ranging from efficient hardware design [2], to efficient placing of communicating codes in the network [3,4]. One of the most well-known approaches to this problem is based on constructing a proper network topology by which energy consumption becomes optimum. Adjusting transmission ranges (TR) of sensors, also called Topology Control (TC), is a classic way to construct an energy efficient topology while preserving some important properties of the resulting network, like connectivity [5]. The main intuition behind such an approach is that the rate of energy consumption in each sensor is highly related to its transmission range.

Yet, we believe there is a shortcoming in the definition of TC problem that negatively affects all existing proposals. Factually, in this problem the optimization goal is solely based on reducing transmission ranges of nodes. Nevertheless, transmission range together with traffic load on a device will determine its energy consumption rate. That is, there may be a node with a very large TR that forwards only a small fraction of the network's load, and consequently consumes much less energy than another node with a smaller TR forwarding more packets per time.

In this paper, we try to consider the above deficiency. More precisely, we formulate a new problem, called *Load Sensitive Topology Control* (*LSTC*), for energy conservation in relatively dense sensor networks. We model the network as an n-dimensional region ($n = 1$, 2, or 3) where sensors are uniformly deployed over the region. Then, the LSTC problem is considered particularly in this model, where minimizing the required energy for delivering data from specific source regions to data collectors, also called Base Stations (BS), is desired. After that, we present analytical solutions to LSTC under three different environmental constraints. Finally, the performance of the proposed methods is evaluated experimentally by simulation. The results indicate that our methods significantly prolong the network lifetime, as compared with traditional TC schemes.

The remainder of this paper is organized as follow: In the next section, we survey the previous works on topology control in wireless ad hoc and sensor networks. Section 3 presents our motivation and also detailed description of the Load Sensitive Topology Control problem. Sections 4 and 5 are devoted to the explanation of our solutions, analytical discussions, and also protocol design considerations. In Section 6, we demonstrate the experimental results and finally, we conclude the paper in Section 7.

2. Related work

Plenty of research activities have been devoted to Topology Control (TC) in wireless ad hoc and sensor networks, as such networks became an important subject of computer communication field. Most of these works have aimed to construct a network that consumes as low energy as possible [6], while TC turned out to be also helpful to obtain other goals, like finding a topology suitable for QoS objectives [7], or augmenting network throughput [8].

The noticeable initial works on topology control are [7,8]. The mutual effects of transmission range and throughput have been studied in [8]. The authors have proposed a model in which nodes should adjust their transmission power to degrade interference which in turn provides higher throughput. On the other hand, Hu suggested a distributed algorithm in [7], where nodes adjust their transmission power independently to gain a high-throughput network. In none of the mentioned works, energy conservation was a concern.

Later works mainly concentrated on minimizing energy consumption through construction and maintenance of a network with some predetermined properties, like connectivity, bi-connectivity, or strong connectivity, such that it utilizes as low energy as possible. A useful summary of these works has been presented in [5]. The authors introduced a 3-tuple $\langle M, P, O\rangle$ to characterize topology control problems, where "M" represents the graph model (either directed or undirected), "P" represents the desired graph property (e.g., 1-connected or 2-connected), and "O" denotes the optimization objective (e.g., minimizing maximum power or total power). They also analyzed the tractability of these problems, and then suggested several practical heuristic methods.

In [9], the authors introduced two centralized algorithms to obtain optimal energy consumption while maintaining connectivity/bi-connectivity. Moreover, they proposed two heuristic methods, called *LINT* (*Local Information No Topology*) and *LILT* (*Local Information Link-state Topology*) to dynamically adjust transmission ranges to keep topology connected, in the presence of environmental changes. Unfortunately, none of the proposed schemes could guarantee connectivity. Based on the well-known *Minimum Spanning Tree* (*MST*) method, Li et al. [10] proposed a localized TC algorithm that could achieve connectivity while minimizing the total energy consumption of nodes. A cone-based distributed TC method was developed in [11], where each node gradually increments its transmission power till at least one neighbor node is found in each α angle (cone). Consequently, the global connectivity is assured with minimum power requirement for each node.

There are numerous more works on energy efficient communication in wireless ad hoc and sensor networks; such as [12,13]. Five important metrics for energy efficient routing has been studied in [13], such as energy consumed per packet, variance in node power levels, cost per packet, and so on. In sensor networks, the problem can be specially stated in a different form that is how to build a broadcast/multicast tree that conserves energy well. This problem has been studied in [14], where authors tried to adjust nodes' power, such that the total energy cost of a broadcast/multicast tree becomes optimized. Heuristics were proposed to address this issue, namely *Broadcast Incremental Power* (*BIP*), *Multicast Incremental Power* (*MIP*), *Minimum Spanning Tree* (*MST*), and *Shortest Path Tree* (*SPT*) algorithms, which were evaluated through simulations. Later, Wanm et al. [15] presented a quantitative analysis to evaluate the performance of these heuristics.

Other practical approaches are *COMPOW* [16] and *CLUSTERPOW* [17], which operate as network-layer protocol. Both rely on the idea that if each node uses the smallest common power required to maintain connectivity, then the capacity of the entire network with respect to carrying traffic is maximized, the battery life is extended, and the MAC-level contention is mitigated. The major drawback of these approaches is their significant message overhead, since each node has to run multiple daemons, each of which has to exchange link-state information with its counterpart at other nodes.

One fundamental shortcoming appeared in all of these proposals is that they try to minimize energy utilization of nodes only through reducing their transmission ranges without taking into consideration the amount of data they send or relay. However, as we will show later, energy utilization in each wireless device is significantly affected by the volume of traffic it forwards. As an evidence, one common negative phenomenon in the process of data forwarding in multihop sensor networks is the appearance of highly-loaded and early-depleted areas around BSs [18,19]. This deficiency is caused, since almost all messages should pass over this small area to reach the BS. Thus far, some techniques have been proposed in the literature to address this problem, from which is the use of mobile BS to change the hot spot area during time [18]. In brief, the main contribution of this paper is to address this deficiency by assigning various ranges to different nodes of the network.

3. Preliminaries, motivation, and problem statement

3.1. Basics of wireless communication

Communication's energy consumption is caused by either transmitting or receiving data. Here, we use the popular modeling also used in [6]. In this model, the required energy for transmitting a bit-stream at rate r over the Euclidian distance of d is assessed by the following formula:

$$E_t(r,d) = r(\alpha_1 + \alpha_2 d^n), \tag{1}$$

in which α_1 is the distance-independent term (i.e., the energy consumed in the transmitter circuit) and α_2 captures the distance-dependent one. Moreover, n is a real value which is usually $2 \leqslant n \leqslant 4$ for the free-space and short-to-medium range communications. Likewise, the amount of energy used to receive a bit-stream again at rate r can be calculated as

$$E_r(r) = r\beta. \tag{2}$$

Therefore, when a node forwards data by rate r over the distance d, it consumes $E(r,d)$ units of energy per time where $E(r,d)$ is:

$$E(r,d) = r(\gamma + \alpha d^n), \tag{3}$$

in which $\gamma = \alpha_1 + \beta$ and $\alpha = \alpha_2$.

3.2. Motivation

The above formula evidently shows that the rate of energy consumption in each wireless node not only depends on its Transmission Range (TR) but also on the amount of data that it sends. Clearly, nodes that only forward a limited number of packets can have a longer TR and, at the same time, consume even less amount of energy than other highly-loaded nodes.

In this paper, we consider both traffic load and transmission range parameters to minimize the maximum energy consumption among wireless sensors to maximize the life-time of the entire network. In this context, the lifetime is the time elapses from the network startup time till the first sensor of the network stops working. Our solution to this problem is primarily based on the observation that in real sensor networks, the nodes near to the BSs usually bear high loads [18]. As described before, the reason is that usually in hop-by-hop communication, all of the messages should pass from the farther nodes to the nearer nodes around the BSs. Therefore, it makes sense to use variable transmission range idea to assign lower ranges to such close sensors, and higher ranges to those placed far from the BSs.

A simple clarifying example is shown in Fig. 1. As the left figure depicts, if all three sensor nodes A, B and C choose a relatively equal and small value for their transmission ranges, then node A which is nearer to the BS will be obligated to receive and transmit a huge volume of traffic generated by two farther nodes B and C. Unfortunately, this overloading of node A quickens its energy expiration and so breaks the network connectivity rapidly. On the other hand, when B and C send their packets directly to the BS, although all nodes will bear a rather even traffic load but due to the selection of long transmission ranges, their energy will drain much earlier than node A. Indeed, as this example demonstrates, the TC process makes a trade-off between the load and the transmission power requirements of sensors in the network. Namely, as much as the ranges of sensors increase, even though the traffic load will be balanced more uniformly over the network, the increase in energy consumption of nodes (arising from the selection of higher ranges) may significantly influence the virtues of such a load balancing approach.

Notice that while managing the topology in this manner (i.e., by adjusting the ranges of nodes) can result in longer network lifetime, it may increase the total energy consumption. Yet, most of the practical works try to increase the lifetime of the network rather than its total energy consumption. Surely, when the network stops working, the amount of energy remained in the still alive sensors is not of importance. Therefore, our algorithms try to maximize the lifetime through minimizing the maximum rate of energy consumption over the entire network.

3.3. Network model and problem statement

We assume a relatively dense sensor network that harvests data from the area it covers. It consists of a set C of sensors (or nodes) and a set B of base stations that collect data from all nodes. Here, it is conceivable to imagine the sensor region as an n-dimensional field F, i.e., $F \subseteq R^n$, $n = 1, 2, 3$. We assume that sensors are uniformly distributed over the region F with density δ. The places of data sources are formally defined as a subset of F (i.e., $S \subseteq F$), sending data to the BS nodes with a constant message rate λ. All sensors of the field are responsible to forward messages from source nodes to BSs. Each message contains the id and the geographical position of its destination, which is an element of B, towards which all intermediate nodes forward it. The forwarding procedure continues until the message enters the BS hole which is defined

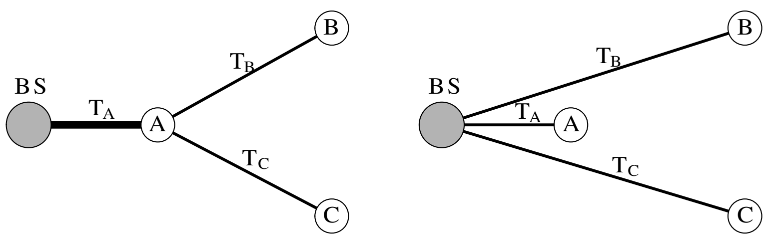


Fig. 1. A simple motivating example. The bold line indicates the highly-loaded link.

as a circular region centered at the destination BS with a tiny radius ε.

