15-121
Introduction to data structures - Fast Sorting

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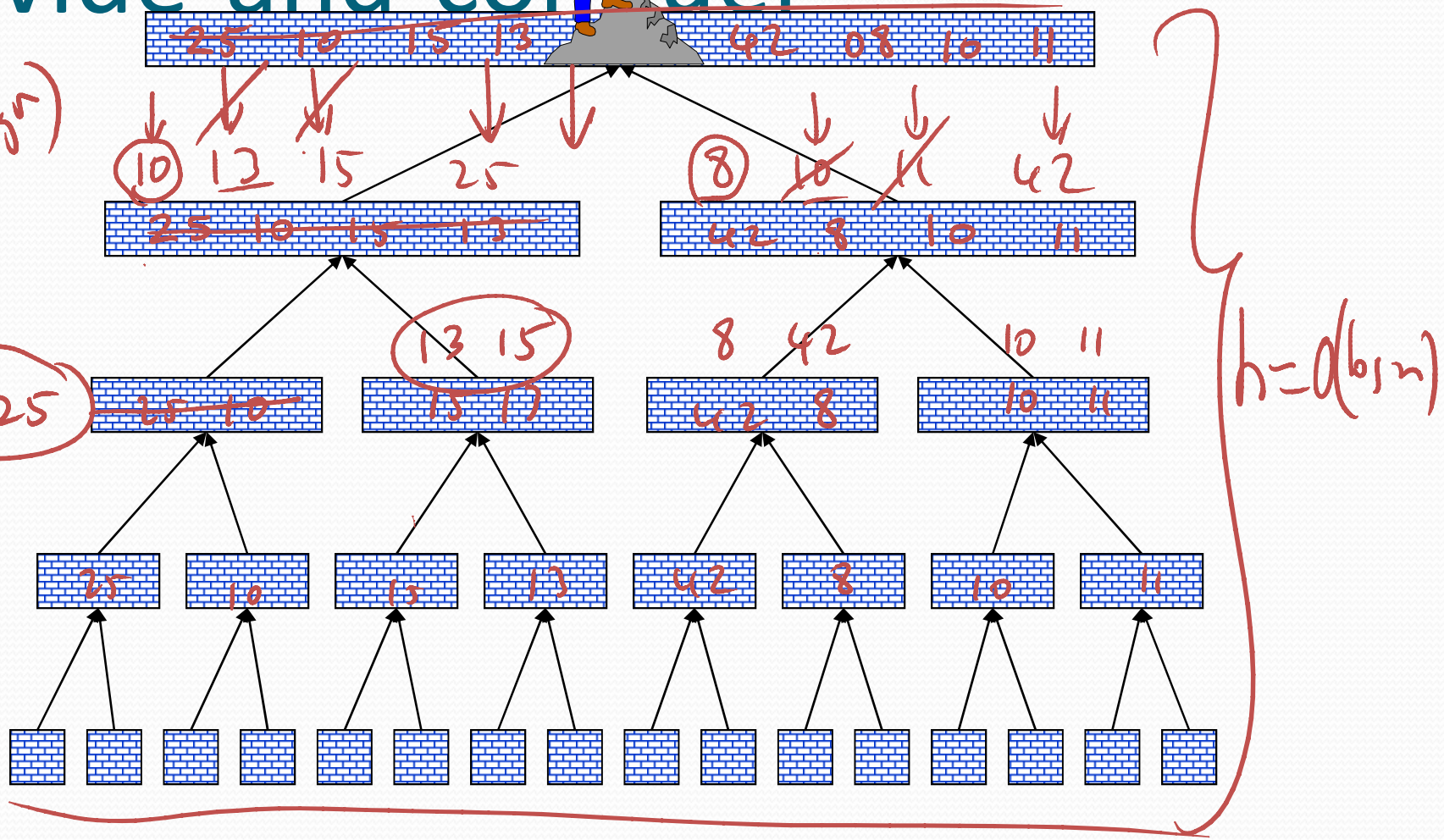
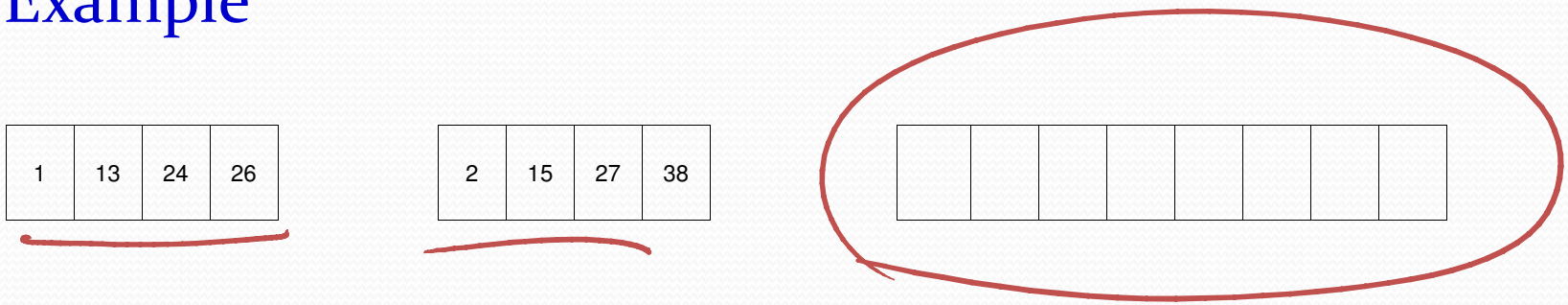
Merge Sort

