17-654
Analysis of Software Artifacts

Midterm Review

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Analysis as an Approximation

- If S doesn’t terminate normally, y cannot be 0
- Problem: undecidable to tell if S terminates!
- In general program analysis must compute an approximation

Safety and Precision

- ***Conservative/Safe Analysis***
  - *Computes a larger set of possibilities than will actually occur in program execution*
- ***Precise Analysis***
  - *Computes as small a set of possibilities for program execution as it can*

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Finding the Fixpoint

- Why should we think we will find an *n* such that $F^{n+1}(\overrightarrow{\mathrm{RD}_{\varnothing}}) = F^{n}(\overrightarrow{\mathrm{RD}_{\varnothing}})$?

Finding the Fixpoint

- Why should we think we will find an *n* such that $F^{n+1}(\overrightarrow{\text{RD}}_{\varnothing}) = F^{n}(\overrightarrow{\text{RD}}_{\varnothing})$?
  - *F* is monotone
  - Therefore, every application of F either:
    - Does not change $\overrightarrow{\text{RD}}$ (and so we have a fixpoint)
    - Or increases the size of a set in $\overrightarrow{\text{RD}}$
  - The set of definitions is finite so the sets in $\overrightarrow{\text{RD}}$ cannot increase in size forever
  - Therefore the algorithm terminates with a fixpoint at some finite *n*

Reaching Defs. vs. Available Exp.

- Reaching Defs. ***May*** analysis
  - Initial dataflow values: *empty* sets
  - *Union* at control flow merge
  - Precision: want *least* fixed point
  - Safety: err on the side of *larger* sets
- Available Exp. ***Must*** analysis
  - Initial dataflow values: *universal* sets
  - *Intersection* at control flow merge
  - Precision: want *greatest* fixed point
  - Safety: err on the side of *smaller* sets

Monotone Framework

$\text{Analysis}_{\circ}(\ell) = \iota$ if $\ell \in E$

$= \bigsqcup \{ \text{Analysis}_{\bullet}(\ell') \mid (\ell', \ell) \in F \}$ otherwise

$\text{Analysis}_{\bullet}(\ell) = f_{\ell}(\text{Analysis}_{\circ}(\ell))$

where:

- $\circ$ means entry (forward) or exit (backward)
- $\bullet$ means exit (forward) or entry (backward)
- $\bigsqcup$ is $\bigcup$ (may) or $\bigcap$ (must)
- F is $\mathit{flow}(S_*)$ (forward) or $\mathit{flow}^R(S_*)$ (backward)
- E is $\{ \mathit{init}(S_*) \}$ (forward) or $\mathit{final}(S_*)$ (backward)
- ι specifies initial or final analysis information, and
- f_{ℓ} is a transfer function
  - Typically $f_{\ell}(x) = x \setminus \text{kill}_{\text{Analysis}}(B^{\ell}) \cup \text{gen}_{\text{Analysis}}(B^{\ell})$

Monotone Framework

$$\text{Analysis}_\circ(\ell) = \begin{cases} \iota & \text{if } \ell \in E \\ \bigsqcup \{ \text{Analysis}_\bullet(\ell') \mid (\ell', \ell) \in F \} & \text{otherwise} \end{cases}$$

$$\text{Analysis}_\bullet(\ell) = f_\ell(\text{Analysis}_\circ(\ell))$$

	RD	**AE**	**LV**
$\bigsqcup$			
F			
E			
ι			

Monotone Framework

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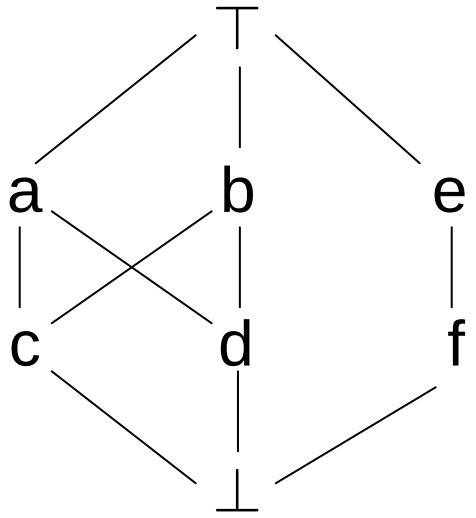
	RD	**AE**	**LV**
$\bigsqcup$	$\bigcup$	$\bigcap$	$\bigcup$
F	$flow(S_*)$	$flow(S_*)$	$flow^R(S_*)$
E	$\{ init(S_*) \}$	$\{ init(S_*) \}$	$final(S_*)$
ι	$\{ (x,?) \mid x \in FV(S_*) \}$	$\emptyset$	$\emptyset$

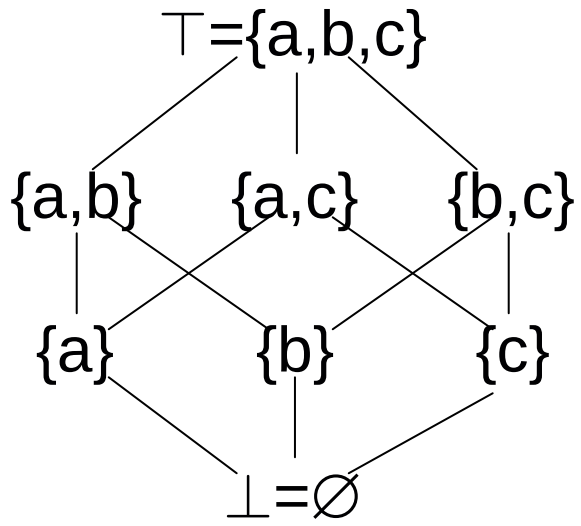
Complete Lattice

- Not all data flow analyses use sets
  - Lattice: a more general concept



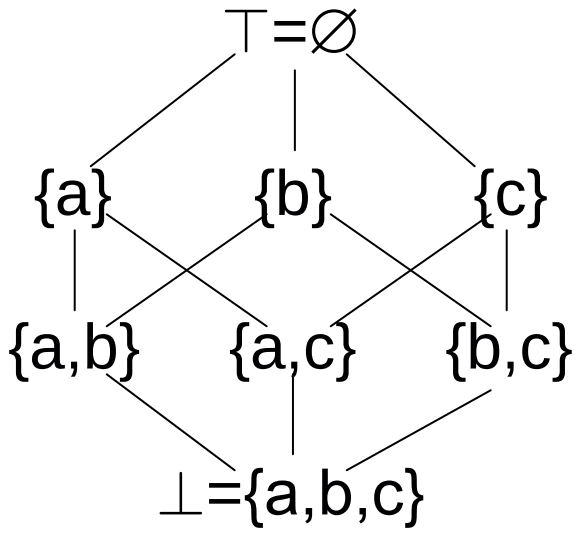
- A set *L* with:
  - A partial order $\sqsubseteq$
  - A combination operator $\sqcup$
  - A least element $\bot = \sqcup(\varnothing)$
  - A greatest element $\top = \sqcup(L)$
  - Each subset *Y* of *L* has a least upper bound $\sqcup(Y)$
- Typically we want the lattice to have finite height
  - A finite number of elements on each path from $\bot$ to $\top$
    - See NNH Appendix A.3