The transmission and sensing ranges of a sensor node s are identical and also configurable based on a transmission range function t. Factually, the transmission range function is defined as a function $t : F \rightarrow R$, that assigns a real number to each point of the field, indicating the value of transmission range for each sensor probably settled on this point. A node can directly communicate with any other node within a distance of its transmission range.

We suppose that there is a link between any two nodes which can communicate directly. It is clear that the choice of the range function t causes a load function $r : F \rightarrow R$ that tells the amount of data passing though specific points of the field per time unit. According to the functions t and r, the rate of energy consumption for a node at point $s \in F$ will be obtained simply by (3). We also assume that each source sends its message via the shortest path to the BS. Recent observations [20] show that in relatively dense ad hoc networks, the shortest path between any source–destination pair is very close to the line segment connecting the pair together. Hence, our computations are all done based on the assumption that each sensor forwards messages through a rather straight line to the nearest BS. In Section 6, we justify the correctness of the straight forwarding assumption through simulation experiments.

Given a sensor field F, a base station set B, and a source set S whose nodes aim to deliver messages with rate λ to nodes of B, the problem of *Load Sensitive Topology Control* (*LSTC*) can be stated as "how to determine the range function t, so that the maximum rate of energy utilization over all nodes inside F becomes minimized". It is worth noting here that deciding on the function t affects energy consumption both directly by changing the range values of individual nodes, and indirectly by affecting the amount of traffic load on each node of the network. This obviously makes the problem of finding the optimal range function extremely complicated.

Fig. 2 demonstrates a sample field F in which sensors of the subspace $S = (S_1 \cup S_2 \cup S_3) \subseteq F$ plays the role of data sources that transmit data to the set of BSs, $B = \{BS_1, BS_2, BS_3\}$. Each point of the area S submits its data to the nearest BS though the shortest path. Consequently, all points of S_2 transmit data to BS_2, but S_1 and S_3 are partitioned according to the BSs their points submit data (e.g., the left and right parts of S_1 submit to BS_1 and BS_2,

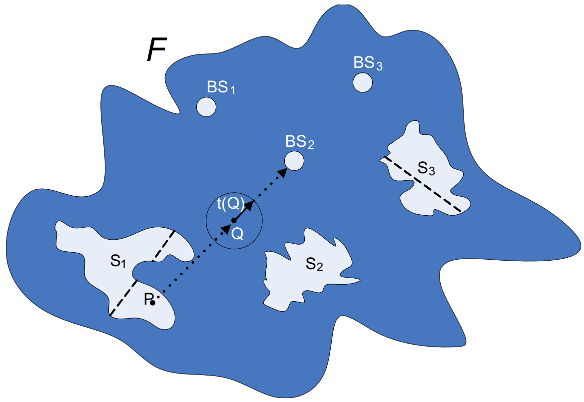


Fig. 2. A sample sensor network field.

respectively). To better understand this issue, consider a node at point P in Fig. 2 which transmits data to BS_2. The intermediate node Q forwards the message of P towards BS_2, using a transmission range of $t(Q)$ units.

In this paper, we concentrate on a sub-problem of what we have explained so far, which better reflects the characteristics of real-world sensor environments. In the confined model, F is a subset of R^2, S is equal to F, and finally, there exists a finite number of BSs in B. A more constrained version of this problem where the network contains only one BS is analytically addressed in the next section. Later, we will show how our methods can be extended to be applied on multi base station sensor networks.

4. Theoretical solutions

In order to determine the range function t to enhance network lifetime as much as possible, the field F is initially partitioned into very thin sectors, like A in the network field of Fig. 3. Using divide

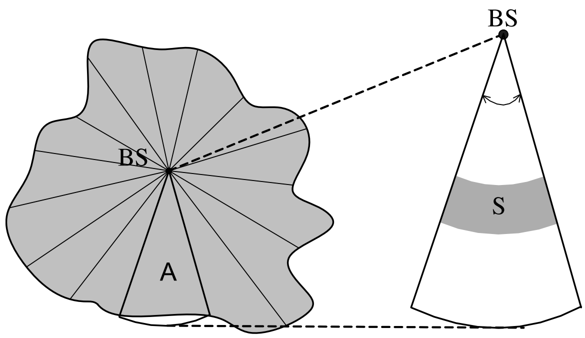


Fig. 3. A sector of the field to simplify computation.

and conquer approach, we first try to find out the best function t for such sectors of the field, and then combine the results to achieve an efficient unified solution for the whole network. The intuition behind such an approach is that because of the straightness property of routes in dense networks, the influence of every sector on traffic loads of its neighbor sectors will be negligible.

Due to the uniformity of both node distribution and traffic demands in our model, we can easily conclude that the network and traffic model inside each sector are symmetric in the sense that all nodes of the same distance from the center of the sector (i.e., the BS) are similar. In other words, the transmission range (TR) and also the amount of load going through all nodes, which are of a fixed distance r from the center, is the same. Conclusively, our reduced problem can be stated as "how to choose a transmission range function $t(x)$, in the sector field A, that defines the transmission range of each sensor according to its distance x from the BS with the goal that the maximum rate of energy utilization becomes optimized".

The remaining of this section is devoted to address the reduced problem under various assumptions, each motivated from different environmental situations. First, we consider the problem of finding a single optimal TR for all sensors of the sector field. Then, a similar problem has been addressed in a more generalized environment where sensors may have different range values while each value is selected from a predetermined set. Finally, we address the problem under no such restriction, in which range values are allowed to be any positive real number.

4.1. Single optimal transmission range

The first problem we are going to address here is how to determine a single transmission range for all sensors of a given sector field to minimize the maximum energy utilization inside the sector. Note that this is a special case of the LSTC problem, named *Single-range LSTC* (*S_LSTC*), where t is a constant positive function.

The assumption that all the sensors have a single range t_u partitions the sector area into multiple virtual ribbons, represented by $R_0, R_1, \ldots, R_N$, each with width t_u. As Fig. 4a depicts, sensors inside every ribbon R_i forwards their traffic to the nodes of the ribbon ahead (i.e., R_{i-1}).

Prior to presenting our solution to S_LSTC, we first have to introduce some important definitions and theoretical fundaments. We call a range assignment mapping two adjacent ribbons thoroughly to each other a *Complete Ribbon Transmission* (*CRT*). As shown in Fig. 4b, two ribbons are associated in every CRT, namely a *Source Ribbon* (*SR*) that transmits data, and a *Destination Ribbon* (*DR*) that receives data. In order to compute the traffic load on individual ribbons, we propose the following lemma.

Lemma 1. *In a CRT, if the range function* $t(x)$ *in SR is linear with respect to* x, *i.e.,* $t(x) = px + q$, *and also the traffic load function of SR has the form of* $r_{\mathrm{SR}}(x) = \lambda(a_{\mathrm{SR}} + \frac{b_{\mathrm{SR}}}{x})$, *for some constants* a_{SR} *and* b_{SR}, *the traffic load function caused by SR on points of DR obeys the following rule*:

$$r_{\mathrm{DR}}^{\mathrm{SR}}(x) = \lambda \cdot \left[a_{\mathrm{DR}}^{\mathrm{SR}} + \frac{b_{\mathrm{DR}}^{\mathrm{SR}}}{x} \right], \tag{4}$$

where $a_{\mathrm{DR}}^{\mathrm{SR}} = \frac{a_{\mathrm{SR}}}{(1-p)^2}$, *and* $b_{\mathrm{DR}}^{\mathrm{SR}} = \frac{a_{\mathrm{SR}} \cdot q}{(1-p)^2} + \frac{b_{\mathrm{SR}}}{1-p}$.

Proof. Consider a very narrow arc-ribbon area inside DR whose boundaries are x_1 and x_2 and its supplier is located inside SR with y_1 and y_2 as its boundaries, as illustrated in Fig. 4b. Obviously, the following equality becomes true when $x_1 = x$, $x_2 = x + \Delta x$, and Δx tends to zero (or equivalently, x_2 tends to x_1):

$$r_{\mathrm{DR}}^{\mathrm{SR}}(x) = \lim_{x_2 \to x_1} \left[\frac{\delta \int_{\circ}^{\alpha} \int_{y_1}^{y_2} r_{\mathrm{SR}}(x) \cdot x \cdot \mathrm{d}x \cdot \mathrm{d}\theta}{\delta \int_{\circ}^{\alpha} \int_{x_1}^{x_2} x \cdot \mathrm{d}x \cdot \mathrm{d}\theta} \right]. \tag{5}$$

By replacing r_{SR} with the function assumed in the lemma, and then simplifying the above formula, we get:

$$\begin{aligned} r_{\mathrm{DR}}^{\mathrm{SR}}(x) &= \lambda \cdot \lim_{x_2 \to x_1} \left[\frac{a_{\mathrm{SR}} \int_{y_1}^{y_2} x \, \mathrm{d}x + b_{\mathrm{SR}} \int_{y_1}^{y_2} \mathrm{d}x}{\int_{x_1}^{x_2} x \, \mathrm{d}x} \right] \\ &= \lambda \cdot \lim_{x_2 \to x_1} \left[\frac{\frac{a_{\mathrm{SR}}}{2} \left(y_2^2 - y_1^2 \right) + b_{\mathrm{SR}} (y_2 - y_1)}{\frac{1}{2} (x_2^2 - x_1^2)} \right]. \end{aligned} \tag{6}$$

On the other hand, from linearity assumption of t_{SR}, we can conclude the following relation:

$$y_i - t_{\mathrm{SR}}(y_i) = y_i - (py_i + q) = x_i \Rightarrow y_i = \frac{x_i + q}{1 - p}, \quad i \in \{1, 2\}. \tag{7}$$

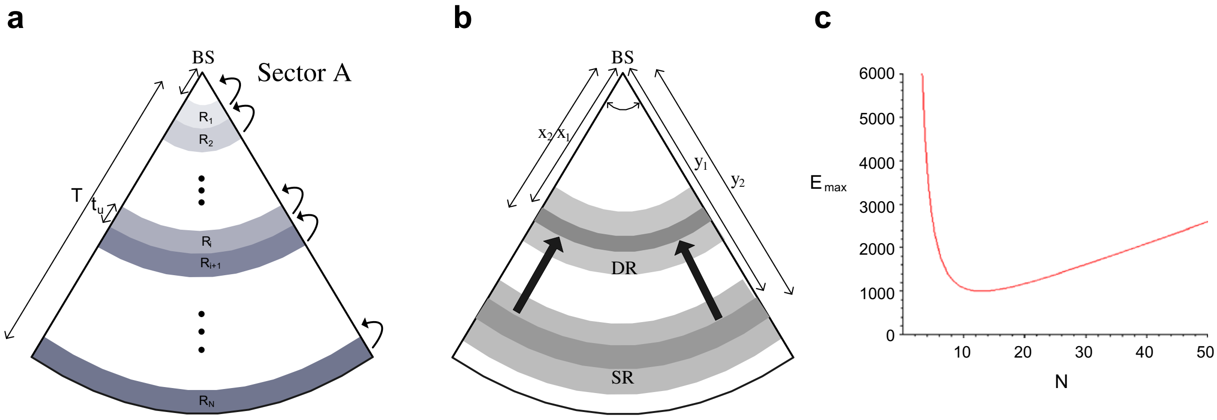


Fig. 4. A solution to Single-range LSTC.