Divide-and-conquer



Merging Two Sorted Arrays

- All the work in merge sort is done at the merge step.
- Example



Why Merge sort is bad

→ Extra memory to merge

→

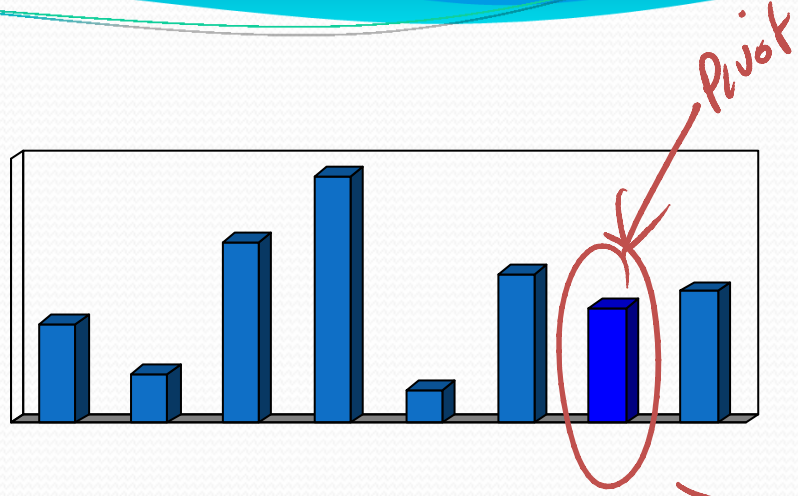
Quick Sort

Quicksort

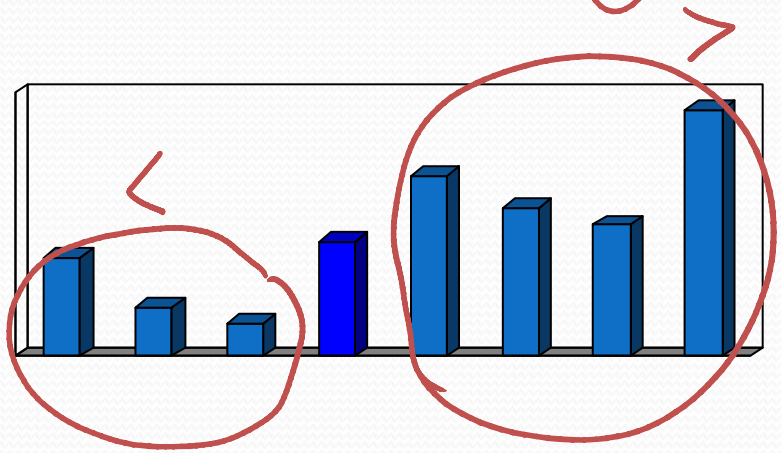
- Quicksort was invented in 1960 by Tony Hoare.
- Quicksort has $O(N^2)$ worst-case performance, and on average $O(N \log N)$.
- *More importantly, it is the fastest known comparison-based sorting algorithm in practice.*

Quicksort idea

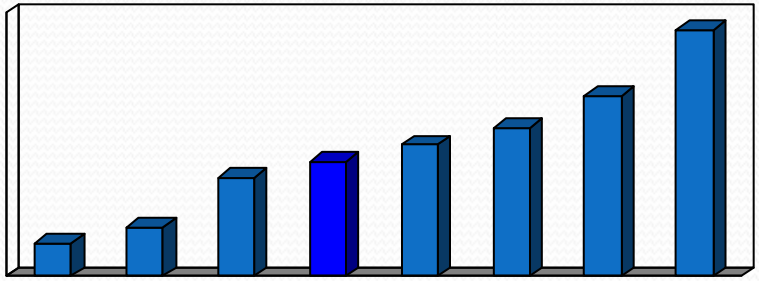
- Choose a pivot.



- Rearrange so that pivot is in the “right” spot.



