Binary Search Trees

15-111

Data Structures

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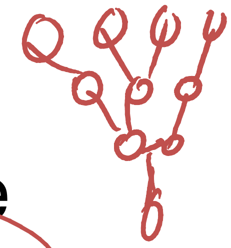
$8 \times 10^9 = 2^3 \times (2^{20}) . (2^{10}) = 2^{33}$ $\log_2 8 \times 10^9 = 33$

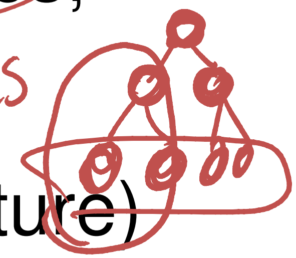
Real



Tree Data Structure

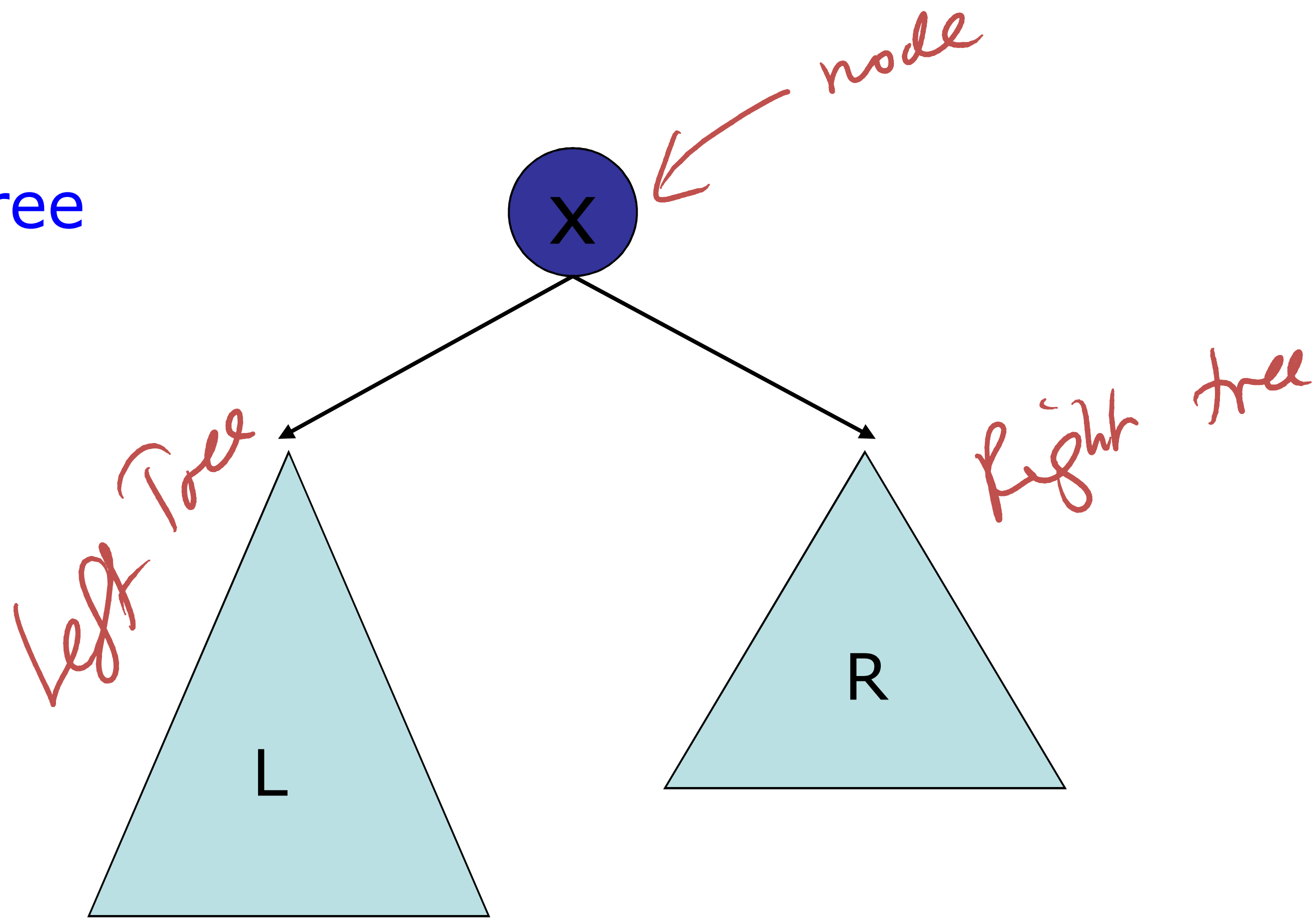
- A non-linear data structure that follows the shape of a tree (i.e. root, children, branches, leaves etc)
- Most applications require dealing with hierarchical data (eg: organizational structure)
- Trees allows us to find things efficiently
  - Navigation is O(log n) for a "balanced" tree with n nodes
- A Binary Search Tree (BST) is a data structure that can be traversed / searched according to an order
- A binary tree is a tree such that each node can have at most 2 children.