By replacing y_i with the one calculated in (7) and also considering $x = x_1 = x_2$ as $\Delta x \to 0$, we can rewrite (6) as

$$r^{\mathrm{SR}}_{\mathrm{DR}}(x) = \lambda \cdot \lim_{x_2 \to x_1} \left[\frac{\frac{a_{\mathrm{SR}}}{2(1-p)^2}\left((x_2+q)^2 - (x_1+q)^2\right) + \frac{b_{\mathrm{SR}}}{1-p}\left((x_2+q)-(x_1+q)\right)}{\frac{1}{2}\left(x_2^2 - x_1^2\right)} \right]$$

$$= \lambda \cdot \lim_{x_2 \to x_1} \left[\frac{\frac{a_{\mathrm{SR}}}{2(1-p)^2}(x_2+x_1+2q)(x_2-x_1) + \frac{b_{\mathrm{SR}}}{1-p}(x_2-x_1)}{\frac{1}{2}(x_2-x_1)(x_2+x_1)} \right] = \lambda \cdot \left[\frac{\frac{a_{\mathrm{SR}}}{2(1-p)^2}(2x+2q) + \frac{b_{\mathrm{SR}}}{1-p}}{\frac{1}{2}(2x)} \right].$$

Furthermore, if we simplify the above equation, the following relation is obtained which is actually equivalent to (4) and hence, the correctness of the lemma is proved, i.e.

$$r^{\mathrm{SR}}_{\mathrm{DR}}(x) = \lambda \cdot \left[\frac{\frac{a_{\mathrm{SR}}}{(1-p)^2}(x+q) + \frac{b_{\mathrm{SR}}}{1-p}}{x} \right]$$

$$= \lambda \cdot \left[\frac{a_{\mathrm{SR}}}{(1-p)^2} + \frac{\frac{a_{\mathrm{SR}} \cdot q}{(1-p)^2} + \frac{b_{\mathrm{SR}}}{1-p}}{x} \right]. \qquad \square$$

Using Lemma 1, we now present the following corollary which helps us to easily compute the traffic load on every points of a ribbon inside the sector field.

Corollary 1. *In a sector area containing N ribbons (as shown in Fig. 4a) where all sensors have an equal TR value, i.e., $t(x) = t_u$, the traffic load function of a point inside an arbitrary ribbon R_i (with distance x from the BS) is computed by*

$$r_i(x) = \lambda \cdot \left[a_i + \frac{b_i}{x} \right]. \tag{8}$$

where $a_i = N + 1 - i$, *and* $b_i = t_u \cdot \sum_{j=1}^{N-i} j = t_u \cdot \frac{(N-i)(N-i+1)}{2}$.

Proof. To prove this proposition, we use a reverse induction. For the last ribbon (i.e., R_N), we have that $r_N(x) = \lambda$. Hence, it apparently obeys (8), because $a_N = 1$, and $b_N = 0$. Now, suppose the proposition is correct for all $i > k$. The traffic load function of any point inside the ribbon R_k is composed of its own traffic load (which is λ), plus the traffic load that the farther ribbon R_{k+1} forwards to it, meaning that

$$r_k(x) = \lambda + \lambda \left[N + 1 - (k+1) + \frac{(N+1-(k+1))t_u + t_u \sum_{j=1}^{N-(k+1)} j}{x} \right] = \lambda \left[N + 1 - k + \frac{t_u \sum_{j=1}^{N-k} j}{x} \right].$$

$r_k(x) = \lambda + r_k^{k+1}(x)$. Note that $r_k^{k+1}(x)$ is the amount of traffic load incurred by sensors of R_{k+1} on nodes inside R_k which are settled at distance of x from the BS. On the other hand, according to Lemma 1, we have that $r_k^{k+1}(x) = \lambda[a_{k+1} + \frac{a_{k+1}t_u + b_{k+1}}{x}]$. Using the induction hypothesis, a_{k+1} and b_{k+1} are replaced with their proper values, i.e., $a_{k+1} = N + 1 - (k+1)$, and $b_{k+1} = \sum_{j=1}^{N-(k+1)} j$. Hence:As an immediate result, we can derive that $a_i = N + 1 - i$, and $b_i = t_u \sum_{j=1}^{N-i} j = t_u \frac{(N-i)(N-i+1)}{2}$, and thus, the proof is completed. □

Through analyzing the load function $r_i(x)$ and by considering the fact that all sensors have a common range value t_u, we can conclude that the most energy consuming point inside the sector is the most highly-loaded one, which is the foremost part of the ribbon R_1. Thus, hereafter, we are going to minimize the value of $E_{\max}$, which is consumed by nodes laid at distance of from the BS. The exact value of $E_{\max}$ can be computed according to corollary 1 and the relation stated in (3), i.e.,

$$\begin{aligned} E_{\max} &= r_1(\varepsilon)(\gamma + \alpha t_u^n) \\ &= \lambda\left(N + \left(\frac{N(N-1)}{2}\right)\frac{t_u}{\varepsilon}\right)(\gamma + \alpha t_u^n). \end{aligned}$$

Now, suppose that the considered sector A is divided into N exact ribbons each with width of t_u, i.e., $T = N \cdot t_u$ where T is the sector length, as depicted in Fig. 4a. In the general case where T is not necessarily an integer multiplier of t_u, the last ribbon becomes narrower than t_u. This clearly lowers the load function in last parts of all ribbons. However, $E_{\max}$ which occurs in the foremost area of ribbon one remains unchanged. By replacing t_u with T/N, we obtain:

$$E_{\max} = \lambda \cdot (\eta N - \varphi) \cdot \left(\gamma + \alpha' \left(\frac{1}{N}\right)^n\right), \quad (9)$$

where $\eta = \left(1 + \frac{T}{2\varepsilon}\right)$, $\varphi = \frac{T}{2\varepsilon}$, and $\alpha' = \alpha T^n$.

The analytical results of the maximum energy $E_{\max}$ with respect to N is plotted in Fig. 4c. The results are obtained when $T = 10$, $\varepsilon = 0.2$, $\gamma = \lambda = \alpha = 1$, and $n = 3$. As we can see from the figure, the minimum value of $E_{\max}$ is attained when N is around 12. Interestingly, by taking the load factor into account for computation of $E_{\max}$, the best possible value for t_u is not necessarily the smallest one.

Finding the optimum value of N is feasible through either classic differentiation from the right side of (9) or simple exhaustive search on natural values of N, which should be done with respect to the overall form of $E_{\max}$, as depicted in Fig. 4c. Finally, our discussions for computing the optimal value of single TR are concluded by the following theorem:

Theorem 1. *Computing Optimum Single Transmission Range (TR). In a sector region where the BS is settled at the center of the sector and all nodes transmit data with an equal TR, the maximum rate of energy consumption, calculated through (9), occurs at the nearest nodes to the BS where $x = \varepsilon$ (x is the distance of the node from the BS). Since $T = N \cdot t_u$, then finding a proper value of N that minimizes (9) results in the optimal value for t_u which is also the best possible one to optimize $E_{\max}$.*

To compute the best value of t_u for the whole network, which may consist of numerous sectors, different energy functions should be considered collectively. More precisely, having m various sectors with different energy functions $E_{\max}^1, \ldots, E_{\max}^m$, we should find t_u so that the value of $\max_{\forall i, 1 \leqslant i \leqslant m}\{E_{\max}^i\}$ becomes minimized.

4.2. Multiple discrete transmission range

Allowing the TR values to be chosen from a set of natural multipliers of a single unit length, i.e., $t : F \to \{t_u, 2t_u, 3t_u, \ldots\}$ defines a more general variant of S_LSTC; named *Multiple Discrete LSTC* (*MD_LSTC*). The intuition behind such a definition is that usually sensors' transmission devices can tune their TRs only to a discrete set of range values. On the other hand, by selecting t_u as small as possible, $t(x)$ practically becomes as deliberate as required, since more permissible multipliers of t_u will be available as TR value. Thus, we consider $t_u = \varepsilon$ which is the lowest possible transmission range in our model.

The overall modeling in MD_LSTC is very similar to the case of S_LSTC, except that here ribbons do not essentially forward the traffic to their adjacent ribbons. As an example, ribbon R_N in Fig. 5 sends its traffic to R_k instead of R_{N-1}. Because the TR values are all multipliers of the unit value t_u, just as S_LSTC, each ribbon is entirely mapped to another ribbon situated nearer to the BS. Therefore, we can imagine that traffic of every ribbon is forwarded through a unified multihop path while each hop represents a ribbon in the sector; for example in the sector of Fig. 5, $\langle R_N, R_k, R_{i+1}, R_2, R_1 \rangle$ is the unified path for transmitting traffic of R_N to the BS. We call R_m a *Midway Ribbon* (*MR*) for R_s if the unified

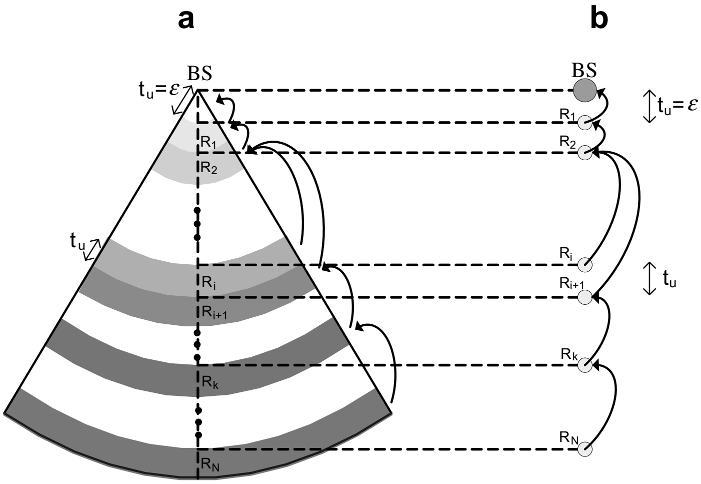


Fig. 5. A solution to multiple discrete LSTC.

path from ribbon R_s to the BS goes over R_m. In this case, R_s is called a *Supplier Ribbon* (*SR*) for R_m; as an instance, R_{i+1} is an *MR* for both R_k, and R_N in sector field of Fig. 5.