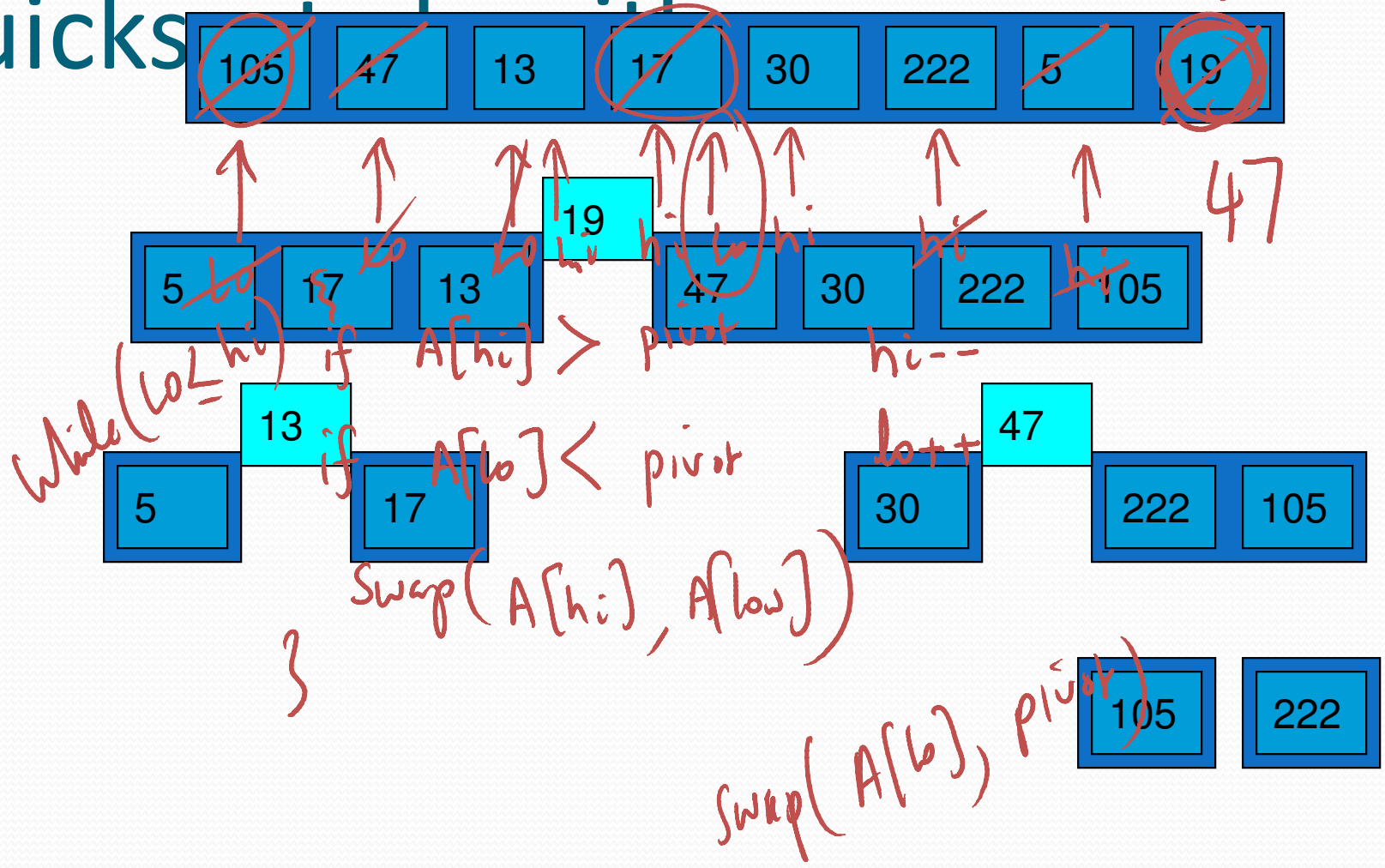
- Recurse on each half and conquer!



Quicksort algorithm

- If array A has 1 (or 0) elements, then done.
- Choose a *pivot element* x from A.
- Divide A-{x} into two arrays:
  - $B = \{y \in A \mid y \leq x\}$
  - $C = \{y \in A \mid y \geq x\}$
- Result is B+{x}+C.
- Recurse on arrays B and C

Quicksort


105
47
13
17
30
222
5
19
19
5
17
13
47
30
222
105
13
5
17
47
30
222
105
105
222
Pivot
while(lo<=hi)
if A[hi] > pivot
hi--
if A[lo] < pivot
lo++
swap(A[hi], A[low])
swap(A[lo], pivot)

