

15-214 - Principles of Software Construction: Objects, Design, and Concurrency
Class Participation Sheet
Hoare Logic, Part 2

Andrew IDs: _____

Question 1 [Practice]. Consider the following program:

```
{ N >= 0 }  
i = 0;  
while ( i < N ) { i = N; }  
{ i == N }
```

Which of the following conditions are loop invariants that are sufficient to prove the postcondition? For those that are incorrect, explain why.

- A) $i = 0$
- B) $i = N$
- C) $N \geq 0$
- D) $i \leq N$

Question 2. For the program above and the invariant $i \leq N$, write the steps of the proof. The form of your answer should be three mathematical implications (third is on the last page).

Invariant is initially true:

Invariant is preserved by the loop body:

Invariant and exit condition imply postcondition:

Question 3. For each of the following loops, is the given variant function correct? If not, why not?

A) Loop: `n = 256;`
`while (n > 1) { n = n / 2; }`

Variant Function: $\log_2 n$

B) Loop: `n = 100;`
`while (n > 0) {`
 `if (random()) then {n = n + 1; }`
 `else { n = n - 1; }`
`}`

Variant Function: n

C) Loop: `n = 0;`
`while (n < 10) { n = n + 1; }`

Variant Function: $-n$