Lemma 2. *Given an instance of the MD_LSTC problem with unit length t_u, if R_s is a supplier of R_m (equivalently, R_m is an MR of R_s), whose traffic load function is $r_s(x) = \lambda(a_s + b_s\frac{1}{x})$, then independent from the path chosen between two ribbons, the traffic load function caused by R_s on R_m is computed by*

$$r^{R_s}_{R_m}(x) = r^s_m(x) = \lambda\left(a^s_m + b^s_m\frac{1}{x}\right), \tag{10}$$

where $a^s_m = a_s$, and $b^s_m = a_s(s-m)t_u + b_s$.

Proof. Consider an arbitrary path $\langle R_o, R_1, \ldots, R_m\rangle$, where $R_0 = \mathrm{SR}$, and $R_m = \mathrm{DR}$. Immediately from Lemma 1, we have $a^{R_{m-1}}_{R_m} = \cdots = a^{R_{t-1}}_{R_t} = \cdots = a^{R_1}_{R_0}$. Assume that q_i is the distance between R_i and R_{i+1}, q is the distance between R_0 and R_m (clearly, $q = (s-m)t_u$). Lemma 1 confirms the equation below: $b^{R_0}_{R_m} = a_{\mathrm{SR}}\cdot q_m + (a_{\mathrm{SR}}\cdot q_{m-1} + \cdots + ((a_{\mathrm{SR}}\cdot q_0) + b_{\mathrm{SR}})) = a_{\mathrm{SR}}\cdot\sum_{i=0}^{m} q_i + b_{\mathrm{SR}} = a_{\mathrm{SR}}\cdot q + b_{\mathrm{SR}}$, which obviously prove the correctness of the lemma. □

Corollary 2. *Let S_i be the set of all supplier ribbons of R_i. The total traffic load on the ribbon R_i follows the relation below:*

$$r_i(x) = \lambda\left(1 + \sum_{s\in S_i} a_s + \left(\sum_{s\in S_i}(a_s(s-i)t_u + b_s)\right)\frac{1}{x}\right). \tag{11}$$

Proof. This proposition is immediately deduced from Lemma 2. □

Since $r_i(x)$ is a decreasing functions of x, the maximum rate of energy consumption in each ribbon definitely occurs at its foremost portions. Motivating from this simple fact, we map MD_LSTC to a new equivalent problem, called *Linear LSTC* (*L_LSTC*), in which ribbons are represented by distinct nodes all laid on a straight line passing through the BS. In this case, each consecutive couple of nodes has the same distance of t_u units from each other, and especially the first node is t_u units far from the BS. In addition, the ith node has itself a traffic load with rate L^i_{int} which is linearly related to i, meaning that $L^i_{\mathrm{int}} = \lambda\times i$ where λ is a constant value. Also, different nodes utilize energy by dissimilar coefficients which are actually $1/i$ for the ith node. These coefficients are obtained by setting $b_s = 0$, $a_s = \lambda$, and $x = i\cdot t_u$ in (11). The reason for assigning different rates of energy consumption for nodes is that the farther nodes to the BS represent wider ribbons, which in turn contains more sensors due to uniform distribution of nodes. In such situations, the goal of the L_LSTC problem is to determine the next hop of each node, so that (1) all nodes become connected to the BS through either direct or indirect unified paths, and (2) the maximum rate of energy consumption among all nodes is optimized. Fig. 5 demonstrates an instance of MD_LSTC, and its correspondent L_LSTC. □

Lemma 3. *An optimum hop assignment for L_LSTC is exactly equivalent to the optimum range assignment for MD_LSTC.*

Proof. Since the amount of energy consumption in each node in L_LSTC equals the one utilized in the foremost points of the corresponding ribbon in MD_LSTC, the correctness of the lemma is clear. □

Fortunately, Lemma 3 allows us to deduce an optimal range assignment for MD_LSTC by presenting an equivalent L_LSTC solution. Therefore, we hereafter argue about the L_LSTC problem. Although the following theorem shows that proving the **NP**-hardness of L_LSTC is unlikely, but our investigation suggests that the optimal polynomial-time solution (if there is any) is yet very hard to obtain. Thus, we focus here on devising a fast and efficient heuristic algorithm to approximate L_LSTC.

Theorem 2. *L_LSTC for a fixed λ and unary encoding for N is not* ***NP****-Hard, unless* $P = \mathbf{NP}$.

Proof. The proof is presented in the appendix section. □

After discussing the tractability of L_LSTC, we propose a heuristic algorithm, named *approx_ L_LSTC*, to approximate it. The pseudo-code of this recursive algorithm is given below. In each step, approx_L_LSTC divides the sensors (from 1st to Mth node, where the first node is the nearest node to the BS) into three segments: (1) the lower half (LH), containing nodes from 1 to k, (2) the intermediate nodes (IN), containing $(k+1)$th and $(k+2)$th nodes, and (3) the upper half (UH), containing nodes from $k+3$ to M. In line 3, nodes of IN are set to forward their traffic directly to the BS. Line 4 sets TR values of nodes in UH so that their next hops are selected one by one from the lower half, respectively. Finally, line 5 is a recursive call to assign TR values of nodes inside the LH, in the next step. Now, we try to obtain an approximation factor for the heuristic approx_L_LSTC. For ease of understanding, here we present our analytical discussions for $n = 2$. For other values of n, similar computations can be carried out to find out the approximation factor.

approx_L_LSTC (M: number of nodes):

1 **if** $M \leqslant 1$ **then set** *Transmission Range of* Mth *node to* t_u;
2 $k = \lceil M/2 \rceil - 1$;
3 **set** *Transmission Range of* $(k+1)$th *and* $(k+2)$th *to* $(k+1).t_u$ *and* $(k+2).t_u$;
4 **set** *Transmission Range of nodes from* $(k+2)$th *to* Mth *node to* $(k+2).t_u$;
5 **call** *approx_L_LSTC (k)*;
end *approx_L_LSTC*;

Lemma 4. *Given an instance of L_LSTC* (*i.e.*, *N nodes settled on a straight line with a fixed step size* t_u *between every pair of consecutive nodes*), *where node i has an initial load* $a + (i-1)b$. *The approx_L_LSTC algorithm makes unified paths* (*from all nodes to the BS*) *in a way that the maximum required energy in segment* $UH \cup IN$ *in step* $t+1$ *is at most* $\frac{k+2}{k+1}$ *times larger than what is necessary in segment* $UH \cup IN$ *in step t.*

Proof. Since all nodes laid on the upper half (UH) segment have the same transmission ranges, i.e., $(k+2)t_u$, then the rate of energy consumption for the ith node (for $i = k+2, \ldots, N$) is computed through:

$$E_i = \frac{a + b(i-1)}{i}(k+2)^2 t_u^2.$$

From another perspective, since $\frac{a+b(i-1)}{i}$ is decreasing for natural values of i, then the $(k+2)$th node consumes more energy than other nodes in UH. The exact amount of this energy is

$$E_{k+2} = \frac{a + b(k+1)}{k+2}(k+2)^2 t_u^2. \tag{12}$$

To complete the proof, it is sufficient to show that nodes of the lower half (LH) segment do not consume energy with higher rates than $\frac{k+2}{k+1}$ times of E_{k+2}. By similar reasoning, we declare that node $k'+2$ (where $k = 2k'+1$ or $k = 2k'+2$) has the highest rate of energy consumption among nodes from 1 to k. We consider two cases here:

Case 1. If k is even (i.e., $k = 2k'+1$), we have:

$$\begin{aligned} E_{k'+2} &= \frac{(a + b(k'+1)) + (a + (k'+k+3)b)}{k'+2} \\ &\quad \times (k'+2)^2 t_u^2 \\ &= \frac{(2a + b(4k'+5))}{k'+2}(k'+2)^2 t_u^2 \\ &= (2a + b(4k'+5))(k'+2)t_u^2. \end{aligned}$$

On the other hand, (12) can be rewritten as

$$E_{k+2} = \frac{a+b(k+1)}{k+2}(k+2)^2 t_u^2 = \frac{a+b(2k'+2)}{2k'+3}(2k'+3)^2 t_u^2 = (a+b(2k'+2))(2k'+3)t_u^2.$$

Thus

$$Ratio = \frac{E_{k'+2}}{E_{k+2}} = \frac{(2a+b(4k'+5))(k'+2)}{(a+b(2k'+2))(2k'+3)} \leqslant \frac{(2a+b(4k'+5))(k'+2)}{(2a+b(4k'+4))(k'+3/2)} = \frac{(2a+b(4k'+5))(k'+2)}{(2a+b(4k'+4))(k'+1)+(2a+b(4k'+4))\frac{1}{2}} \leqslant \frac{(2a+b(4k'+5))(k'+2)}{(2a+b(4k'+4))(k'+1)+b(k'+1)} = \frac{(2a+b(4k'+5))(k'+2)}{(2a+b(4k'+5))(k'+1)} = \frac{(k'+2)}{(k'+1).}$$

Case 2. If k is odd (i.e., $k = 2k'+2$), then:

$$E_{k'+2} = \frac{(a+b(k'+1))+(a+(k'+k+3)b)}{k'+2} \times (k'+2)^2 t_u^2 = \frac{(2a+b(4k'+6))}{k'+2}(k'+2)^2 t_u^2. \quad (13)$$

On the other hand, (12) can be rewritten as

$$E_{k+2} = \frac{a+b(k+1)}{k+2}(k+2)^2 t_u^2 = \frac{a+b(2k'+3)}{2k'+3}(2k'+4)^2 t_u^2. \quad (14)$$

Through dividing (13) by (14), we get:

$$Ratio = \frac{E_{k'+2}}{E_{k+2}} = \frac{\frac{(2a+b(4k'+6))}{k'+2}(k'+2)^2}{\frac{a+b(2k'+3)}{2k'+3}(2k'+4)^2} = \frac{(2a+b(4k'+6))(2k'+3)}{4(a+b(2k'+3))(k'+2)} = \frac{(2k'+3)}{(2k'+4)} \leqslant 1 \leqslant \frac{k'+2}{k'+1.}$$

Hence, by addressing both cases, the proof is completed. □

With the help of Lemma 4, we can now calculate an upper bound on energy consumption rates of nodes in the approx_L_LSTC algorithm.

Lemma 5. *Approx_L_LSTC assigns TR values in a way that the energy consumption rate of every node does not exceed* $3\lambda_u \cdot (\lceil \frac{N}{2} \rceil + 1)^2 t_u^2$.