Place the pivot algorithm

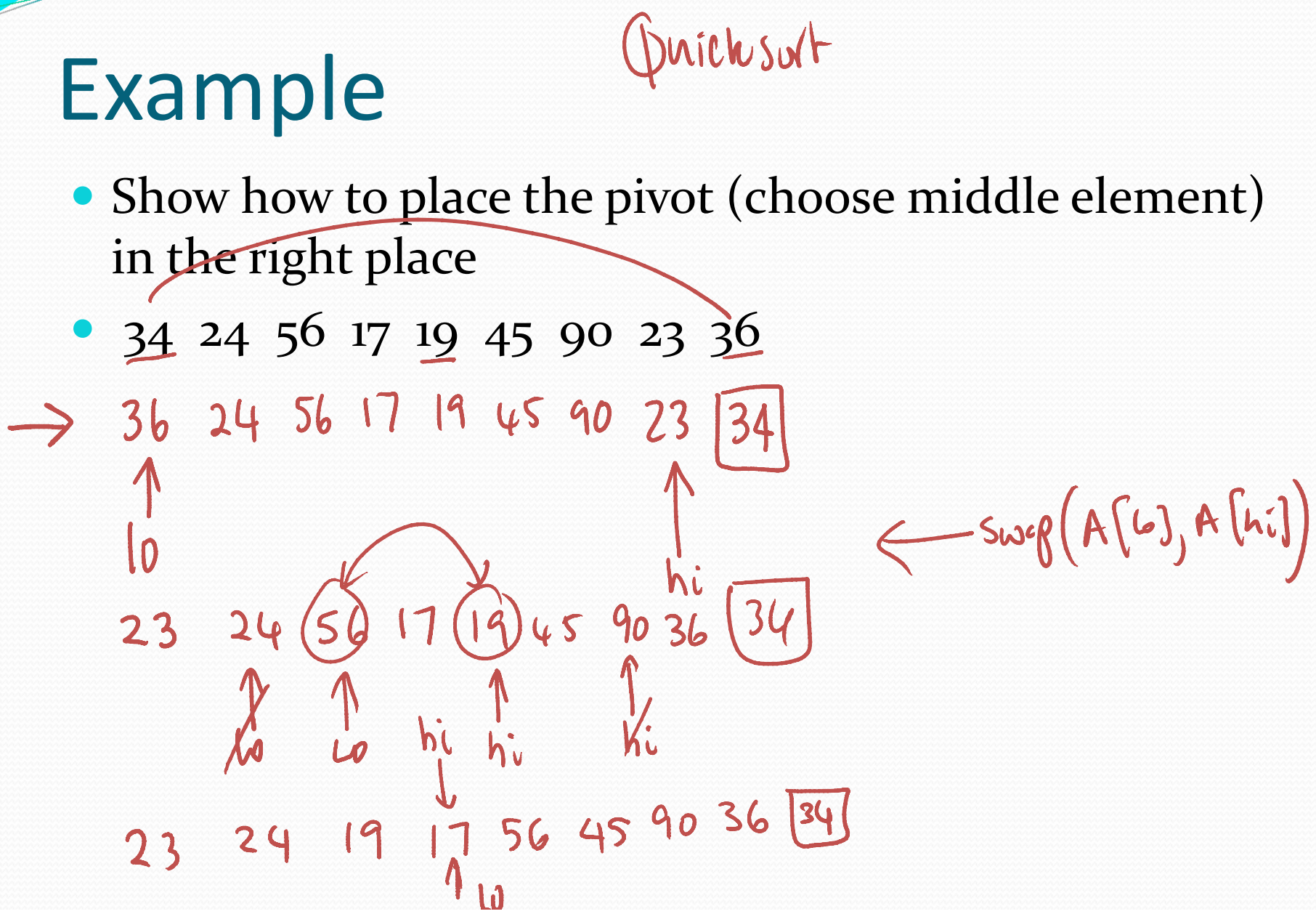
- Assume we have an array of size n
- Pick a pivot x
- Place the pivot x in the last place of the array
- Have two pointers i = 0, j = n-2

