Question 1 [Practice]. Consider the following program:

\[
\{ N \geq 0 \} \\
\text{i } = \text{ 0}; \\
\text{while } (\text{i } < \text{ N}) \{ \text{i } = \text{ N}; \} \\
\{ \text{i } = \text{ N} \}
\]

Which of the following conditions are loop invariants that are sufficient to prove the postcondition? For those that are incorrect, explain why.

A) \( i = 0 \)

B) \( i = N \)

C) \( N \geq 0 \)

D) \( i \leq N \)

Question 2. For the program above and the invariant \( i \leq N \), write the steps of the proof. The form of your answer should be three mathematical implications (third is on the last page).

Invariant is initially true:

Invariant is preserved by the loop body:
Invariant and exit condition imply postcondition:

**Question 3.** For each of the following loops, is the given variant function correct? If not, why not?

A) Loop: \( n = 256; \)
   \[\text{while } (n > 1) \{ n = n \div 2; \}\]
   
   Variant Function: \( \log_2 n \)

B) Loop: \( n = 100; \)
   \[\text{while } (n > 0) \{ \]
   \[\quad \text{if } \text{random()} \text{ then } \{ n = n + 1; \} \]
   \[\quad \text{else } \{ n = n - 1; \} \]
   \[\}\]
   
   Variant Function: \( n \)

C) Loop: \( n = 0; \)
   \[\text{while } (n < 10) \{ n = n + 1; \}\]
   
   Variant Function: \( -n \)