Proof. The amount of energy required in nodes of the upper half (from $\lfloor N/2 \rfloor$th to Nth node) is $E_{\max}^{\text{upper}} = \lambda_u \cdot (\lceil \frac{N}{2} \rceil + 1)^2$, which is determined in the first step of the algorithm and never changes again (as nearer nodes do not transmit their messages backward). Lemma 4 told us that in each recursive call of approx_L_LSTC, the maximum rate of energy consumption is at most multiplied by the factor of $\frac{k+2}{k+1}$. To provide a clear proof, we here assume that $N = 2^i$, otherwise we substitute this value by the first larger power of two and the computations remain unchanged. In this case, in step s of the algorithm, we are sure that no node consume more energy than $\frac{2^{i-s}+2}{2^{i-s}+1}$ times the largest energy consumption occurred in $(s-1)$th step. Thus, after completion of the assignments, no node utilizes more energy than $C \times E_{\max}^{upper}$, where C is

$$C = \frac{2^{i-1}+2}{2^{i-1}+1} \times \frac{2^{i-2}+2}{2^{i-2}+1} \times \cdots \times \frac{2^0+2}{2^0+1} = \frac{2(2^{i-2}+1)}{2^{i-1}+1} \times \frac{2(2^{i-3}+1)}{2^{i-2}+1} \times \cdots \times \frac{2^0+2}{2^0+1} = \frac{2^{i-1}}{2^{i-1}+1} \times 3 < 3.$$

Form the above bound, we immediately conclude:

$$E_{\max} < 3 \times E_{\max}^{\text{upper}} = 3\lambda_u \cdot \left(\left\lceil \frac{N}{2} \right\rceil + 1 \right)^2 t_u^2. \qquad \square$$

To continue the computation of the approximation ratio for approx_L_LSTC, it seems essential to analyze characteristics and limitations of the perfect solution. The following lemma establishes a lower bound on the maximum energy obtained by any range assignment for an instance of L_LSTC.

Lemma 6. *Given an instance of L_LSTC (with N nodes) and a range assignment R, the maximum amount of energy consumption among all nodes is at least* $\frac{N(N+1)}{2H_N}$ *where HN is the sum of the first N elements of the Harmonic series.*

Proof. Let FR be the set of nodes (with size of k) that access the BS directly, i.e., by one hop. Also, assume that nodes of FR have distances of $\alpha_1, \alpha_2, \ldots, \alpha_k$ units from the BS, and their traffic loads are $\lambda_1^*, \lambda_2^*, \ldots, \lambda_k^*$, respectively. Since nodes of FR must forward the whole traffic of the network to the BS, we have:

$$\sum_{i=1}^{k} \lambda_i^* = \sum_{i=1}^{k} i \cdot \lambda_u = \frac{N(N+1)}{2}\lambda_u. \tag{15}$$

The maximum required energy among nodes of FR becomes optimum when all its nodes consume an equal amount of energy. This situation clearly happens once the total load is scattered fairly among the nodes. This condition is formulated as

$$\frac{\lambda_1^*(1)^2 t_u^2}{1} = \frac{\lambda_i^*(\alpha_i)^2 t_u^2}{\alpha_i} \Rightarrow \lambda_i^* = \frac{\lambda_1^*}{\alpha_i} \quad \forall i \in \{1, 2, \ldots, k\}. \tag{16}$$

Now, using (15) and (16), we conclude:

$$\lambda_1^* \sum_{i=1}^{k} \frac{1}{\alpha_i} = \frac{N(N+1)}{2}\lambda_u \Rightarrow \lambda_1^* = \frac{\frac{N(N+1)}{2}\lambda_u}{\sum_{i=1}^{k}\frac{1}{\alpha_i}} > \frac{\frac{N(N+1)}{2}\lambda_u}{\sum_{i=1}^{k}\frac{1}{i}}. \tag{17}$$

Consequently, the following relation is obtained which confirms the correctness of the lemma:

$$\lambda_1^* > \frac{\frac{N(N+1)}{2}\lambda_u}{\sum_{i=1}^{N}\frac{1}{i}} = \frac{N(N+1)\lambda_u}{2H_N} \Rightarrow E_{\max} \geqslant \lambda_1^* \cdot t_u^2 > \frac{N(N+1)\lambda_u}{2H_N} \cdot t_u^2. \qquad \square$$

Now, as a direct result of Lemmas 5 and 6, we present the main theorem of this part below.

Theorem 3. *Approx_L_LSTC is an* $O(\ln(N))$*-approximation algorithm for L_LSTC when n* = 2.

Proof. The proof of the lemma is directly obtained from Lemmas 5 and 6 (note that $H_N \leqslant \ln(N) + 1$). $\square$

4.3. Multiple continuous transmission range

The final part of this section is devoted to solving the LSTC problem under the idealistic assumption that allows the TR value to be any positive real value; this version of the problem is called *Multi Continuous LSTC* (*MC_LSTC*). To solve MC_LSTC, we here suggest an iterative algorithm which improves the estimation of the perfect $t(x)$ in each step. Prior to description of our algorithm, some theoretical fundament should be explained.

First, let us compute the average energy consumption in the arc-shaped area S located in the sector A in Fig. 6. Considering $E(s)$ as the amount of energy consumed by a sensor $s \in S$, the average energy consumption of nodes in S is computed by

$$\overline{E(S)} = \frac{\sum_{s \in N(S)} E(s)}{|N(S)|} \tag{18}$$

in which $N(S)$ is the set of all sensors placed inside the area S. Since sensors have a uniform distribution with density δ, $N(S)$ is equal to $|S| \times \delta$ where $|S|$ is the area of S. To compute (18), we divide the area S into many tiny elements according to Fig. 6, so that each element is determined by $\bar{x}$ (i.e., the distance from the center of the element to the BS), Δx, and $\Delta\theta$. Thus, we obtain:

$$\sum_{s \in N(S)} E(s) = \lim_{\substack{\Delta x \to 0 \\ \Delta\theta \to 0}} \sum_{0 < \theta < \alpha} \sum_{d_1 < x < d_2} E(x) \cdot x \Delta x \Delta\theta \cdot \delta. \tag{19}$$

From Riemann integral theorem, we can rewrite (19) as:

$$\sum_{s \in N(S)} E(s) = \delta \cdot \int_{\circ}^{\alpha} \int_{d_1}^{d_2} E(x) \cdot x \, \mathrm{d}x \, \mathrm{d}\theta. \tag{20}$$

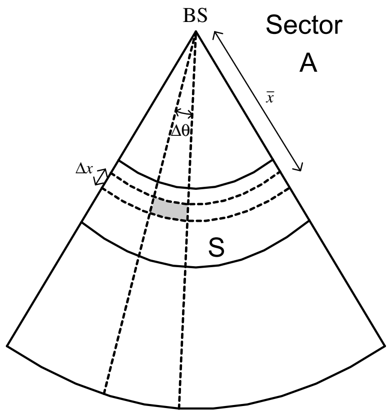


Fig. 6. Element of integration.

Then, from (18) and (20), we have:

$$\overline{E(S)} = \frac{\int_{\circ}^{\alpha} \int_{d_1}^{d_2} E(x) \cdot x\mathrm{d}x\,\mathrm{d}\theta}{|S|}. \tag{21}$$

Through applying (3) while paying attention to the independence of the integral from $d\theta$, we can draw the following formula:

$$\overline{E(S)} = \frac{\alpha \cdot \int_{d_1}^{d_2} r(x) \cdot (\gamma + \alpha t^n(x)) \cdot x\,\mathrm{d}x}{|S|}. \tag{22}$$

If the sector A in Fig. 6 is partitioned into some tiny arc-shaped areas (like S), in a way that the average energy consumptions in all these sub-areas become equal and minimal, then we can guarantee that the energy consumption is well distributed among all sensor nodes.

To calculate the transmission range for all the sensors according to their distances from the BS, initially we start from the case where all nodes transmit their messages directly to the BS, i.e., the transmission range of each sensor equals to its distance to BS (see Fig. 7a). Then, by iteratively dividing the range function $t(x)$ into more line segments in each step, we try to get better approximations of optimal TR. Three iterations of such a process are illustrated below:

Precisely speaking, in each step a segment whose maximum energy consumption is larger than other segments is selected and partitioned into two parts. This partitioning is done in a way that the average energy consumptions of both newly-emerged areas, which can be computed through (22), become equal. Suppose we are to divide the area S_1 in Fig. 8. Equivalently, we should find an appropriate value of c (i.e., the location of the border arc between new sub-areas S_1' and S_1''), such that on average the same amount of energy is consumed in two sub-areas S_1' and S_1''. Note that sensors of each area can relay packets only to the nodes of the adjacent area. Thus, it is very important to keep the range function $t(x)$ continuous, and also to update the load function $r(x)$ correctly at the end of partitioning.

Hence, our problem is now transformed to finding an optimal value for c together with properly

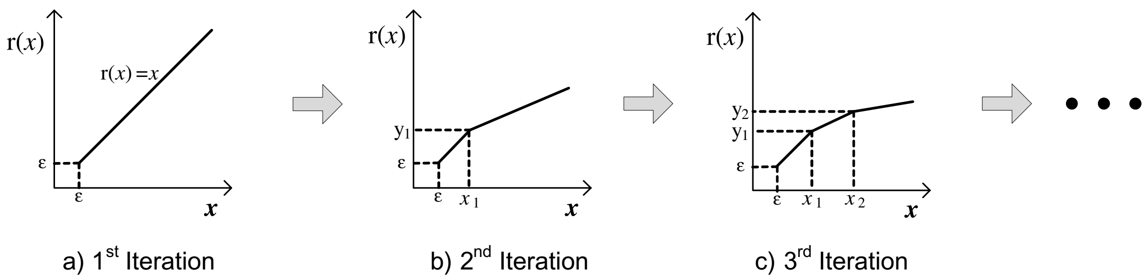


Fig. 7. Three iterations of $t(x)$ estimation process.