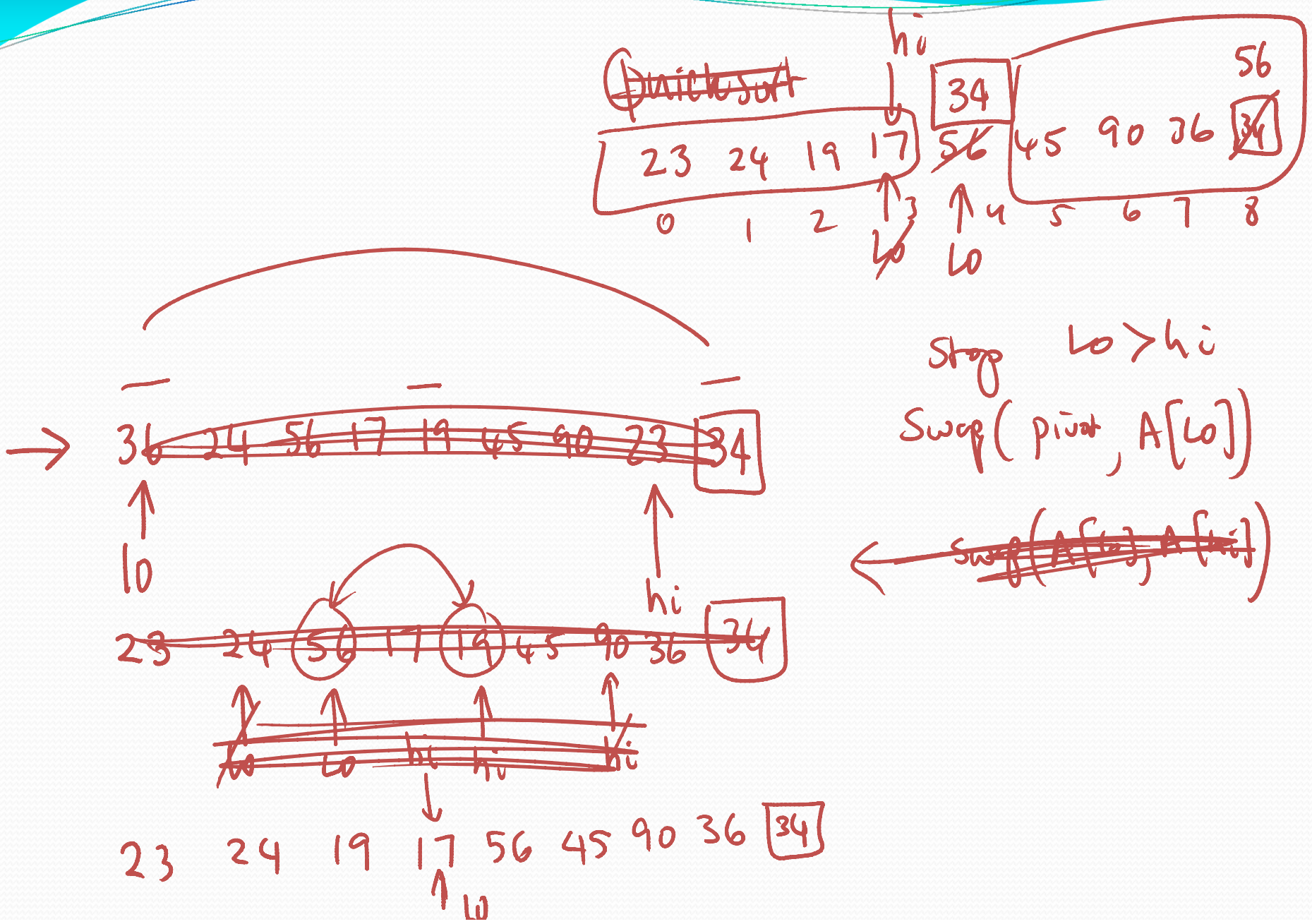
```
while ( i <= j) {
   while (A[j] > pivot)  j--;
   while (A[i] < pivot) i--;
   swap (A[i], A[j]);
}
swap (A[i], pivot);
```