Example: Subset Lattice



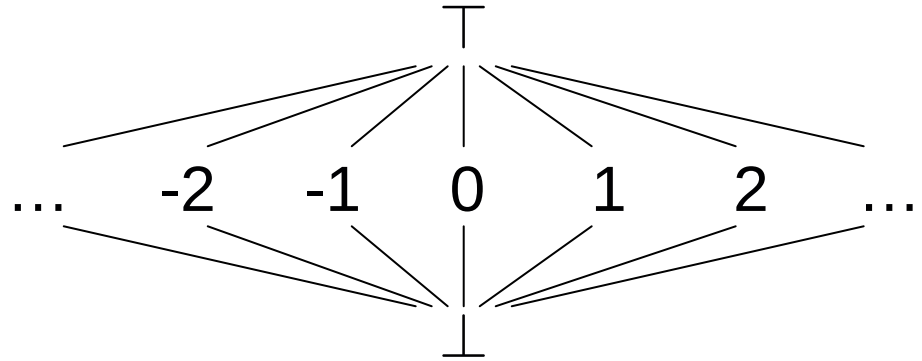
- Reaching Definitions
- The set $L=\mathcal{P}(\{a,b,c\})$ with:
  - $\sqsubseteq = \subseteq$
  - $\sqcup = \cup$ (may analysis)
  - $\bot = \varnothing$ (the most precise and starting element)
  - $\top = \{a,b,c\}$ (the least precise element)

Example: Superset Lattice



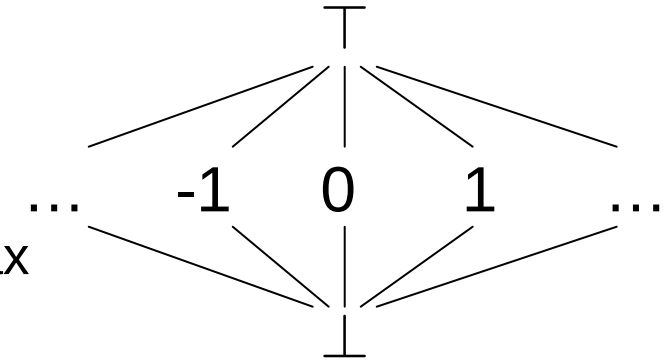
- Available Expressions
- The set $L=\mathcal{P}(\{a,b,c\})$ with:
  - $\sqsubseteq = \supseteq$
  - $\sqcup = \cap$ (must analysis)
  - $\bot = \{a,b,c\}$ (the most precise and starting element)
  - $\top = \varnothing$ (the least precise element)

Constant Propagation Lattice



- More efficient than the set of possible values
  - Don't want to store sets
  - If more than one value, give up and assume any (⊤)
- The set L={⊥,⊤} ⋃ NAT with:
  - x ⊑ ⊤, ⊥ ⊑ x, x ⊑ x
  - x ⊔ ⊥ = x, x ⊔ ⊤ = ⊤, n ⊔ m = ⊤ (for n ≠ m)
- ι = ⊤

Constant Propagation Transfer Fns



- Can't use gen and kill sets
  - Data flow values aren't sets anymore!
- Instead, define function by cases on syntax
  - Input is incoming data flow value σ

- $f^{CP}([x := a],\sigma) = \sigma\,[x \mapsto CP(a,\sigma)]$
- $f^{CP}([skip],\sigma) = \sigma$
- $f^{CP}([b],\sigma) = \sigma$

- $CP(n,\sigma) = n$
- $CP(x,\sigma) = \sigma(x)$
- $CP(a_1\ op_a\ a_2,\sigma) = CP(a_1, \sigma)\ \widehat{op_a}\ CP(a_2, \sigma)$

- $z_1\ \widehat{op_a}\ z_2$ $= z_1\ \widehat{op_a}\ z_2$ if $z_1, z_2 \in NAT$
  $= \top$ if $z_1 = \top$ or $z_2 = \top$
  $= z_1\,(z_2)$ if $z_2\,(z_1) = \bot$

Execution Traces

- Sequence of <pp,mem> pairs
  - pp is a program point
    - Just before statement pp
  - mem is the state of variables in memory

$[y := x]^1$;
$[z := 1]^2$;
while $[y>1]^3$ do
    $[z := z * y]^4$;
    $[y := y - 1]^5$;
$[y := 0]^6$;

pp	x	y	z
1	2	0	0
2	2	2	0
3	2	2	1
4	2	2	1
5	2	2	2
3	2	1	2
6	2	1	2
-	2	0	2

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pp	x	y	z
1	1	0	0
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6	1	1	1
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3	3	2	3
4	3	2	3
5	3	2	6
3	3	1	6
6	3	1	6
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pp	x	y	z

Repeat for all possible initial values of x,y,z!

Abstraction

- Abstraction function α
  - maps traces to data flow values at a certain time *t* in the trace
- $\alpha_{CP}(<p_1,m_1>\ldots<p_n,m_n>,t) = m_t$
- $\alpha_{SA}(<p_1,m_1>\ldots<p_n,m_n>,t) = sign(m_t)$

- Also define program point function *pp*
- $pp(<p_1,m_1>\ldots<p_n,m_n>,t) = p_t$

t	pp	x	y	z
0	1	3	0	0
1	2	3	3	0
2	3	3	3	1
3	4	3	3	1
4	5	3	3	3
5	3	3	2	3
6	4	3	2	3
7	5	3	2	6
8	3	3	1	6
9	6	3	1	6
10	-	3	0	6

$\alpha_{CP}(T,0) = (x=3,y=0,z=0)$
$\alpha_{CP}(T,10) = (x=3,y=0,z=6)$

$\alpha_{SA}(T,10) = (x=+,y=0,z=+)$

WHILE Traces, Formally

- A trace for program S_1 and initial state σ_1 is either:
  - a finite sequence $(S_1, \sigma_1), \ldots, ([], \sigma_n)$, where $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1})$ for $i \in 1, \ldots, n\text{-}1$
  - an infinite sequence $(S_1, \sigma_1), \ldots, (S_i, \sigma_i), \ldots$ where $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1})$ for $i \geq 1$

- Slight notational simplification
  - We will abbreviate $(S_1, \sigma_1), \ldots, (S_n, \sigma_n)$ as $(\mathit{first}(S_1), \sigma_1), \ldots, (\mathit{first}(S_n), \sigma_n)$
    - Uses program counter labels instead of complete programs

Local Soundness

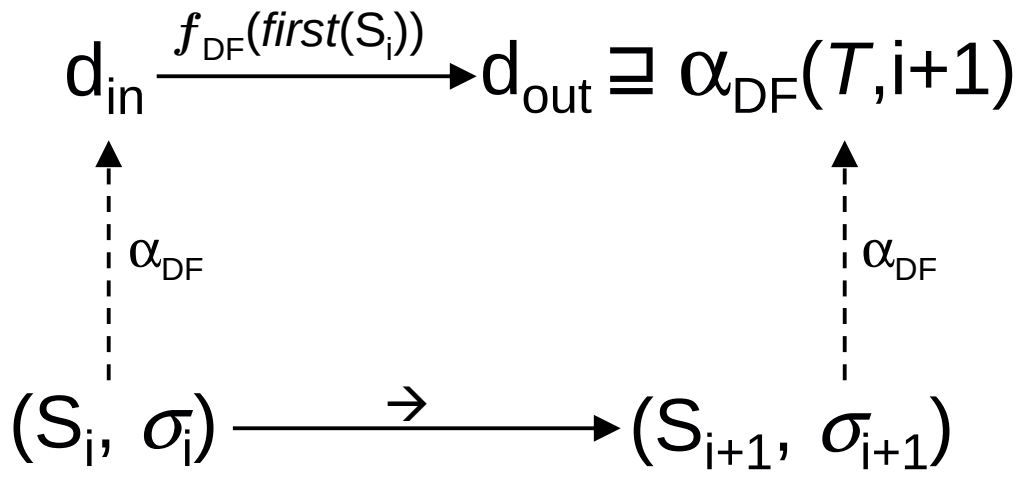
To show:

if $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1}) \in T$

and $d_{in} = \alpha_{DF}(T,i)$

and $d_{out} = f_{DF}(first(S_i), d_{in})$

then $\alpha_{DF}(T,i+1) \sqsubseteq d_{out}$



Intuitively, translating from concrete to abstract and applying the flow function will safely approximate (⊒) taking a step in the trace and translating from concrete to abstract

What does Correctness Mean?