CS



BST In Pictures

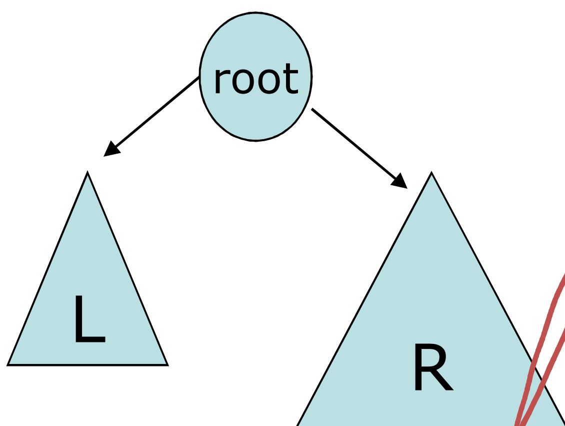
Empty Tree



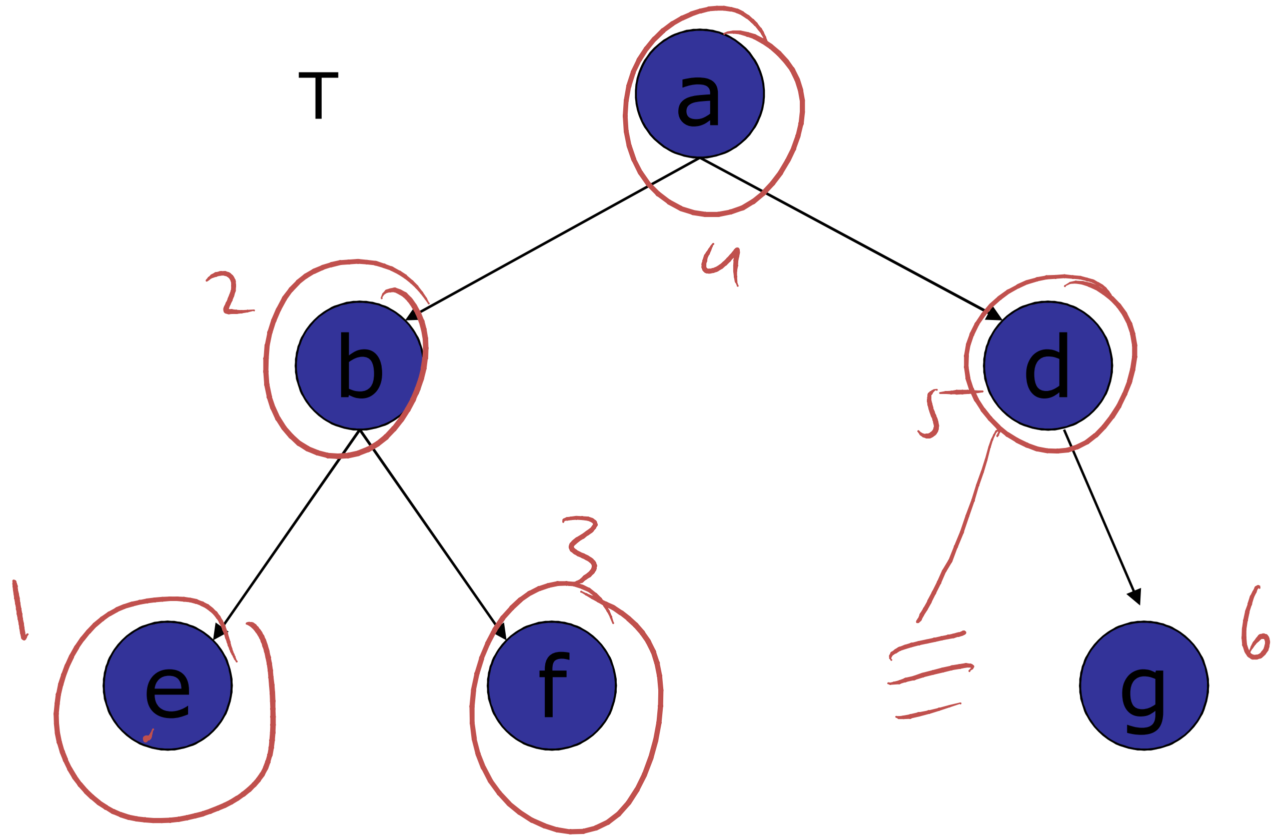
Definition of Flat T

- Given a Binary Tree T, Flat(T) is a sequence obtained by traversing the tree using inorder traversal
  - That is for each node, recursively visit
    - Left Tree, Then Root, then Right Tree