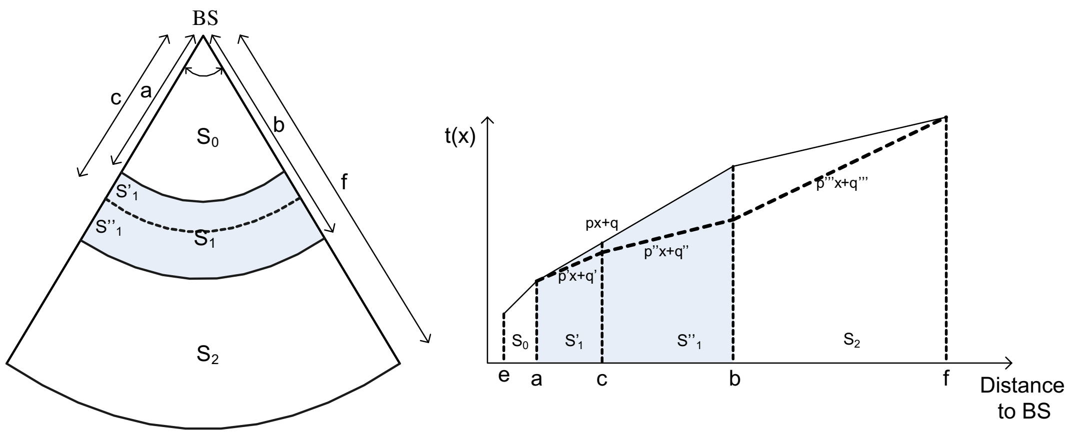


Fig. 8. Refining $t(x)$.

configuring both transmission range and traffic load functions in $[a, c]$ and $[c, b]$ intervals. As depicted in the right diagram of Fig. 8, this configuration consists of determining the line equations $p'x + q'$, $p''x + q''$ and $p'''x + q'''$. Here, there exist some conditions in the problem that can be translated into multiple mathematical equations. Then, we can use them as a system of equations (SE) to find the desired values of parameters $c, p', q', p'', q'', p''', q'''$. First, the following six equations are obtained from the simple fact that each arc-ribbon area should be exactly mapped into its next neighbor:

$$\begin{aligned} &p'' \cdot b + q'' = b - c, \\ &p'' \cdot c + q'' = c - a, \\ &p' \cdot c + q' = c - a, \\ &p' \cdot a + q' = pa + q, \\ &p''' \cdot b + q''' = b - c, \\ &p''' \cdot f + q''' = f - b. \end{aligned} \tag{23}$$

On the other hand, by using (22) and also considering the fact that an equal average energy consumption in the sub-areas S_1' and S_1'' is desired, the seventh equation is drawn as

$$\begin{aligned} &\overline{E(S_1')} = \overline{E(S_1'')} \\ &\quad \Rightarrow \frac{\int_a^c r_{S_1''}(x) \cdot (\gamma + \alpha t_{S_1''}^n(x)) \cdot x \cdot \mathrm{d}x}{(c^2 - a^2)} \\ &\quad = \frac{\int_c^b r_{S_1'}(x) \cdot (\gamma + \alpha t_{S_1'}^n(x)) \cdot x \cdot \mathrm{d}x}{(b^2 - c^2)}. \end{aligned} \tag{24}$$

Although it seems that we have enough equations to solve SE, there exist one step to complete the solution which is the computation of the load functions in new areas S_1' and S_1'' (i.e., $r_{S_1'}(x)$, and $r_{S_1''}(x)$), and also the load function of the neighbor region S_0 (i.e., $r_{S_0}(x)$) that may be affected by the partitioning operation. Notice that $r_{S_2}(x)$ will not be altered, since the supplier areas of S_2 remain unchanged. On the other hand, regarding the linearity of the range function in all segments together with the fact that communications between any two segments are CRT, all load functions $r_{S_1'}(x)$, $r_{S_1''}(x)$, and $r_{S_0}(x)$ can be easily computed according to Lemma 1. More precisely, assuming $r_{S_2}(x) = \lambda \cdot [a_{S_2} + \frac{b_{S_2}}{x}]$, the load functions in all areas S_0, S_1', S_1'' have the common form $r_S(x) = \lambda \cdot [a_S + \frac{b_S}{x}]$ for $S \in \{S_1', S_1', S_0\}$, where:

$$a_{S_1''} = \frac{a_{S_2}}{(1 - p''')^2}, b_{S_1''} = \frac{a_{S_2} \cdot q'''}{(1 - p''')^2} + \frac{b_{S_2}}{1 - p'''},$$

$$a_{S_1'} = \frac{a_{S_1''}}{(1 - p'')^2}, \quad b_{S_1'} = \frac{a_{S_1''} \cdot q''}{(1 - p'')^2} + \frac{b_{S_1''}}{1 - p'''},$$

$$a_{S_0} = \frac{a_{S_1'}}{(1 - p')^2}, \quad \text{and} \quad b_{S_0} = \frac{a_{S_1'} \cdot q'}{(1 - p')^2} + \frac{b_{S_1'}}{1 - p'}.$$

Starting from the line $t(x) = x$, we continue breaking it step by step using the above equations. In each step, it is required to keep track of all segments' range and load functions. Simulations confirm that this iterative scheme gradually degrades the maximum amount of energy consumption in the network.

5. Protocol design issues

Up to here, three methods have been introduced to provide well estimations about the perfect range function $t(x)$. Now, the main issue is how each node should determine its own TR value.

To inform the sensors about their proper range function, a simple protocol is designed as follows: In the first step, each node starts sending a Hello message to the BS, using a small transmission range. Each Hello message contains the position of its original sender. During forwarding of Hello messages to the BS, every sensor determines the value of the sector radius (i.e., R) by selecting the message whose original sender has a maximum distance to the BS among the ones laid inside the same sector. Receiving different radius values from diverse directions, the BS starts computing $t(x)$ for all radiuses. Remind that the BS is not so limited in power, thus computing the range functions is feasible there. Before starting the computation of $t(x)$, the BS should wait enough to become sure that it has received the messages from all boundary points. For each direction, the BS finds the proper radius value, by computing the maximum value of R received from the messages of the mentioned sector. After that, the BS starts to send the computed function of each sector to the appropriate direction, to inform all the corresponding sensors. Once a node receives more than one advertisement of $t(x)$, it selects the one whose radius is closer to its registered R, obtained in the first step.

We should emphasize here that the calculation of $t(x)$ function is done statically (i.e., once forever) and in a centralized manner. Then, all nodes will be informed about the computed range function to obtain their own TR value. The rate of energy

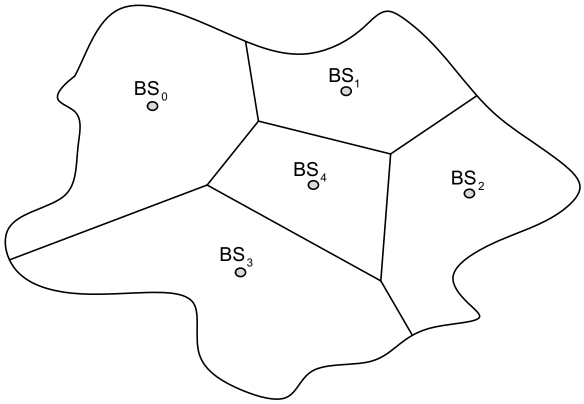


Fig. 9. An example of Voronoi diagram for a multi base station network.

consumption during the initial phase of our protocol is equal to what the regular TC schemes, which use the smallest range value, achieve throughout the network lifetime. However, as this phase is so short, it does not affect the overall rate significantly.

The complexity of the problem would not change much if the network contains more than one BS. However, prior to running of the algorithm, each sensor has to recognize its own BS, which is logically the nearest BS, and then must forward all arriving Hello messages to it. From a centralized viewpoint, the execution of our algorithm in a multi base station environment actually turns out to the multiple running of the original algorithm in several mutual-exclusive areas determined by the Voronoi diagram. As depicted in Fig. 9, the Voronoi diagram of the BSs divides the whole network into several smaller sub-areas such that (1) each area contains exactly one BS, and (2) all points inside an area are closer to the corresponding BS of that area than any other BS in the network.

$$Error(x) = \frac{\text{Path length from } x \text{ to BS-Euclidean distance from } x \text{ to BS}}{\text{Euclidean distance from } x \text{ to BS}}. \quad (25)$$

6. Simulation results

In the preceding sections, we proposed an analytical model, along with three separate solutions to handle different variants of the LSTC problem. In this section, we justify the assumptions made in our model and then analyze the performance of the proposed algorithms through computer simulations.

6.1. Simulation model

Our experiments were carried out using a customized implementation with an experimental setup, similar to the one described in [18]. The simulations are conducted by varying the node density (δ) from 15 to 90 node/sq unit in a circle area with radius $R = 10$, where the BS laid at the center, and the sensors are uniformly distributed in the area. In each experiment, after generating a placement of the sensors, we run the algorithms on the network consisting of those sensors. For each algorithm, we measure both the energy consumption rate, as well as the traffic load for different nodes in the resulting topology. Also, we compare the maximum rate of energy consumption obtained by running each TC algorithm over the whole network. We assume that the energy consumption in each node can be computed according to (3) with $n = 2$, $\alpha = 1$, and $\gamma = 1$. Also, we fix the traffic rate λ and the minimum range ε to 0.1 and 0.2, respectively. Note that the results of energy consumption and traffic load are the average over 10 trials.

6.2. Justifying the straight forwarding assumption

All of our analytical discussions provided in the previous sections are based on the simple assumption that nodes forward messages through a straight line towards the BS. This fact was justified by intuitive reasons there. Now, we present some experimental results to show that this hypothesis is conceivable and realistic.

To measure the correctness of straight forwarding assumption, the Error metric for a node x is defined as

In fact, this metric tells how much the actual path between x and the BS is longer than the straight line connecting two nodes together.

To show the effect of density and transmission range on the accuracy of straight forwarding assumption, two experiments have been done. In the first one, TR values of all nodes are fixed to

0.1*R*, and then the average of Error metric among all sensors is computed under various node densities. The result of this experiment is shown in Fig. 10a. Interestingly, the Error rate degrades rapidly by the increase in the number of sensors. For example, this error becomes very small for more than 700 nodes and rather negligible for more that 1000 nodes. On the other hand, Fig. 10b demonstrates how TR value may influence Error rates. As the length of TR values decreases, the resulting topology becomes evidently sparser, making the paths more divergent from the straight line. Similarly, the mean of Error metric drops quickly for large TR values. More specifically, it remains acceptable for all $TR > 0.13 \times R$ and surprisingly becomes negligible for $TR > 0.2 \times R$. Finally, Fig. 10c shows the distribution of the exact Error among all sensors of the network. As the figure shows, the number of nodes that contribute large Errors is very small in comparison with those transmitting approximately through a straight line to the BS. In general, as we can see from Fig. 10, the assumption of straight forwarding becomes closer to reality when the node density or transmission range of sensors increase in the network.

6.3. Performance analysis through simulation results

In this section, we experimentally evaluate the proposed methods. For each topology, we have measured the energy consumption rate obtained by using one of the following four TC schemes: (1) regular TC (R_TC), which is the traditional TC method that only minimizes the transmission ranges so that the resulting topology becomes connected, (2) single-range LSTC or S_LSTC (proposed in Section 4.1), (3) multiple discrete LSTC or MD_LSTC (proposed in Section 4.2), and (4) multiple continuous LSTC (proposed in Section 4.3).