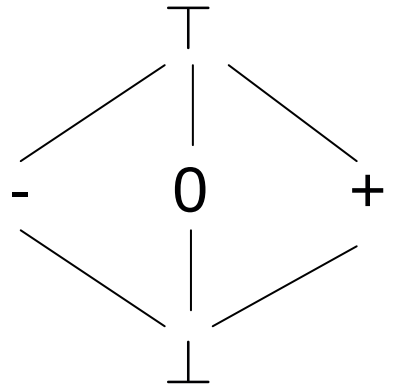
- Intuition
  - At a fixed point, analysis results are a *conservative abstraction* of *program execution*
- *Soundness* condition
  - When data flow analysis reaches a fixed point *F*, then for all traces *T* and all times *t* in each trace, $\alpha(T,t) \sqsubseteq F(pp(T,t))$

Global Soundness

- Intuition
  - We begin with initial dataflow facts ι that safely approximate ($\sqsupseteq$) all initial stores σ_1
  - By local soundness, each transfer function when given safe input information yields safe output information
  - By induction, any fixed point of the analysis is sound

Soundness Example: Sign Analysis

Custom Lattice



- Transfer functions
  - σ *is input data flow value*
  - $f^{SA}([x := a],\sigma) = \sigma\,[x \mapsto SA(a,\sigma)]$
  - $f^{SA}([skip],\sigma) = \sigma$
  - $f^{SA}([b],\sigma) = \sigma$
  - $SA(n,\sigma) = sign(n)$ *// returns sign of n*
  - $SA(x,\sigma) = \sigma(x)$
  - $SA(a_1 + a_2,\sigma) = +$ ***// is this sound?***
  - $SA(a_1\ op_a\ a_2,\sigma) = \top$ *for* $op_a \neq +$

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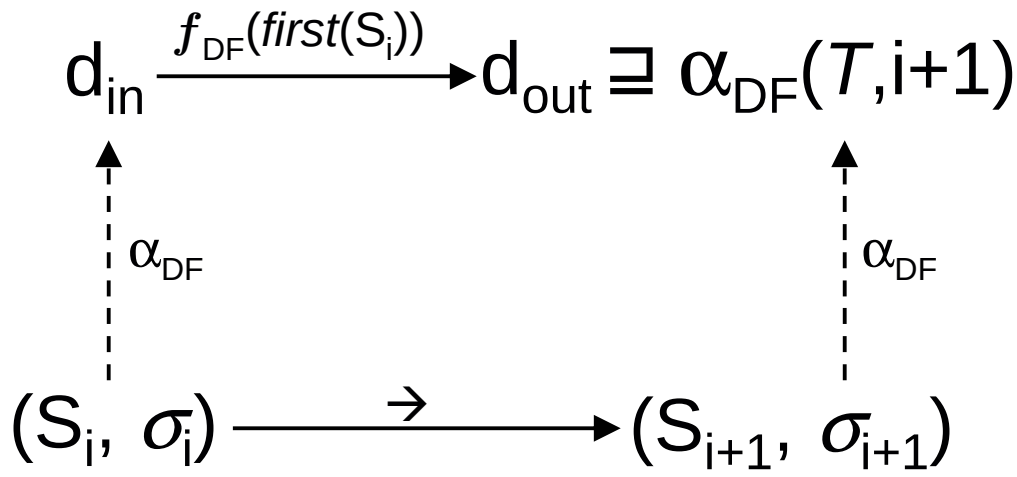

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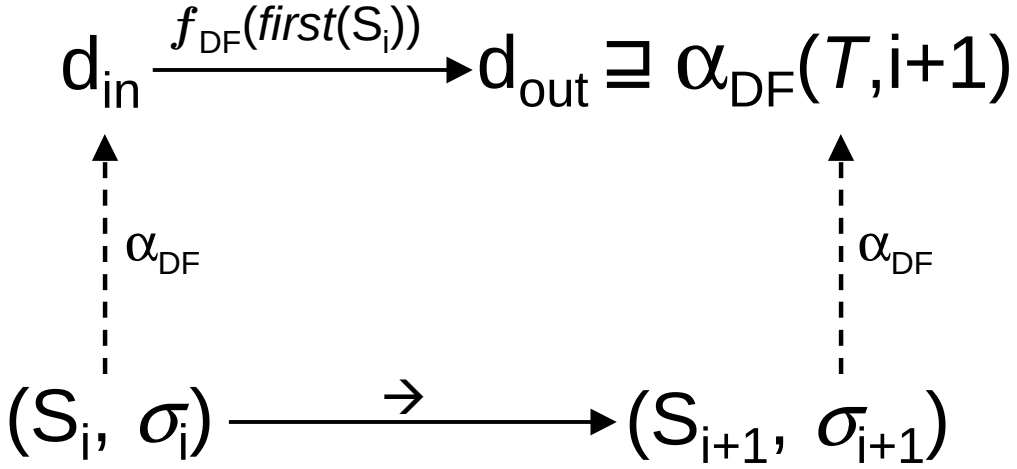
Local Soundness Fails



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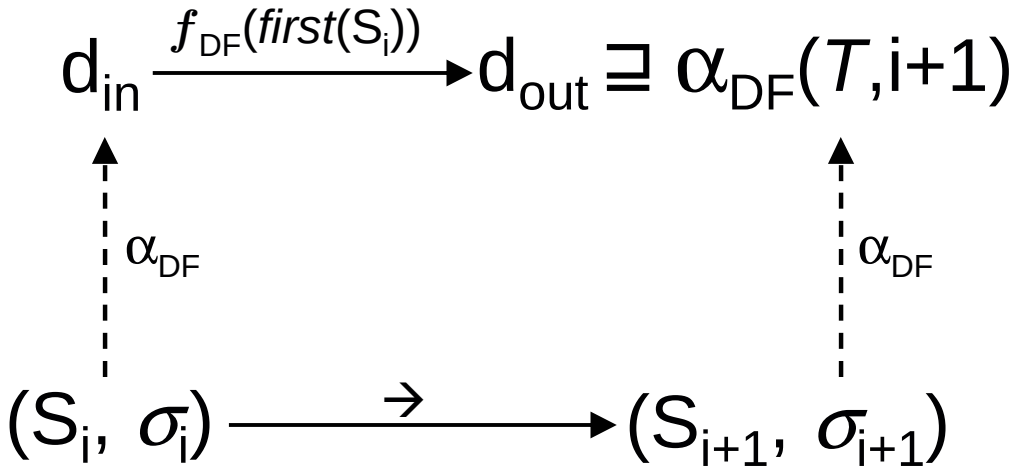

Program:

$[z := y-7]^1$

$[x := y+z]^2$

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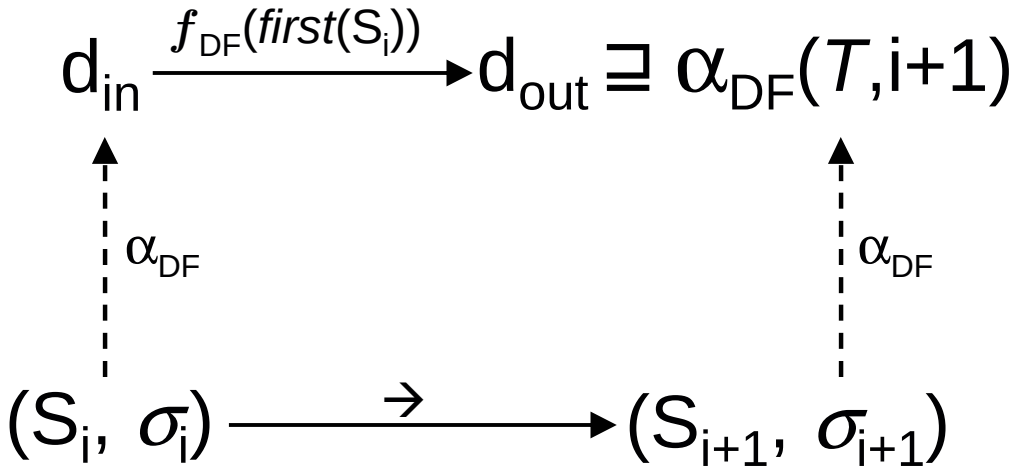

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Trace *T*:

t	pp	x	y	z
0	1	0	3	0
1	2	0	3	-4
2	-	-1	3	-4

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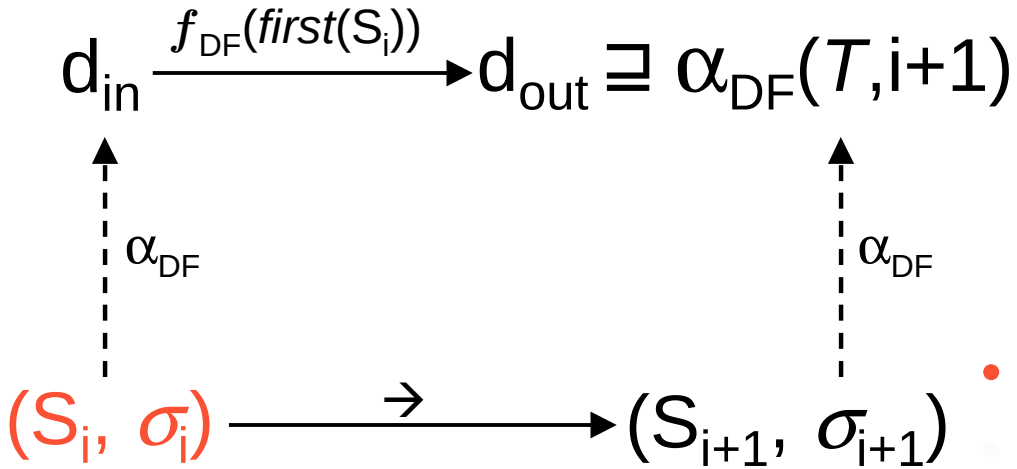

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- $\sigma_1 = (x=0, y=3, z=-4)$
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- $d_{in} = \alpha_{DF}(T,1) = (x=0, y=+, z=-)$
- $\alpha_{DF}(T,2) = (x=-, y=+, z=-)$
- $d_{out} = f^{SA}([x := y+z]^2, d_{in})$
  $= d_{in}[x \mapsto SA(y+z, d_{in})]$
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  $= (x=+, y=+, z=-)$
- $(x=+, y=+, z=-) \not\sqsupseteq (x=-, y=+, z=-)$

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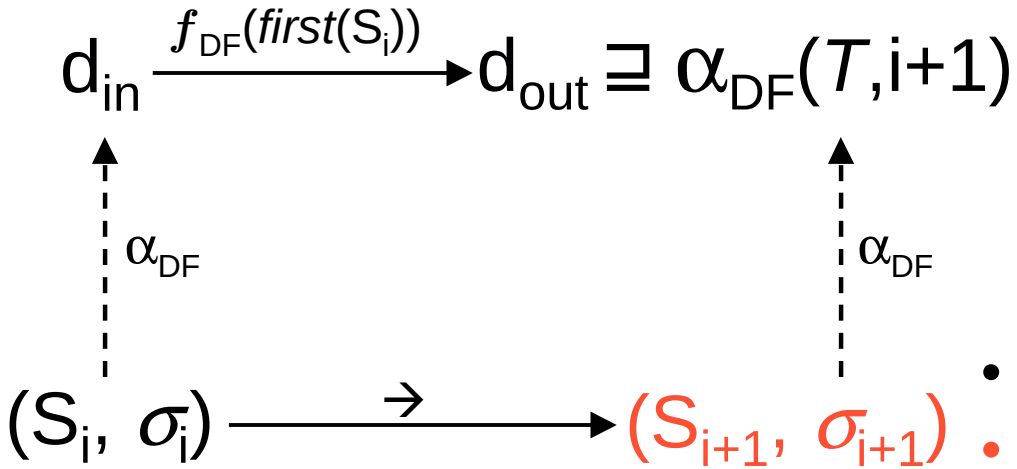

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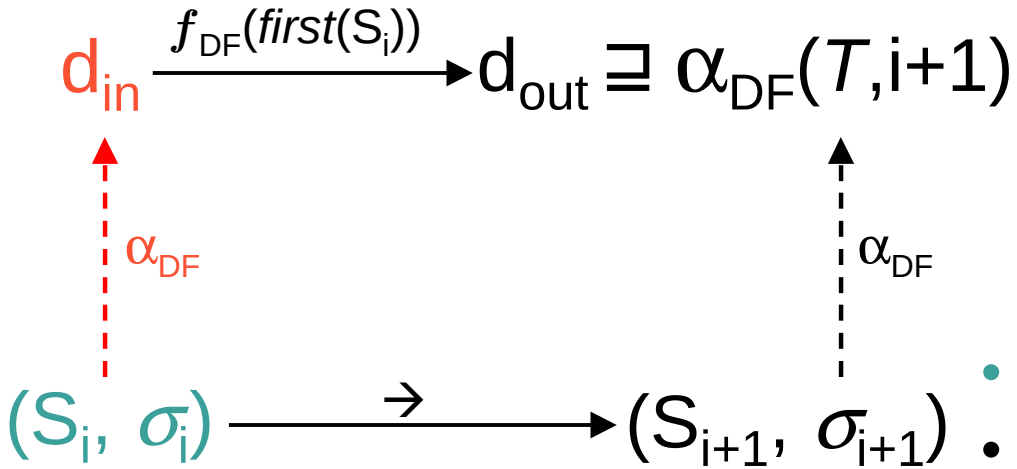