{7 4 2 5 1 6 8 3}

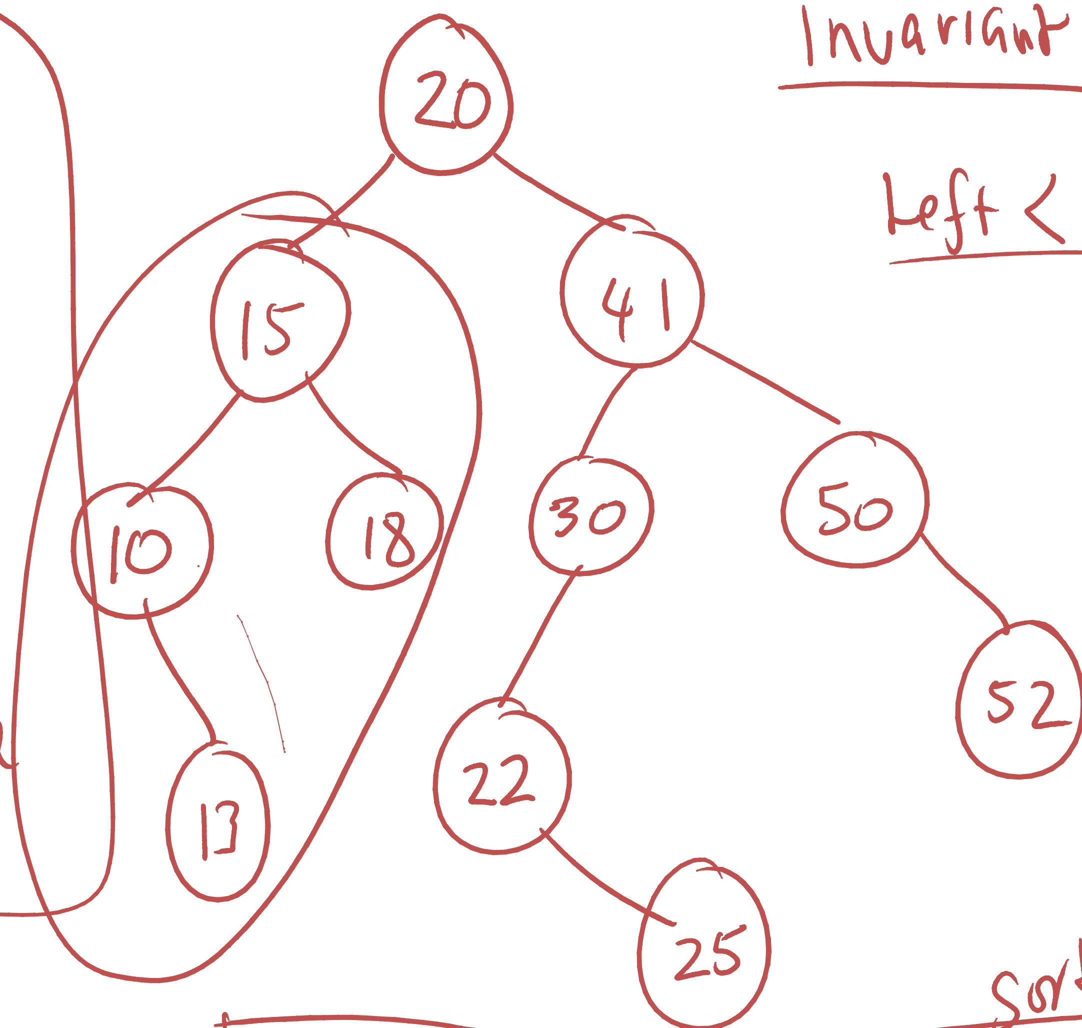


Flattening a BT



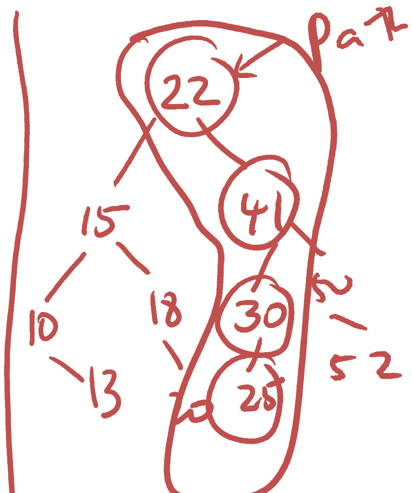
flat(T) = e, b,f,a,d,g

Given n elements how many BST's can be produced



Invariant

Left < Parent < Right



25?

Sorted

$Flat(T) = \{10, 13, 15, 18, 20, \cancel{25}, 22, \cancel{22}, 25, 30, 41, 50, 52\}$

Def: Binary Search Tree

A binary Tree is a binary search tree (BST)
if and only if

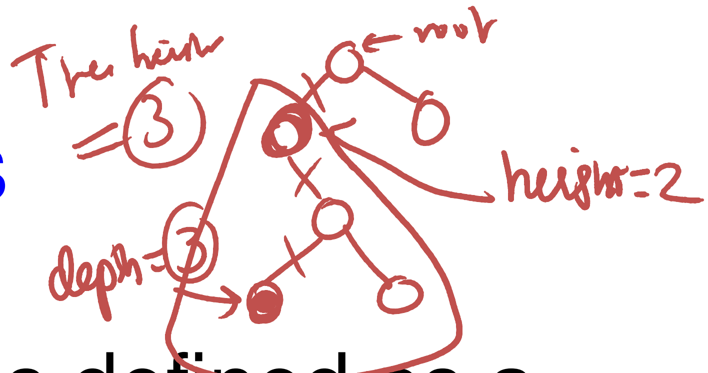
flat(T) is an ordered sequence.

