Problem 1

Given a list of integer and rational numbers, write a function inserts the mediant between each two numbers of the list. The mediant of two rationals \( \frac{p_1}{q_1} \) and \( \frac{p_2}{q_2} \) is \( \frac{p_1}{q_1}. \frac{p_2}{q_2} \).

Commands: Partition, Numerator, Denominator, Flatten

\[ \text{Solution} \]

\[
\text{mediant}[[x_, y_]] := \{x, \\
(Numerator[x] + Numerator[y]) / (Denominator[x] + Denominator[y]), y\}
\]

\[
\text{mediant}[[0, 1]]
\]

\[
\{0, \frac{1}{2}, 1
\]

\[
\text{Partition}[[0, 1/2, 1], 2, 1]
\]

\[
\{\{0, \frac{1}{2}, \frac{1}{2}, 1
\]

\[
\text{mediant} /\@ \%
\]

\[
\{\{0, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{2}{3}, 1
\]

\[
\text{mediant} /\@ \%
\]

\[
\text{mediant} \{0, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{2}{3}, 1
\]

\[
\text{insert[e_List]} := \text{Flatten[mediant /@ Partition[e, 2, 1]]}
\]

\[
\text{insert}[[0, 1]]
\]

\[
\{0, \frac{1}{2}, 1
\]
Problem 2

Five sailors stranded on an island gather coconuts all day long. They decide to divide the pile the next morning. However, one of the sailors wakes up in the middle of the night and decides to make sure he gets his fair share: he divides the coconuts into 5 piles of equal size, with one nut left over -- which he gives to a monkey. He hides his share, and then goes back to sleep. A while later, another sailor wakes up, and does exactly the same. And so on, they all wake up at some point and go through the same procedure. In the morning, they split the remaining nuts into 5 piles of equal size, with none left over.

What is the least possible number of coconuts in the original pile?

√ Solution

Clear[f, a];
f[x_] := 4/5 (x - 1)

Expand[Nest[f, a, 5]]

- \frac{8404}{3125} + \frac{1024 a}{3125}

\frac{8404}{3125} + \frac{1024 a}{3125} == 5 n

\frac{8404}{3125} + \frac{1024 a}{3125} == 5 n

Solve[%, a]

\{a \rightarrow \frac{8404 + 15625 n}{1024}\}
Problem 3

The following function \texttt{dasz} (decrement, append, sort, remove zero) is defined on lists of non-negative integers. It performs the following actions, in this order:

1. subtract 1 from all the numbers in the list,
2. append the length of the list to the list,
3. sort everything,
4. remove all 0 entries from the front of the list.

For example, 
\{1, 3, 5\} \rightarrow \{0, 2, 4\} \rightarrow \{0, 2, 4, 3\} \rightarrow \{0, 2, 3, 4\} \rightarrow \{2, 3, 4\}

A) Implement the function \texttt{dasz} in Mathematica.
B) Find all the fixed points of \texttt{dasz}. Points such that \texttt{dasz list = list}:
C) Consider lists of the form \{i\} and find such \texttt{i} which leads to the fixed point.

Commands: Append, DeleteCases

\textbf{Solution}

\begin{verbatim}
Clear[dasz];
dasz[e_] := DeleteCases[Sort[Append[e - 1, Length[e]]], 0]
dasz[{-3, 3, 3}]
{-4, 2, 2, 3}

B)
dasz[{1, 2, 3, 4}]
{1, 2, 3, 4}

C]
\end{verbatim}
Table[FixedPoint[dasz, {k}, 50], {k, 1, 15}]

{{{1}, {2}, {1, 2}, {2, 2}, {2, 3}, {1, 2, 3}, {1, 1, 2, 3},
{1, 2, 2, 3}, {1, 2, 3, 3}, {1, 2, 3, 4}, {1, 1, 2, 3, 4},
{1, 2, 2, 3, 4}, {1, 2, 3, 3, 4}, {1, 2, 3, 4, 4}, {1, 2, 3, 4, 5}}

{1, 3, 6, 10, 15}

Table[k (k + 1) / 2, {k, 1, 5}]

{1, 3, 6, 10, 15}