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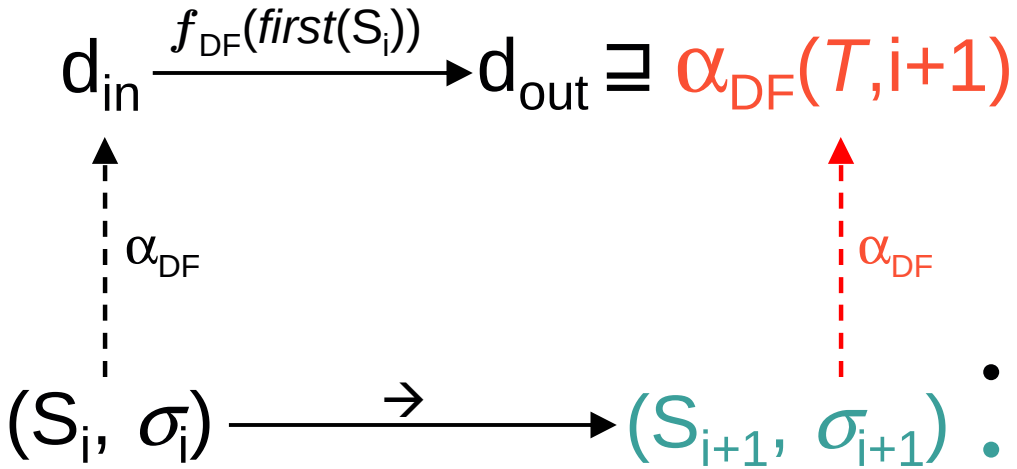

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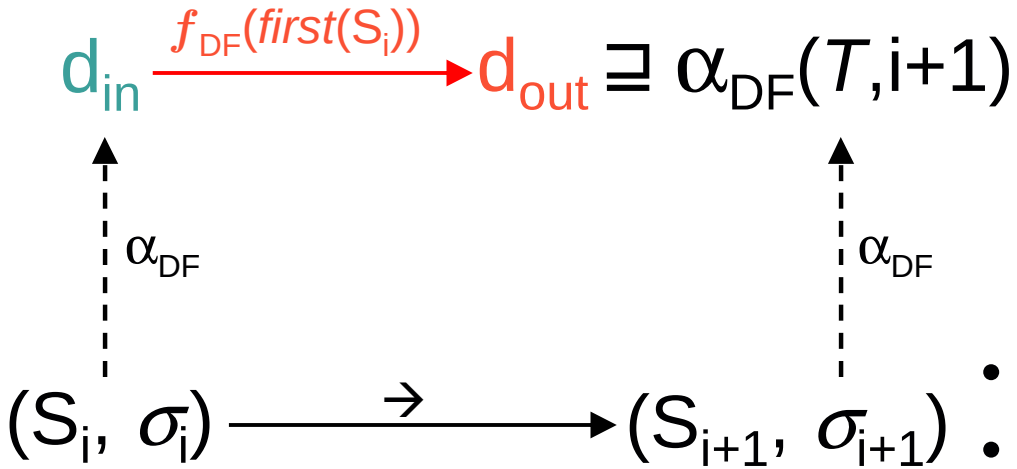

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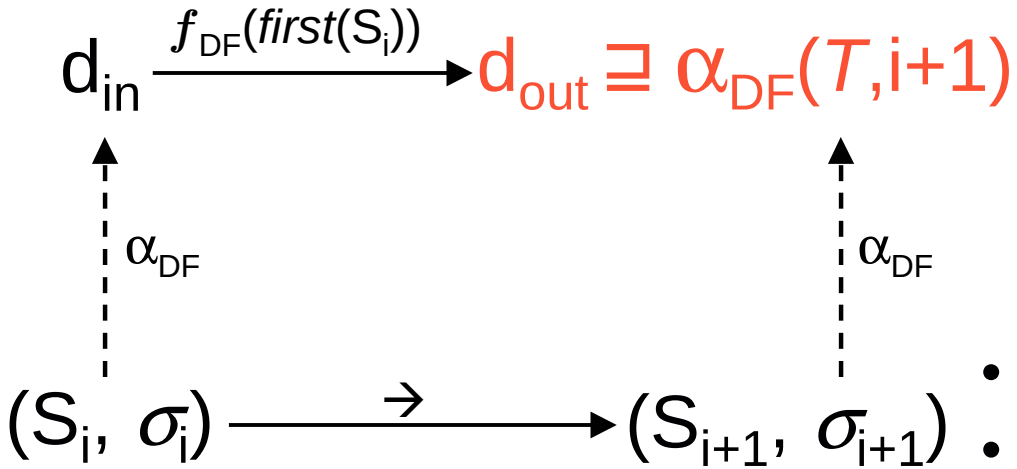

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Local Soundness for Constant Propagation

To show:

if $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1}) \in T$

and $d_{in} = \alpha_{CP}(T,i)$

and $d_{out} = f_{CP}(\mathit{first}(S_i), d_{in})$

then $\alpha_{CP}(T,i+1) \sqsubseteq d_{out}$

- Case: $S_i = [x := a]^\ell$
  - $\sigma_{i+1} = \sigma_i [x \mapsto \mathcal{A}(a, \sigma_i)]$
  - $d_{in} = \alpha_{CP}(T,i) = \sigma_i$
  - $d_{out} = f_{CP}([x := a]^\ell, \sigma_i)$
    $= \sigma_i [x \mapsto CP(a, \sigma_i)]$
  - $\alpha_{CP}(T,i+1) = \sigma_{i+1}$
    $= \sigma_i [x \mapsto \mathcal{A}(a, \sigma_i)]$
  - Lemma: $\mathcal{A}(a, \sigma_i) = CP(a, \sigma_i)$
  - Thus $\sigma_i [x \mapsto \mathcal{A}(a, \sigma_i)]$
    $\sqsubseteq \sigma_i [x \mapsto CP(a, \sigma_i)]$

Abstraction for Reaching Definitions

- $\alpha_{RD}(\langle p_1,m_1\rangle \ldots \langle p_n,m_n\rangle, t) =$
  $\{ (x, p_k) \mid x \in FV(S_*)$
  and $k < t$
  and $stmt(p_k) = [x := a]$
  and $\forall j, k<j<t\ stmt(p_j) \neq [x := a']\}$

Local Soundness for Reaching Definitions

To show:

if $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1}) \in T$
and $d_{in} = \alpha_{RD}(T,i)$
and $d_{out} = f_{RD}(\mathit{first}(S_i), d_{in})$
then $\alpha_{RD}(T,i+1) \sqsubseteq d_{out}$

- Case: $S_i = [x := a]^\ell$
  - $d_{in} = \alpha_{RD}(T,i)$
  - $d_{out} = f_{RD}([x := a]^\ell, d_{in})$
    $= (\alpha_{RD}(T,i) \setminus \{(x,*)\}) \cup \{(x,\ell)\}$
  - Lemma: $\alpha_{RD}(T,i+1)$
    $= (\alpha_{RD}(T,i) \setminus \{(x,*)\}) \cup \{(x,\ell)\}$
  - So $\alpha_{RD}(T,i+1) = d_{out}$
  - Thus $\alpha_{RD}(T,i+1) \sqsubseteq d_{out}$

Abstraction for Live Variables

- $\alpha_{LV}(\langle p_1,m_1\rangle \ldots \langle p_n,m_n\rangle, t) =$
  $\{ x \mid x \in \mathit{FV}(\mathit{stmt}(p_k))$ where $k > t$
  and $\forall j,\ t<j<k\ \mathit{stmt}(p_j) \neq [x := a']\}$

