# Introduction to Graphs

*15-121*

*Introduction to Data Structures*

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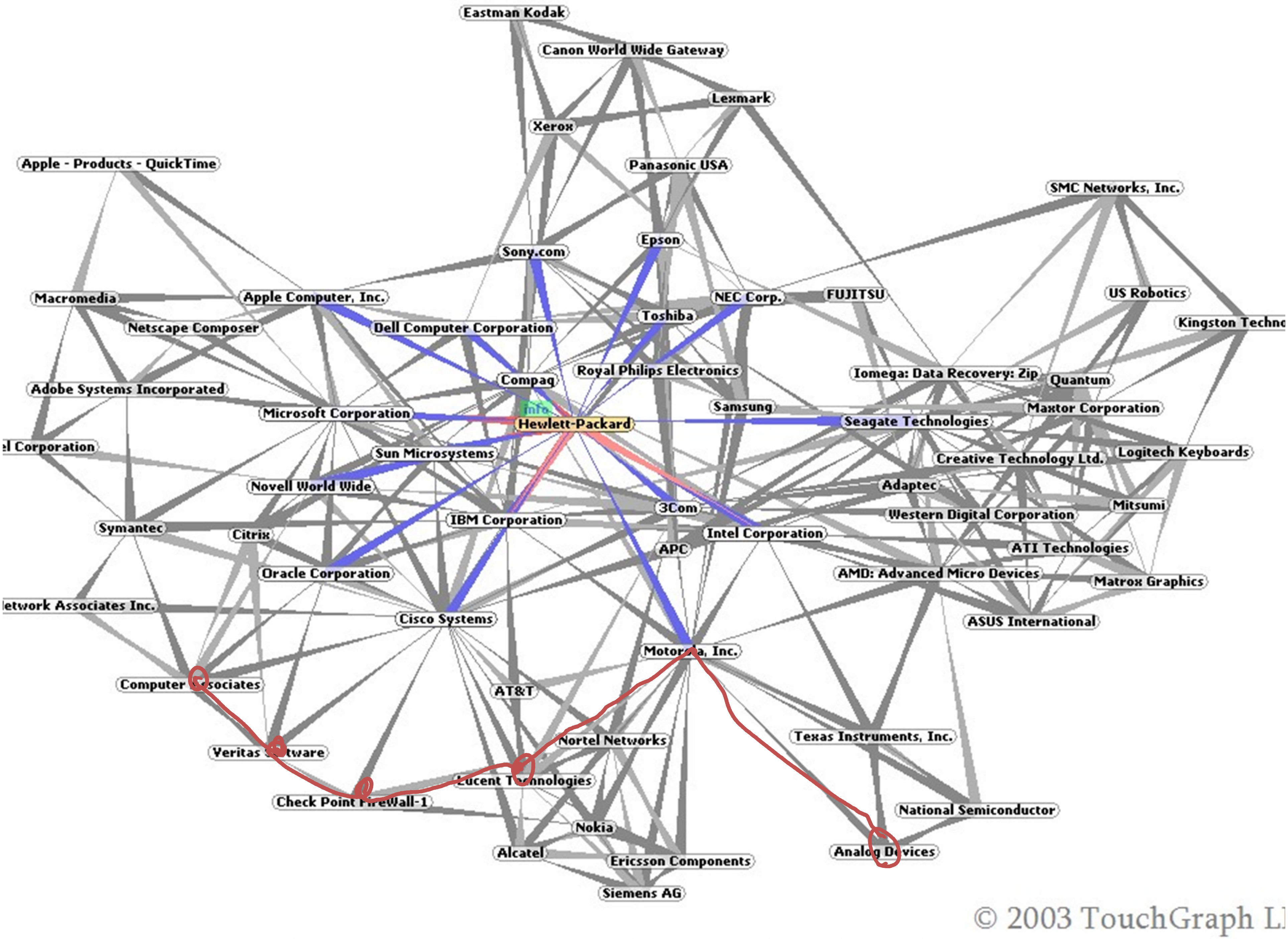
# Graphs are everywhere

# An Airline route Map

edge (cost?)



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# Finding the Shortest Path
# Lots of applications

# Many real world problems can be modeled using graphs

- **Airline Route Map**
  - What is the fastest way to get from Pittsburgh to St Louis?
  - What is the cheapest way to get from Pittsburgh to St Louis?

- **Electric Circuits**
  - Circuit elements - transistors, resistors, capacitors
  - is everything connected together?
    - Depends on interconnections (wires)
  - If this circuit is built will it work?
    - Depends on wires and objects they connect.