Equivalently, in (x,L,R) all the nodes in L are less than x, and all the nodes in R are larger than x.

$L < x < R$

BT Definitions

Definitions



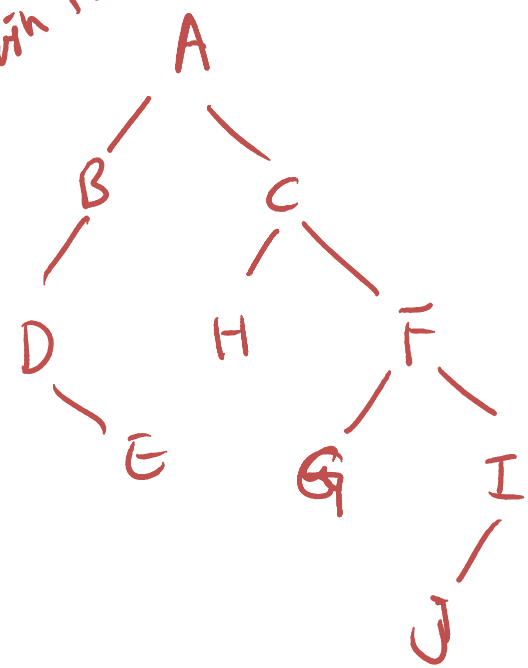
- A path from node n_1 to n_k is defined as a path $n_1, n_2, \ldots, n_k$ such that n_i is the parent of n_{i+1}

total edges in the path

- Depth of a node is the length of the path from root to the node.
- *Height of a node* is length of a path from node to the deepest leaf.
- Height of the tree is the height of the root