Local Soundness for Live Variables

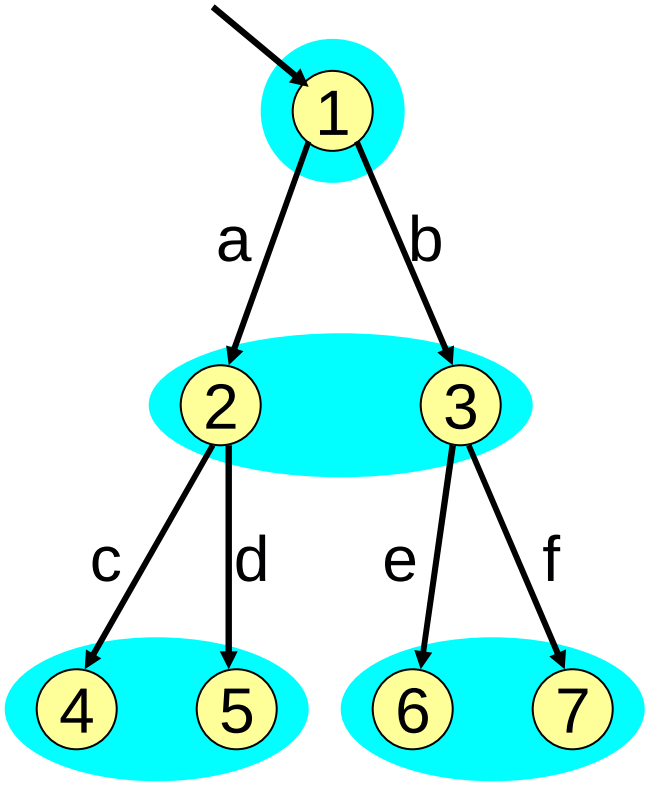
To show:

if $(S_i, \sigma_i) \rightarrow (S_{i+1}, \sigma_{i+1}) \in T$
and $d_{in} = \alpha_{LV}(T,i+1)$
and $d_{out} = f_{LV}(first(S_{i+1}), d_{in})$
then $\alpha_{LV}(T,i) \sqsubseteq d_{out}$

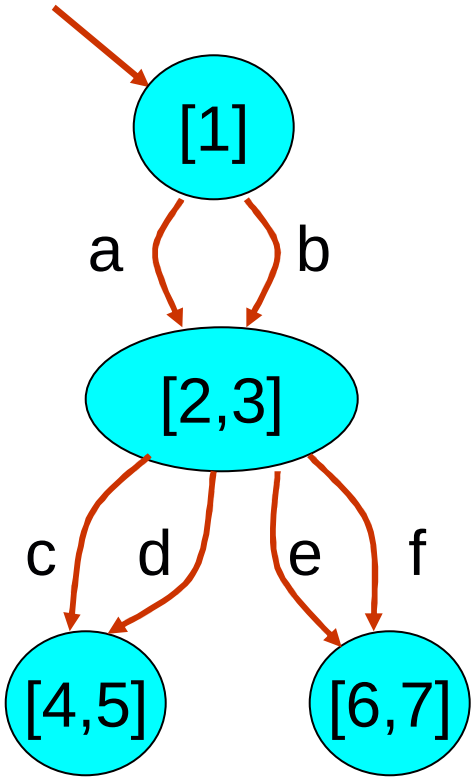
Note: i and i+1 are swapped due to reverse analysis

- Case: $S_{i+1} = [x := a]^\ell$
  - $d_{in} = \alpha_{RD}(T,i+1)$
  - $d_{out} = f_{RD}([x := a]^\ell, d_{in})$
    $= (\alpha_{RD}(T,i+1) \setminus \{x\}) \cup FV(a)$
  - Lemma: $\alpha_{RD}(T,i)$
    $= (\alpha_{RD}(T,i+1) \setminus \{x\}) \cup FV(a)$
  - So $\alpha_{RD}(T,i) = d_{out}$
  - Thus $\alpha_{RD}(T,i) \sqsubseteq d_{out}$

Conservative Abstraction



M

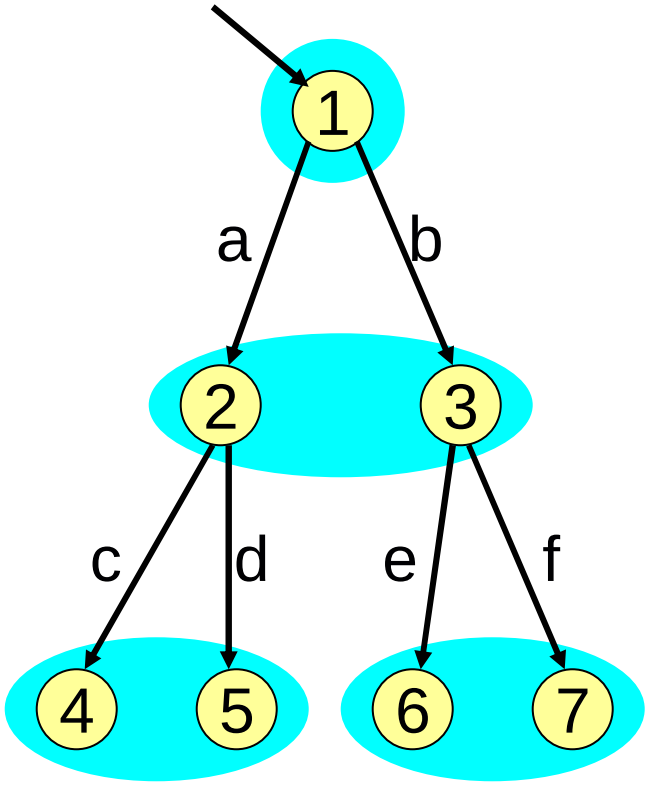


A

Conservative Abstraction

- Every trace of M is a trace of A
  - A over-approximates what M can do
    (Preserves safety properties!): $A \models \phi \Rightarrow M \models \phi$
- Some traces of A may not be traces of M
  - May yield spurious counterexamples - $\langle a, e \rangle$
- Eliminated via abstraction refinement
  - Splitting some clusters in smaller ones
  - Refinement can be automated