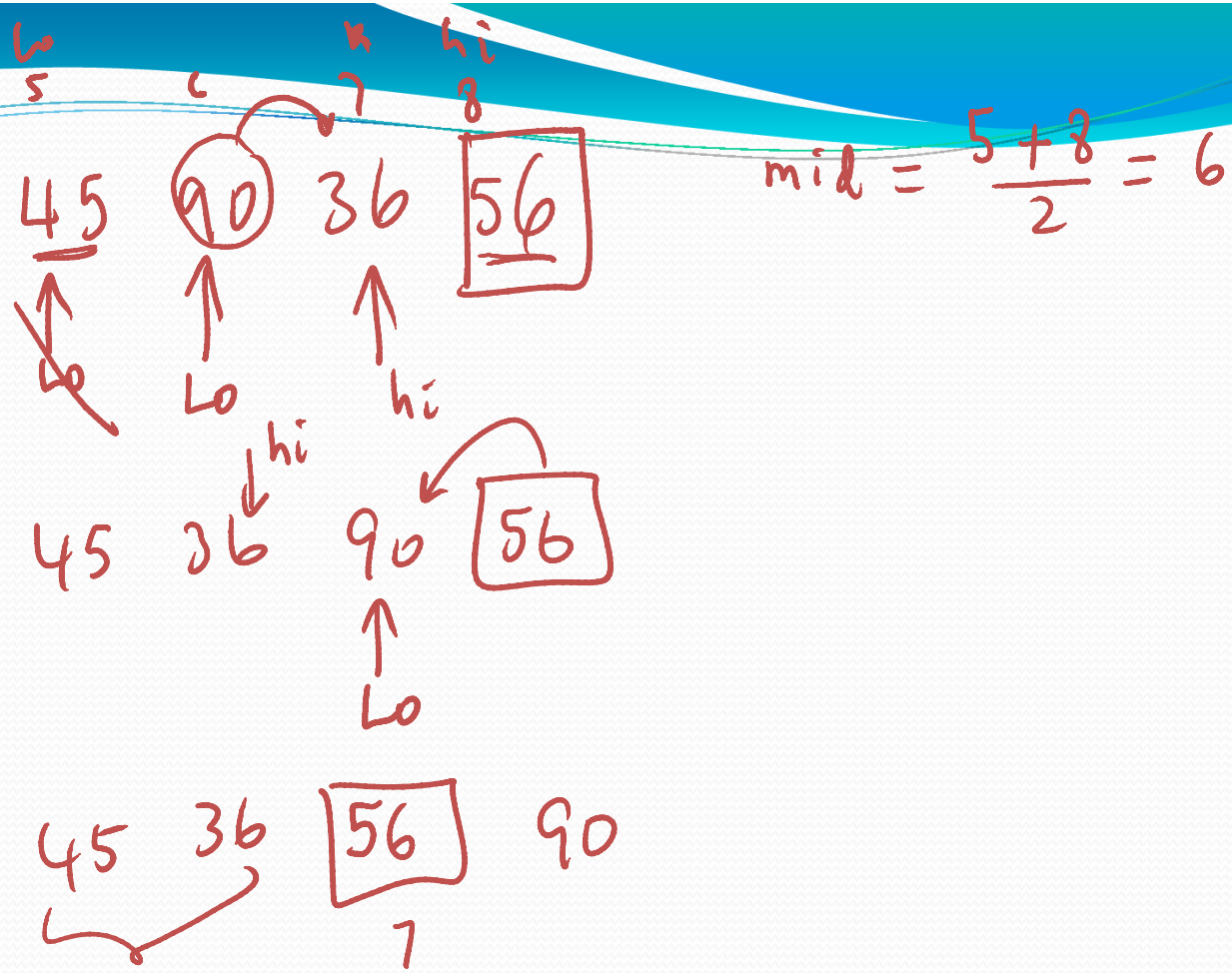
Example

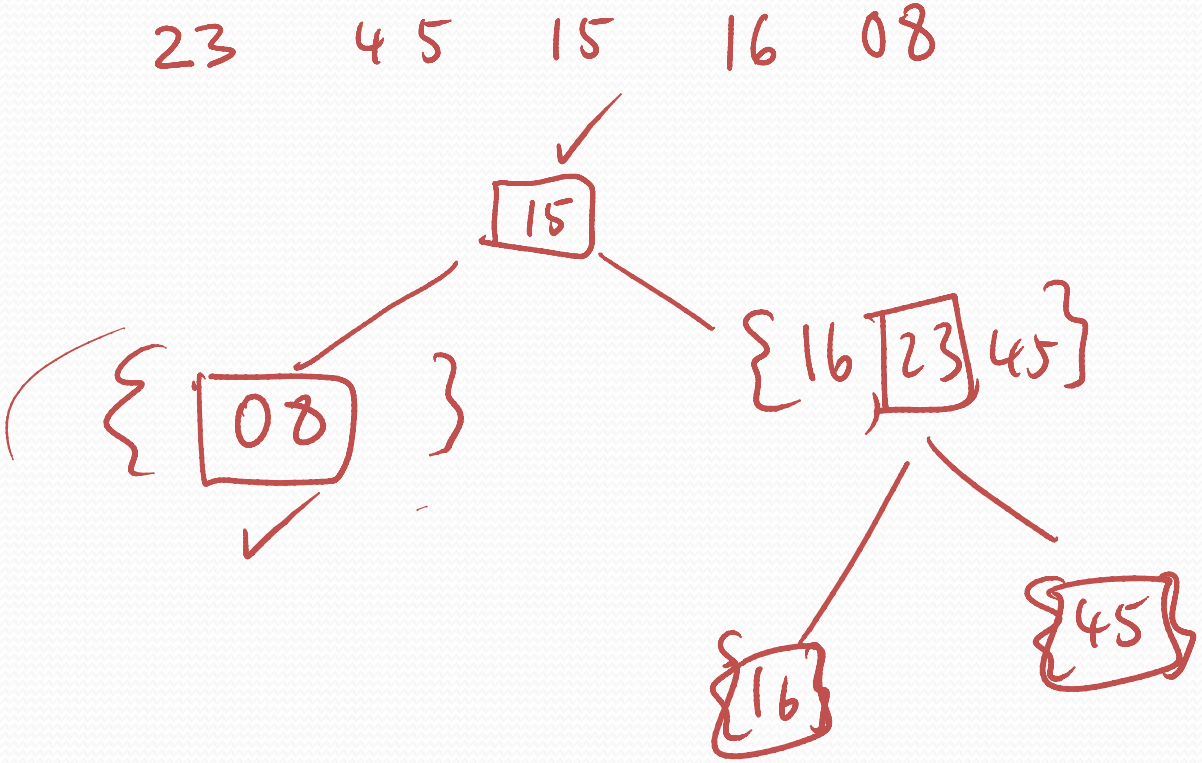
Quicksort

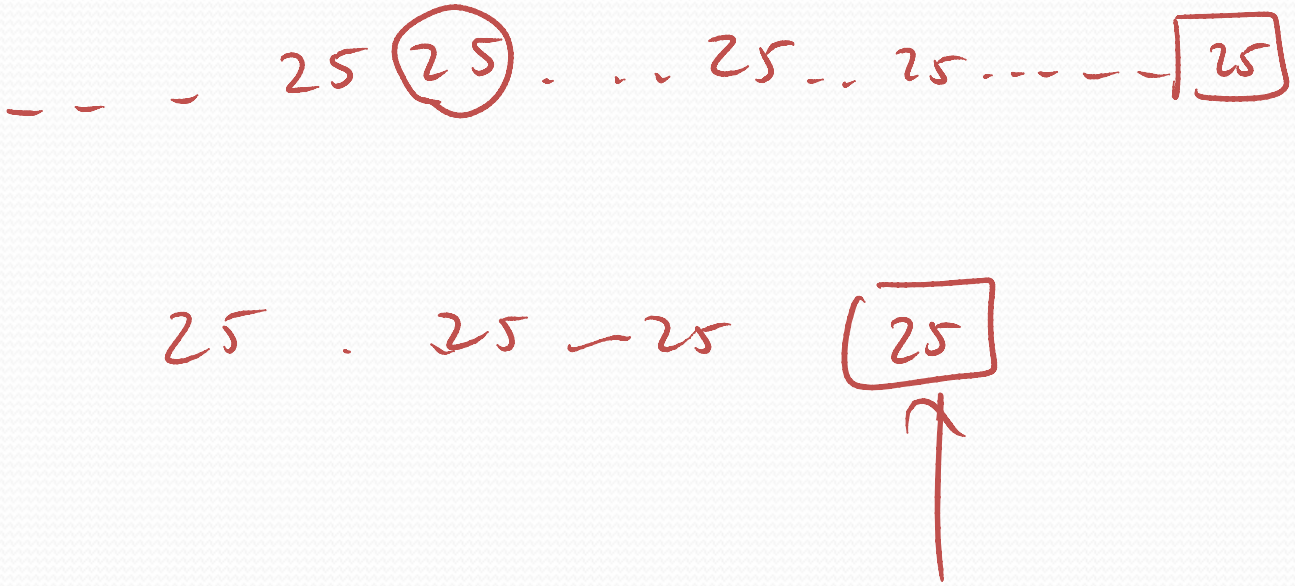
- Show how to place the pivot (choose middle element) in the right place
- 34 24 56 17 19 45 90 23 36




hi
34
56
23 24 19 17 56 45 90 36 34
0 1 2 3 4 5 6 7 8
Lo
Stop Lo > hi
Swap(pivot, A[Lo])
36 24 56 17 19 45 90 23 34