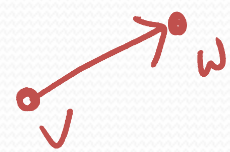
# Graph Definitions

- Graph
  - A set of vertices(nodes) $V = \{v_1, v_2, \ldots, v_n\}$
  - A set of edges(arcs) that connects the vertices $E=\{e_1, e_2, \ldots, e_m\}$
  - Each edge $e_i$ is a pair $(v, w)$ where v, w in V
  - $|V|$ = number of vertices (cardinality)
  - $|E|$ = number of edges

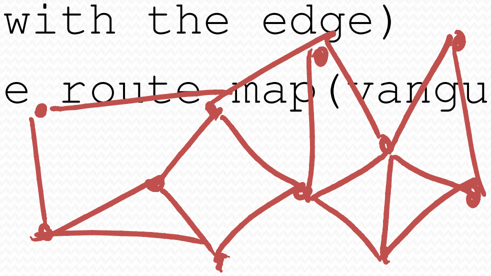
$O(|V|)$

$O(|E| \log |E|)$

- Graphs can be
  - directed (order (v,w) matters)
  - Undirected (order of (v,w) doesn't matter)
- Edges can be

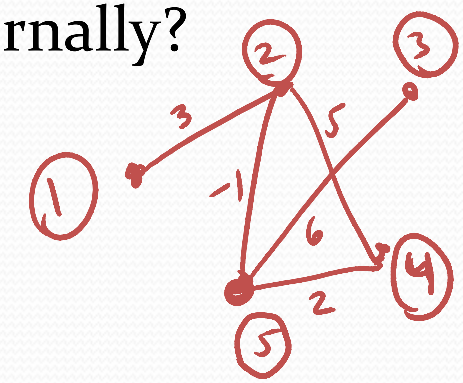

  - weighted (cost associated with the edge)
  - eg: Neural Network, airline route map(vanguard airlines)



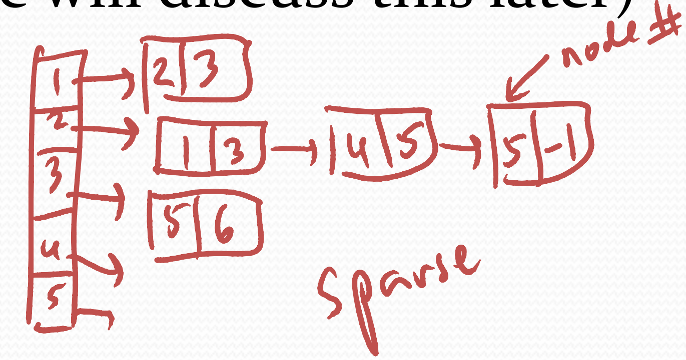
# Graph Representations

# Graph Representation

- How do we represent a graph internally?
- Two ways
  - adjacency matrix
  - Adjacency list




- Adjacency Matrix
  - Use matrix entries to represent edges in the graph
- Adjacency List
  - Use an array of lists to represent edges in the graph (we will discuss this later)




sparse


dense

# Adjacency Matrix

- Adjacency Matrix
  - For each edge (v,w) in E, set A[v][w] = edge_cost
  - Non existent edges with logical infinity
- Cost of implementation
  - $O(|V|^2)$ time for initialization
  - $O(|V|^2)$ space
    - ok for dense graphs
    - unacceptable for sparse graphs

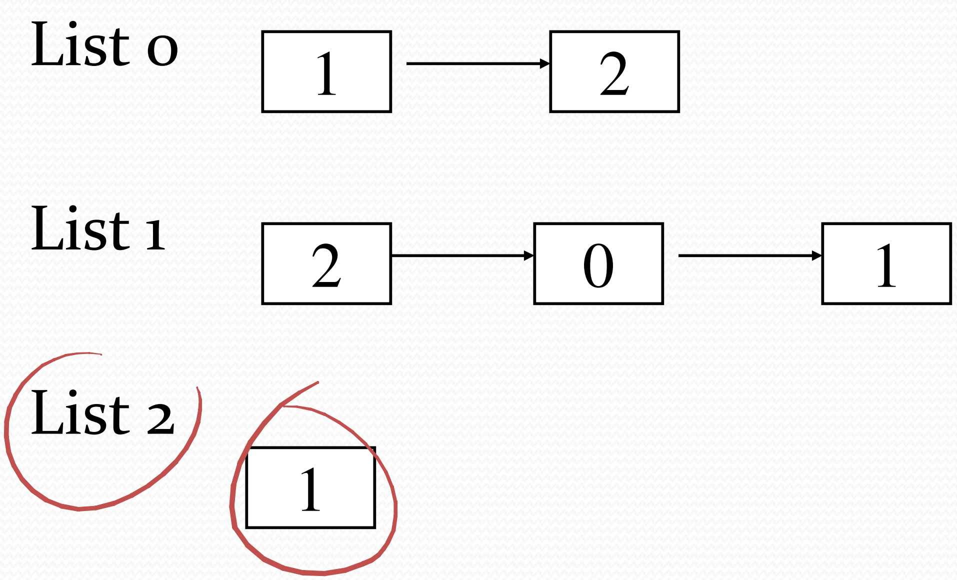
$\infty = 2^{31} - 1$

$O(n^2)$ $|V| = n$

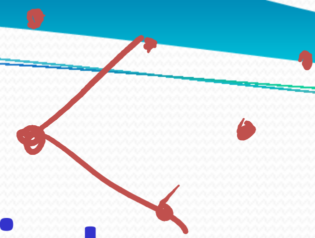
$$\begin{bmatrix} \infty & \infty & \infty & \infty \\ & & & \\ & & & \end{bmatrix}_{n \times n}$$