Original Abstraction



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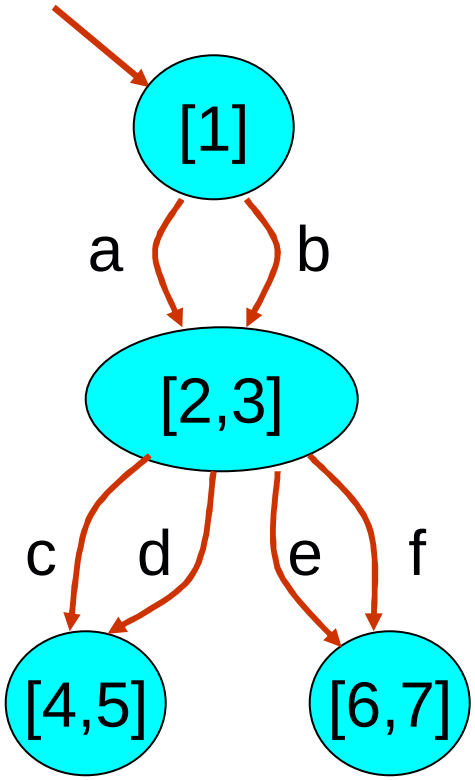
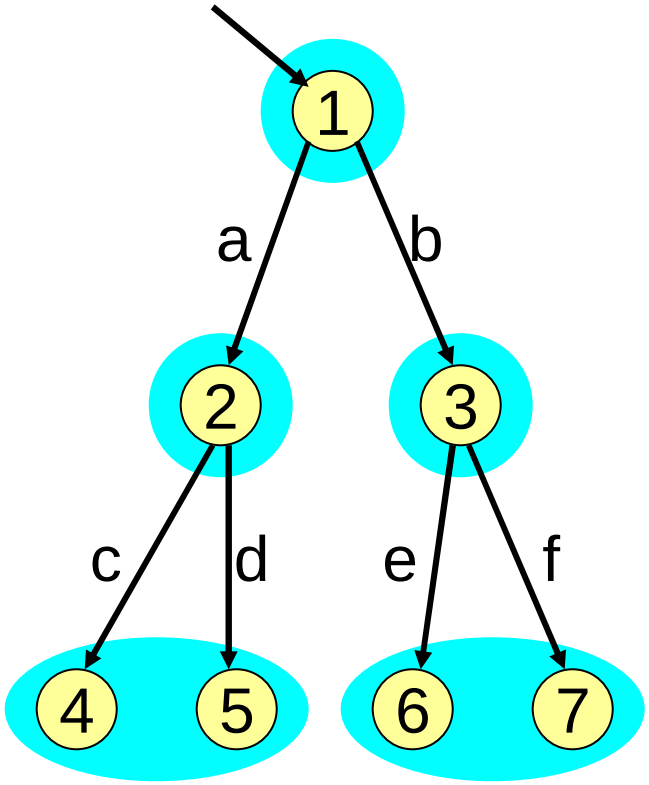
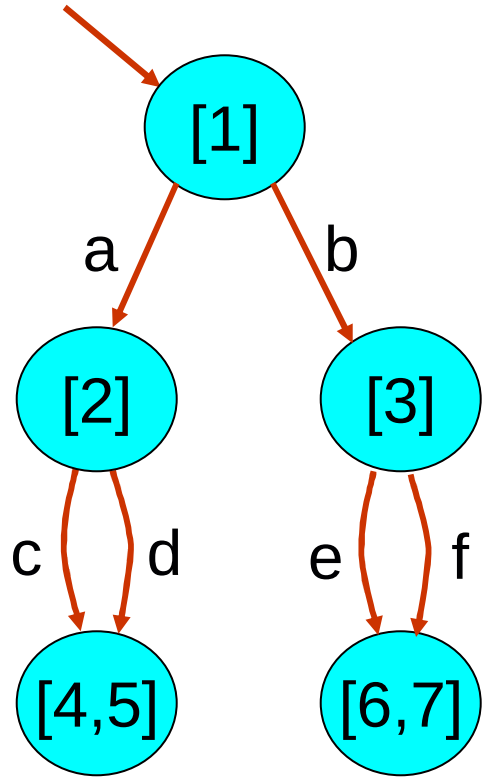


A

Refined Abstraction



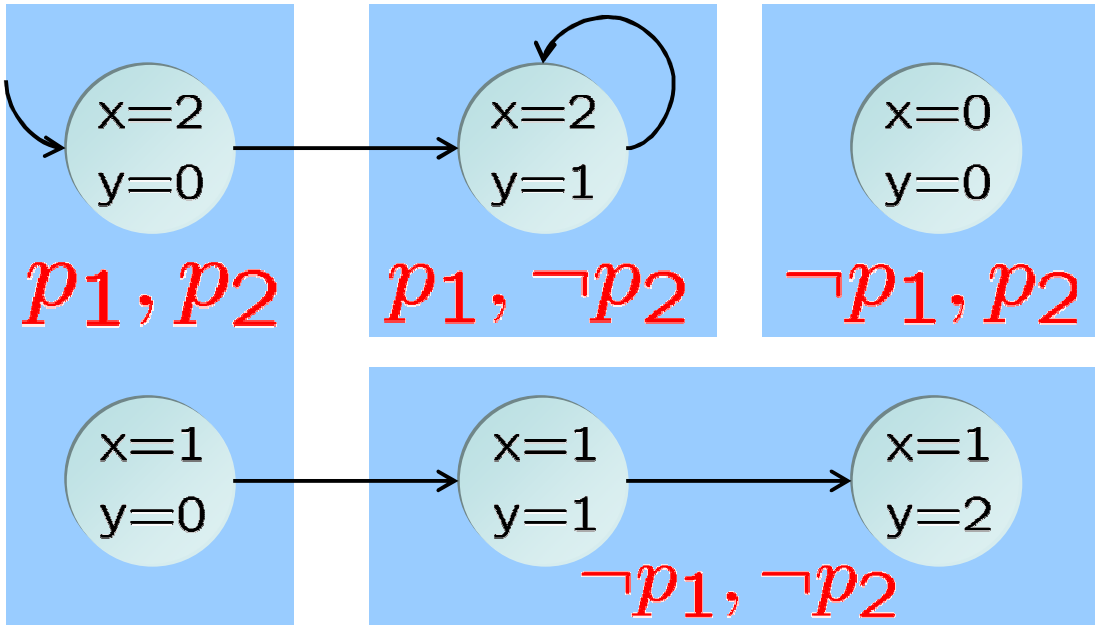
M



A

Predicate Abstraction

Concrete States:



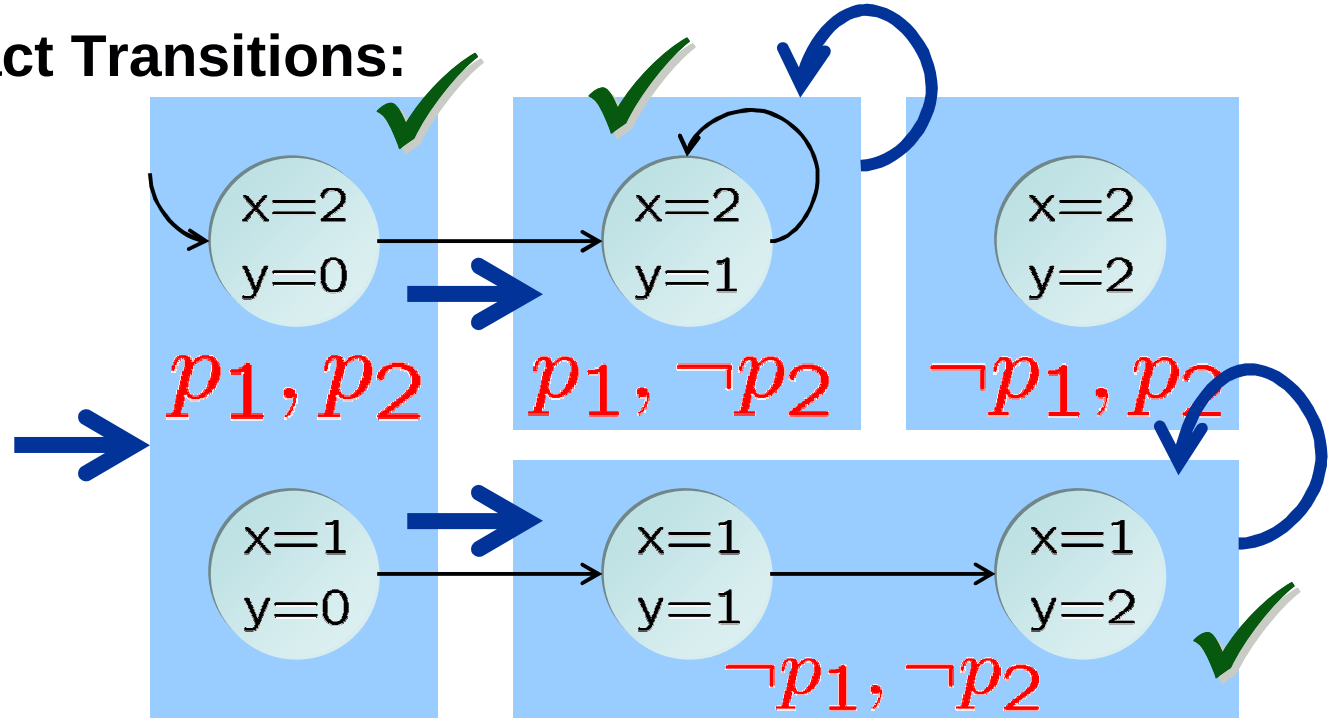
Predicates:

$$p_1(s) = (s.x > s.y)$$
$$p_2(s) = (s.y = 0)$$

Abstract transitions?