Lo
hi
23 24 56 17 19 45 90 36 34
23 24 19 17 56 45 90 36 34
Lo


Lo
5
c
k
7
hi
8
45
90
36
56
Lo
hi
mid = 5+8/2 = 6
hi
45
36
90
56
Lo
45
36
56
90
7


23 45 15 16 08
15
08
16 23 45
16
45


25 25 25 25 25
25 25 25
25

Survey of Sorting

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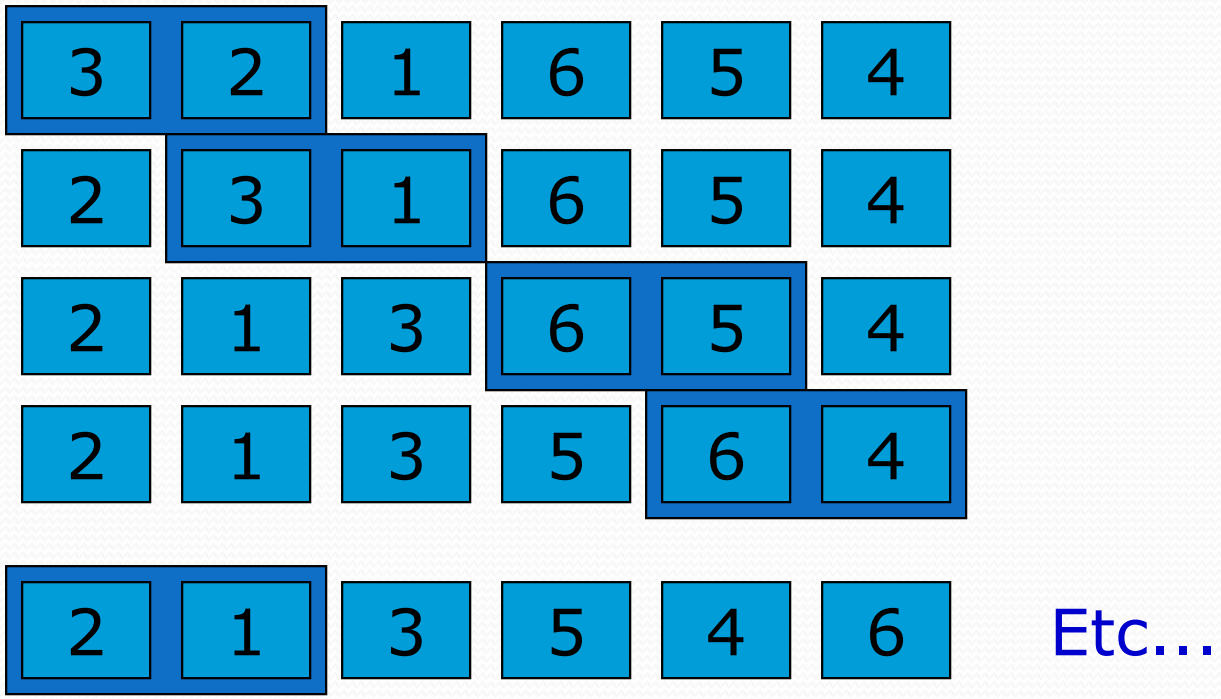
Naïve sorting algorithms