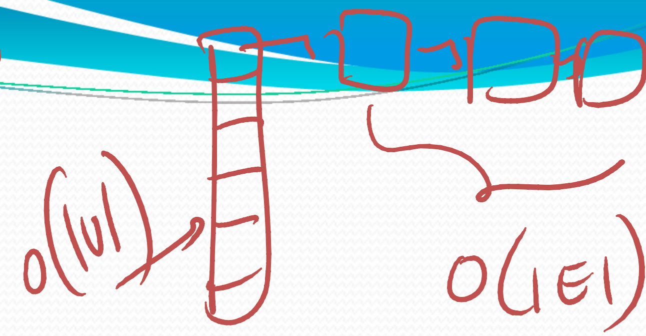
# Adjacency List

- Adjacency List
  - Ideal solution for sparse graphs
  - For each vertex keep a list of all adjacent vertices
  - Adjacent vertices are the vertices that are connected to the vertex directly by an edge.
  - Example



Cost



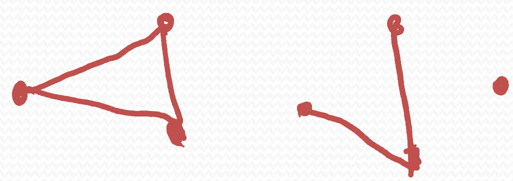



# Adjacency List

- The number of list nodes equals to number of edges
  - O(|E|) space
- Space is also required to store the lists
  - O(|V|) for |V| lists

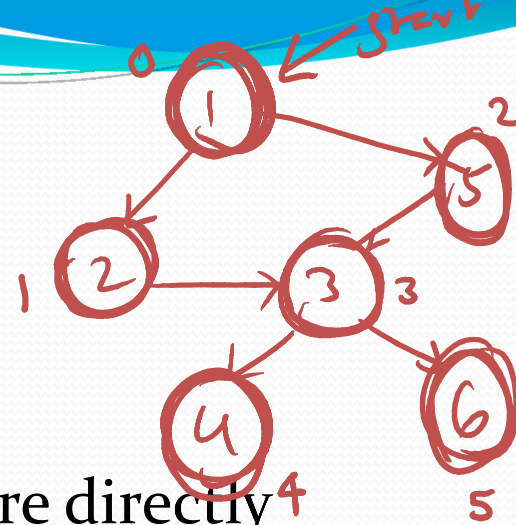
Space complexity

- Note that the number of edges is at least round(|V|/2)
  - assuming each vertex is in some edge
  - Therefore disregard any O(|V|) term when O(|E|) is present
- Adjacency list can be constructed in linear time (wrt to edges)

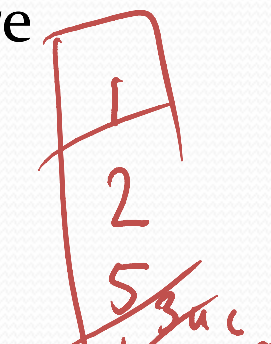


level order

# Breadth First Traversal

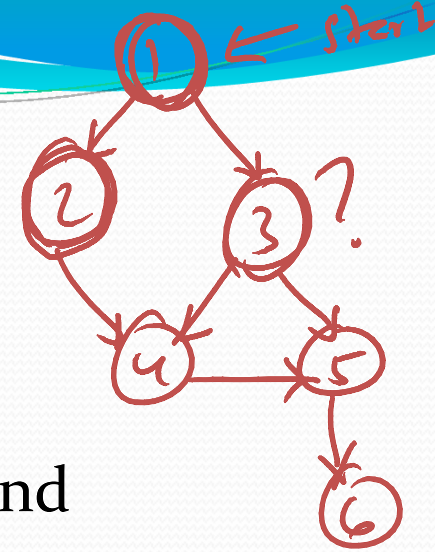



- Algorithm
  - Start from any node in the graph
  - Traverse to its neighbors (nodes that are directly connected to it) using some heuristic
  - Next traverse the neighbors of the neighbors etc.. Until some limit is reach or all the nodes in the graph are visited
  - Use a queue to perform the breadth first traversal




enque(Start);
While (!Q.empty()) {
deque(); process()
enque its children if its not in the queue AND !processed
}

# Depth First Traversal




- Algorithm
  - Start from any node in the graph
  - Traverse deeper and deeper until dead end
  - Back track and traverse other nodes that are not visited
  - Use a stack to perform the depth first traversal

1 2 4 5 6 3

# Next: Graph Algorithms