Predicate Abstraction

Abstract Transitions:



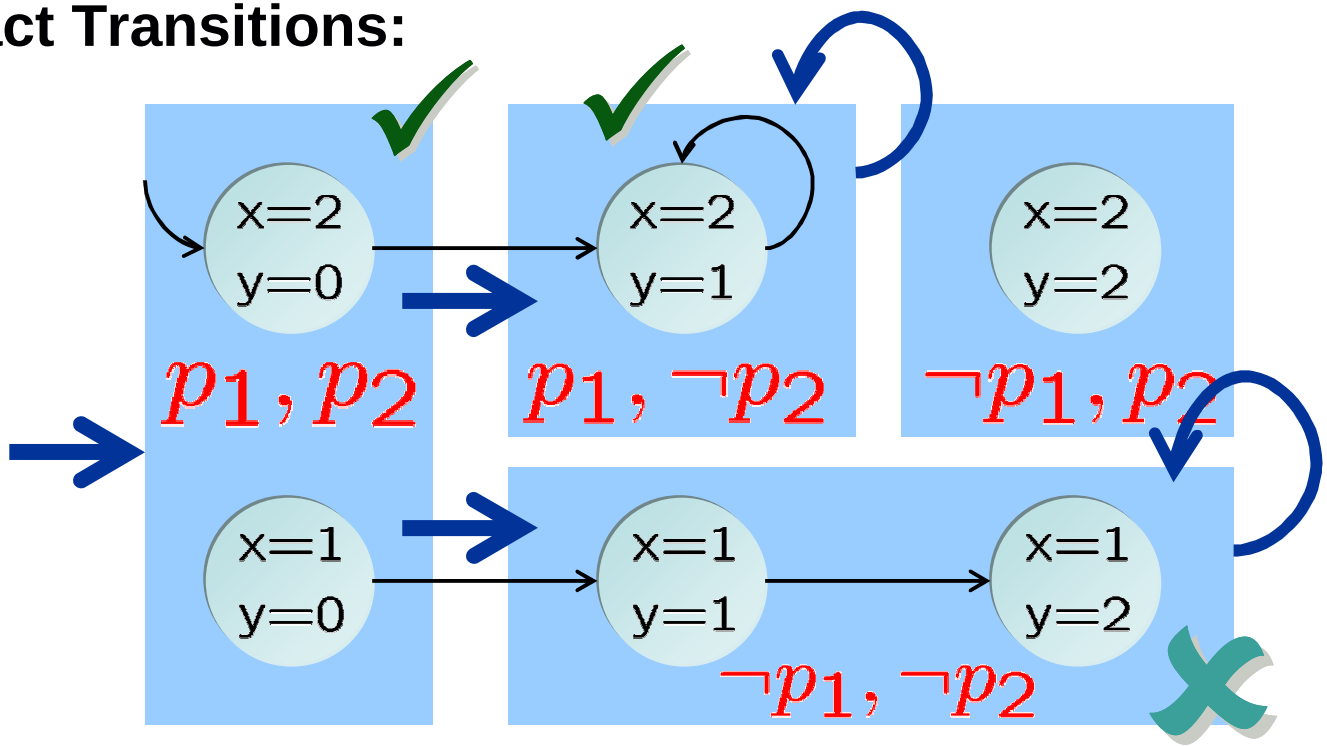
Property:

$$p_1 \vee \neg p_2 \quad \Longleftrightarrow \quad (s.x > s.y) \vee (s.y \neq 0)$$

Property holds. Ok.

Predicate Abstraction

Abstract Transitions:



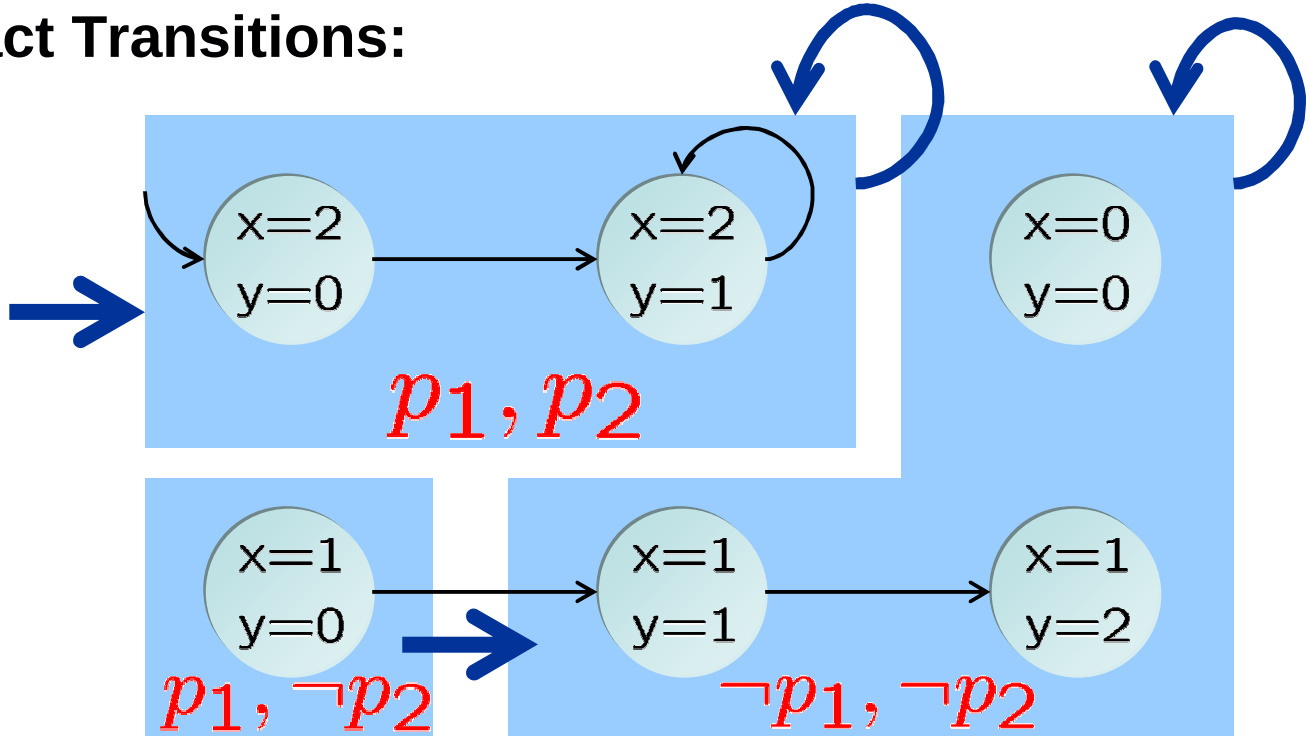
Property:

$$p_1 \iff (s.x > s.y)$$

This trace is spurious!

Predicate Abstraction

Abstract Transitions:



Property:

$$p_1 \iff (s.x > s.y)$$

New Predicates:

$$p_1(s) = (s.x > s.y)$$
$$p_2(s) = (s.x = 2)$$