- Bubble sort: *scan for flips, until all are fixed*



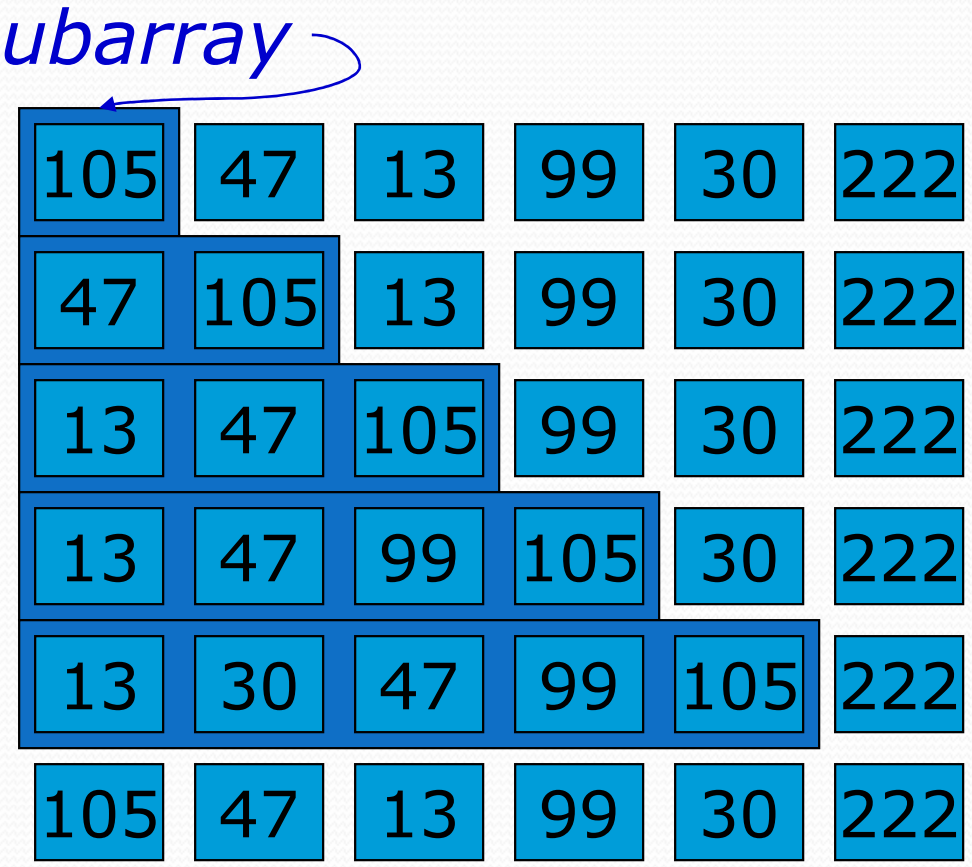
Naïve Sorting

```
for i=1 to n-1
  { for j=0 to n-i-1
      if (A[j].compareTo(A[j+1])>0)
          swap(A[j], A[j+1]);
    if (no swaps) break;
  }
```

- What happens if
  - All keys are equal?
  - Keys are sorted in reverse order?
  - Keys are sorted?
  - keys are randomly distributed?

- Exercise: Count the number of operations in bubble sort and find a Big O analysis for bubble sort

Insertion sort



Insertion sort

- **Algorithm**

```
for   i = 1   to   n-1   do
      insert a[i] in the proper place
           in   a[0:i-1]
```

- **Correctness**
  - Note: after i steps, the sub-array A[0:i] is sorted

How fast is insertion sort?

To insert a[i] into a[0:i-1], **slide** all elements larger than a[i] to the right.

```
tmp = a[i];
for (j = i; j>0 && a[j-1]>tmp; j--)
      a[j] = a[j-1];
a[j] = tmp;
```

# of slides = O(#inversions)

very fast if array is nearly sorted to begin with

Selection sort

- **Algorithm**

```
for  i = n-1  to  1  do
     Find the largest entry in the
          in  the subarray A[0:i]
     Swap with A[i]
```

What is the runtime complexity of selection sort?

Sorting Comparison

- Discuss the pros and cons of each of the naïve sorting algorithms

Advanced Sorting

Quick Sort

- Fastest algorithm in practice
- Algorithm
  - Find a pivot
  - Move all elements smaller than pivot to left
  - Move all elements bigger than pivot to right
  - Recursively sort each half
  - O(n log n) algorithm

Merge Sort

- Divide the array into two equal halves
- Divide each half recursively until each array is of size 1
- Merge two (sorted) arrays of size 1
- Complete the process recursively

Heap Sort

- Build a max heap
- Delete Max (attach to end of array) until heap is empty
- Resulting array is sorted
- Complexity

Radix sort characteristics

- Each sorting step can be performed via bucket sort, and is thus $O(N)$.
- If the numbers are all b bits long, then there are b sorting steps.
- Hence, radix sort is $O(bN)$.

What about non-binary?

- Radix sort can be used for decimal numbers and alphanumeric strings.