Since previous TC approaches have completely ignored the impact of traffic load parameter, they try to make TR values so small that only the connectivity is preserved. More precisely, they choose the smallest possible range for each node such that the resulting topology remains connected.

S_LSTC, on the other hand, tries to find a single TR value for all nodes to reduce their energy consumption. Amazingly, as we mathematically showed in Section 4.A, this value is not necessarily the smallest possible one. This fact is confirmed through

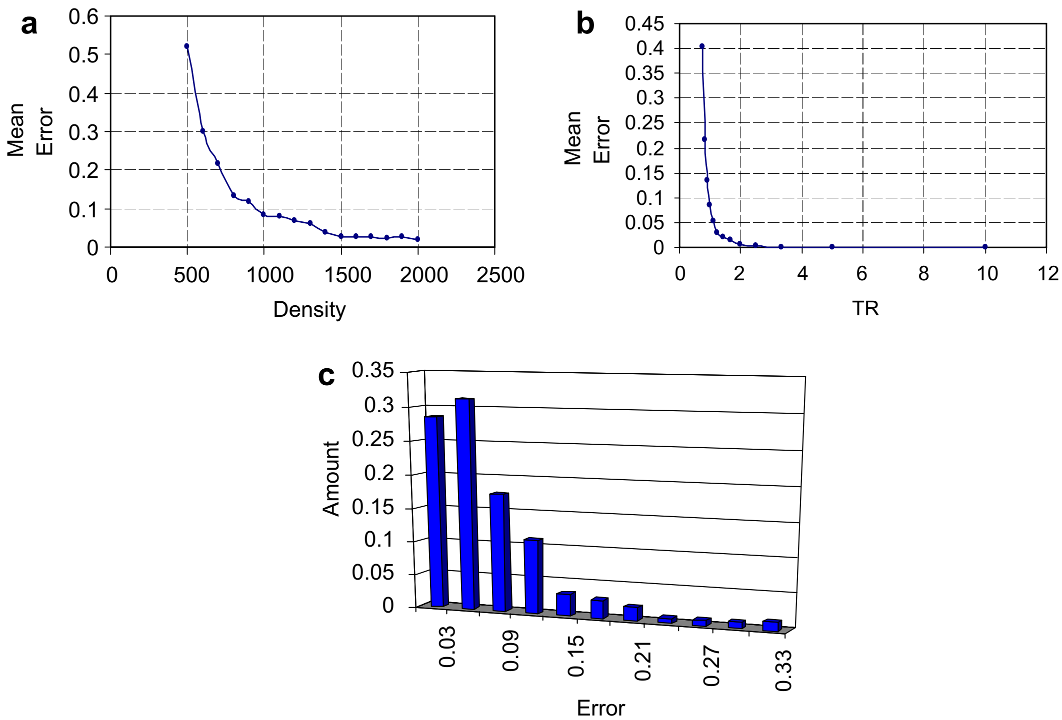


Fig. 10. The results of simulation for justifying the straight forwarding assumption: (a) Mean error vs. density ($TR = 0.1*R$), (b) mean error vs. CT ($density = 1000/R2$, $TR = CT*R$) and (c) error distribution ($density = 1000/R2$, $TR = 0.1*R$).

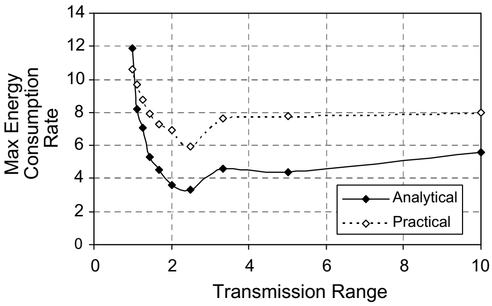
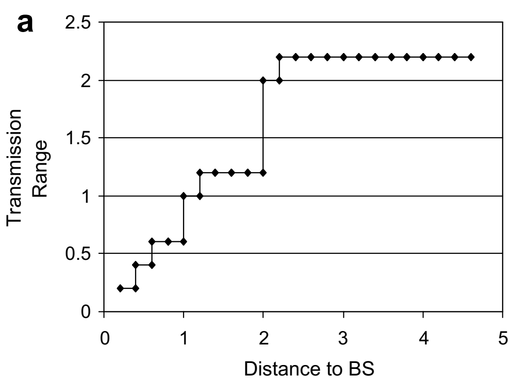


Fig. 11. Max energy consumption vs. transmission range obtained by S_LSTC.

simulation results in Fig. 11 where we compare the max energy consumption obtained by using different range values. The figure also shows acceptable similarity between analytical and experimental results. The reason for the difference between two results is that in our analytical method, load is assumed to be distributed evenly among all foremost nodes, but in experiments, this assumption may not be completely true. Moreover, energy consumption in analytical evaluation has been measured for the foremost points of every sector area (i.e., points settled on the arc with distance ε from the BS) which cannot be computed through experiments, because in practice there may be no sensor laid on this arc. Besides, Fig. 12a and b demonstrate the range functions achieved by two methods MD_LSTC, and MC_LSTC (up to seven iterations), respectively.

Fig. 13 shows the amount of energy utilization, and traffic load, in different points of the field according to their distance from the BS, for all methods under different node densities. In R_TC, and S_LSTC, the part having the most energy consumption is the area nearest to BS, because it has to forward all messages of the network. Note that in both methods, the TR values of all nodes are the same. Thus, the area experiencing the maximum load will have the maximum energy utilization. However, the load functions of the other two methods (i.e., MD_LSTC and MC_LSTC) are smoother, as farther nodes also cooperate in sending messages directly to the BS. This fact becomes especially apparent for denser networks (i.e., networks with a higher node density), where the assumption of straight forwarding becomes more realistic.

At last, Fig. 14 compares the maximum energy consumption rate obtained in networks with diverse node densities. It is clear that when $\delta = 15$, all the introduced methods outperforms R_TC. In comparison with S_LSTC which assigns an equal TR to all nodes, the range assignment made by R_TC consumes more energy by a factor ranging from 1.2 to 5.1. In average, S_LSTC enhances the maximum amount of energy consumption by the factor of 3.2. The right diagram of Fig. 14 particularly compares our three proposed methods in a closer view. Although S_LSTC degrades energy consumption significantly, using different TR values results in better achievements. Different from R_TC, the other methods' energy consumption rate do not vary drastically by the change in density, which justifies why our equations were independent of δ. Overall S_LSTC consumes more energy that MD_LSTC with the factor of 1.16, which in turn utilize the energy 1.03 times what MC_LSTC uses.



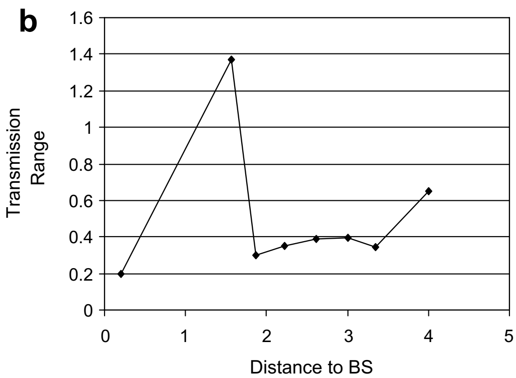
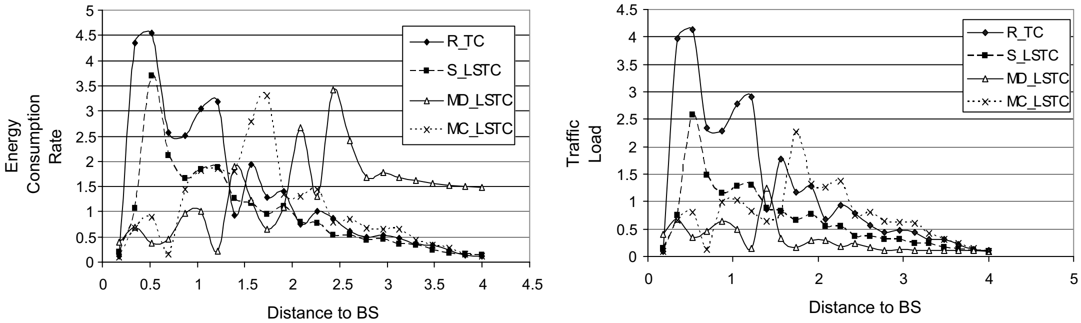
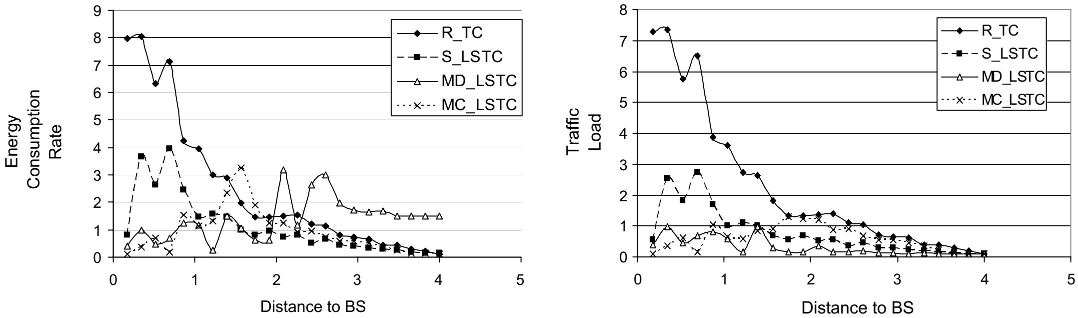


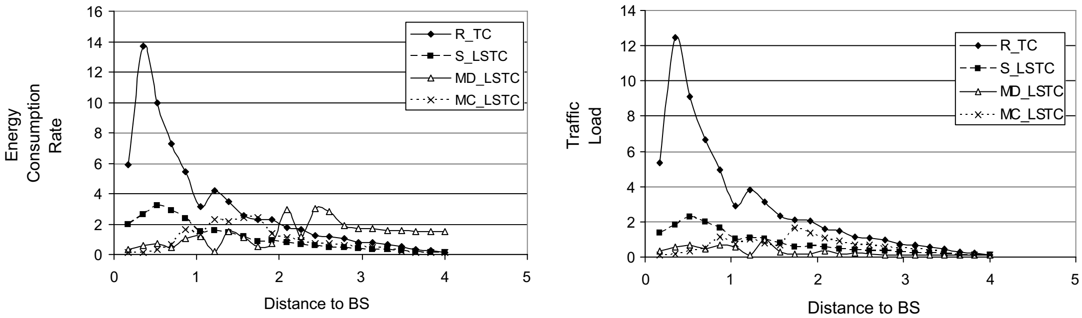
Fig. 12. Transmission range function (obtained by different algorithms) vs. distance of nodes to BS: (a) TR function obtained by MD_LSTC, (b) TR function obtained by MC_LSTC (after 7 iterations).



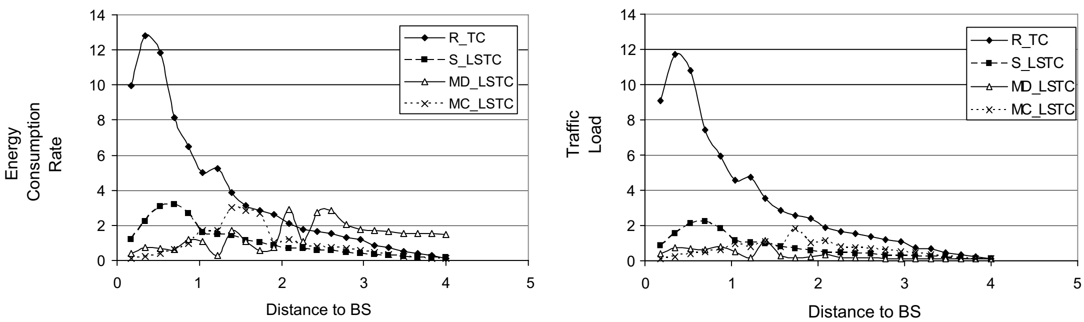
a) δ =15



b) δ =40



c) δ =65



d) δ =90

Fig. 13. Energy consumption rate and traffic load vs. distance to BS under different node densities (δ).

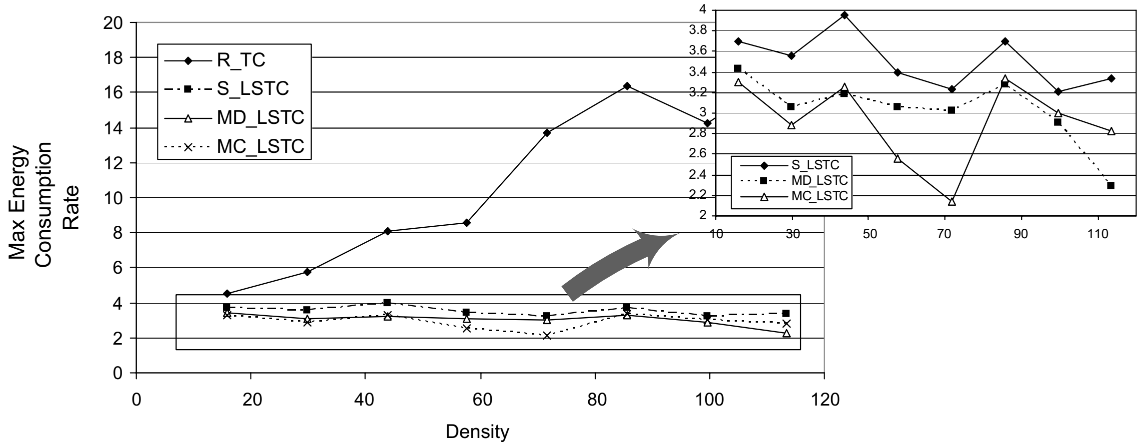


Fig. 14. Max energy consumption vs. node density.

7. Conclusion and future work

In this paper, we suggested a new approach to the topology control problem for the purpose of energy conservation in wireless sensor networks. Criticizing classical topology management schemes that consider transmission range (TR) as the only factor which affects power utilization, we also emphasized on the total load that sensors experience. Consequently, a new problem, called *Load Sensitive Topology Control* (*LSTC*), was formally defined for sensor networks with one or more base stations. Then, we investigated the problem under three different range constraints: (1) when TR values of all nodes are the same, (2) when TR values of nodes are selected from a finite predetermined set, and (3) when there is no restriction on TR values. Using mathematical modeling, elegant relations were obtained to compute the optimal range values for each problem according to the distance of sensors from the nearest base station. We also proved the correctness of this modeling through extensive experiments. Simulation results confirmed the effectiveness of the proposed methods in comparison with previous solutions.

The ideas introduced in this paper can be applied for general wireless ad hoc networks, where unlike sensor networks, different source–destination pairs exist. Another possible extension of this work is formulizing LSTC as a graph theory problem, and providing either centralized or distributed algorithms to solve it. Moreover, other important objectives (like reducing interference of nodes) may be added to the problem definition. Applying similar techniques to the regions of the higher dimension (i.e., 3D environments) is another possible extension of this work.

Appendix. Proof of Theorem 2

Theorem 4. *Let L_LSTC*(N,k) *be an instance of the L_LSTC problem with N as the number of nodes and k as the constraint of the problem, indicating the maximum rate of energy consumption. For a fixed value of* λ, *if we represent L_LSTC*(N,k) *by* $(1^N, k)$ (*instead of the binary representation* (N,k)), *then the problem is not* **NP**-*Hard, unless P* = **NP**.

Discussion. First, we notice that for a fixed value of λ, the only thing needed for the description of an instance of the problem is N and k. We can either represent N in binary format with $c \cdot \log(N)$ bits, or we can represent it in unary format. As polynomial-time algorithms have to run in polynomial time over their input size, the difference between two representations is on the allowed running time for an efficient algorithm which solves the problem. The difference in our theorem shows up when the reduction (from some NP-complete problem) tries to create an instance of our problem (which is described by some (N,k)). In the binary representation, the reduction is able to use instances with huge N (while keeping $\log(N)$ reasonably small), but in the unary format the value of N could not be very big (that is because the reduction itself has to run in polynomial time). The main idea of the proof comes from [21] in which they show that sparse sets can not be **NP**-Hard.

Proof. For sake of contradiction, suppose that there is a polynomial time reduction from some **NP**-Complete problem to L_LSTC. Without loss of generality we assume that the **NP**-Complete problem used in the reduction is the boolean satisfiability problem (SAT). An instance of the SAT problem is described as follows. There are m boolean variables $x_1,\ldots,x_m$. Each variable x_i or its negation $\neg x_i$ is called a literal. Each clause is the disjunction of three literals (e.g. $x_1 \vee \neg x_2 \vee x_3$). We are given n clauses, and we are asked if there is a truth assignment to the n boolean variables satisfying all of the clauses at the same time. For simplicity, we assume that the size of a SAT instance is the number of its variables m. This is possible by some standard padding argument. We show that if the problem for a fixed λ (and unary encoding for N) is **NP**-Hard, then we can solve SAT in polynomial time which means $P = \mathbf{NP}$.

Suppose we are given a SAT instance I which has m variables $x_1, x_2, \ldots, x_m$. It is obvious that if we substitute the value of some variables in I, the size of the resulting SAT instance will reduce. Let us define $I|_{\{x=b\}}$ to be the SAT instance made out of I, by substituting the variable x by the constant value b (which is $b = 0$ or 1).

Suppose $R(\cdot)$ is the reduction which takes a SAT instance and gives out an L_LSTC instance such that the answer for the L_LSTC instance equals to the answer for the SAT instance (we remind that problem instances for L_LSTC are of kind $(1^N, k)$ for some N and k, because λ is fixed).

Assume $q(\cdot)$ is a polynomial such that given a SAT instance of m variables, if $R(J) = (1^N, k)$, then the length of the representation of $R(J)$ is at most $|R(J)| \leqslant q(m)$ and therefore $N \leqslant q(k)$. There must exist such a $q(\cdot)$ since the reduction runs in polynomial time. Therefore, if we assume I' to be a partial substitution of I after substituting t variables, we have $|R(I')| \leqslant q(m - t) \leqslant q(m)$.

Now, we show how to solve the SAT problem in polynomial time. The algorithm runs in m iterations. After the ith iteration, we have a set of SAT instances $C_i = \{J_1, J_2, \ldots, J_r\}$ such that $r \leqslant q(m)$, and for all j, $1 \leqslant j \leqslant r$, I_j is a SAT instance on variables $x_{i+1}, x_{i+2}, \ldots, x_m$. The important property of C_i which will be guaranteed by our algorithm is that I is satisfiable if and only if one of the members of C_i is satisfiable (we call this property as "C_i being equivalent to I"). Assuming the mentioned properties, after the mth iteration, the set C_i can have only constant values: True or False, and I will be satisfiable iff True $\in C_m$.

Before the first iteration (just at the beginning), we have $C = \{I\}$. At the beginning of the ith iteration, C_i is empty. For any member of C_{i-1} like J, we add $J|_{\{x_i=0\}}$ and $J|_{\{x_i=1\}}$ to C_i. Till now, we have $|C_i| \leqslant 2r \leqslant 2q(m)$. It is obvious that one of the members of C_i is satisfiable iff one of the members of C_{i-1} is so. Hence, C_i is equivalent to I. The problem is that maybe $|C_i| > q(m)$. We show how to remove some members of C_i such that C_i will preserve its equivalence to I. For all the members of C_i like J we calculate $R(J)$. So, if $C_i\{J_1, J_2, \ldots, J_s\}$, we get $R(J_1) = (N_1, k_1), R(J_2) = (N_2, k_2), \ldots, R(J_s) = (N_s, k_s)$.

Now, we know that for all i, $1 \leqslant i \leqslant s$, we have $N_i \leqslant q(m)$. If for two indexes $i \neq j$, we have $N_i = N_j$, then we compare k_i and k_j. If $k_i \leqslant k_j$, then we remove J_i from C_i, otherwise we remove J_j. If we repeat this removing procedure, at the end, we will get $|C_i| \leqslant q(m)$. But the question is, why when we remove a member of C_i, still I is equivalent to C_i? That is because when $k \leqslant k'$, if only one of the instances (N, k) and (N, k') is a YES instance, that instance will be (N, k'). By keeping the size of C_i small enough in each iteration, the algorithm will run in polynomial time. Now, if we finish the mth iteration, we can have only two possible members in C_m True and False. That is because all variables are already substituted. Thus, if True was in C_m, I is satisfiable, and vice versa.

You can interpret the algorithm as a back-track routine for SAT which cuts some branches using the reduction $R(\cdot)$. Hence, it shows that if L_LSTC is **NP**-Hard (for a fixed value of λ, and unary representation for N), then we can solve SAT efficiently which is widely believed to be impossible. □

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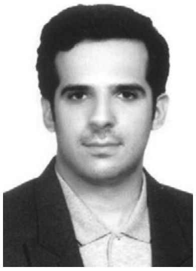

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