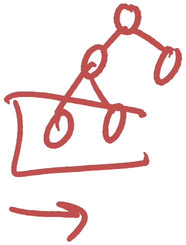
height Tree

Node	depth	height
A	0	4
B	1	2
C	1	3
D	2	1
E	3	0
F	2	2
G	3	0
I	3	1
J	4	0

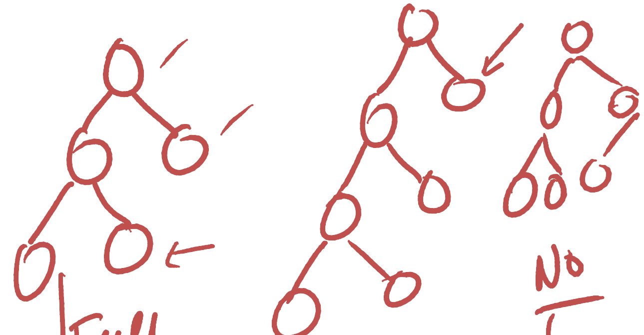


No Children

= leaf Nodes



Definitions

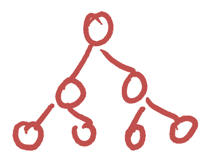


Full Full No

- **Full Binary Tree** Huffman coding
  - Each node has zero or two children
- **Complete Binary Tree** Heaps
  - All levels of the tree is full, except possibly the last level, where nodes are filled from left to right

Complete = yes NO yes

- **Perfect Tree**
  - A Tree is perfect if total number of nodes in a tree of height h is 2^{h+1} -1

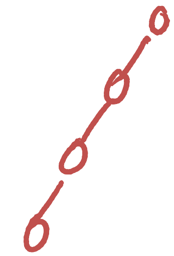


$h=2$ $n=2^{2+1}-1$

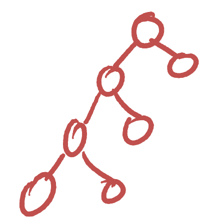
$=7$

$$1+2+2^2+2^3+\cdots+2^h=\frac{2^{h+1}-1}{2-1}$$

Binary Tree Questions



- What is the maximum height of a binary tree with n nodes? What is the minimum height? n-1 $O(\log n)$
- What is the minimum and maximum number of nodes in a binary tree of height h? $h+1$ $2^{h+1}-1$
- What is the minimum number of nodes in a full tree of height h? $h+1+h=2h+1$



- Is a complete tree a full tree? No
- Is perfect tree a full and complete tree? yes

Binary Tree Properties

- Counting Nodes in a Binary Tree
  - The max number of nodes at level *i* is 2^i (i=0,1,…,h)
  - Therefore total nodes in all levels is

$$n = 1 + 2^1 + 2^2 + \dots + 2^h$$

  - Find a relation between *n* and *h.*

- A **complete tree** of height, *h*, has between 2^h and $2^{h+1}-1$ nodes.

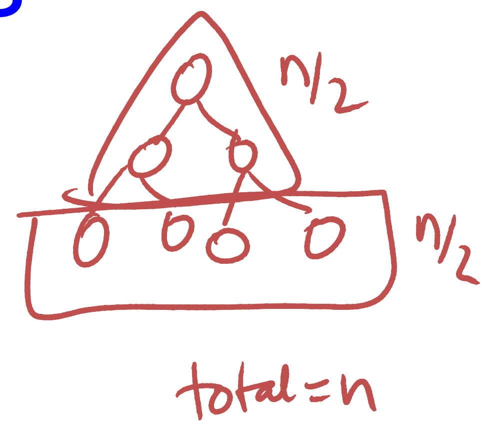
- A **perfect** tree of height h has $2^{h+1}-1$ nodes

Binary Tree Questions

- What is the maximum number of nodes at level k? (root is at level 0)

BST Operations

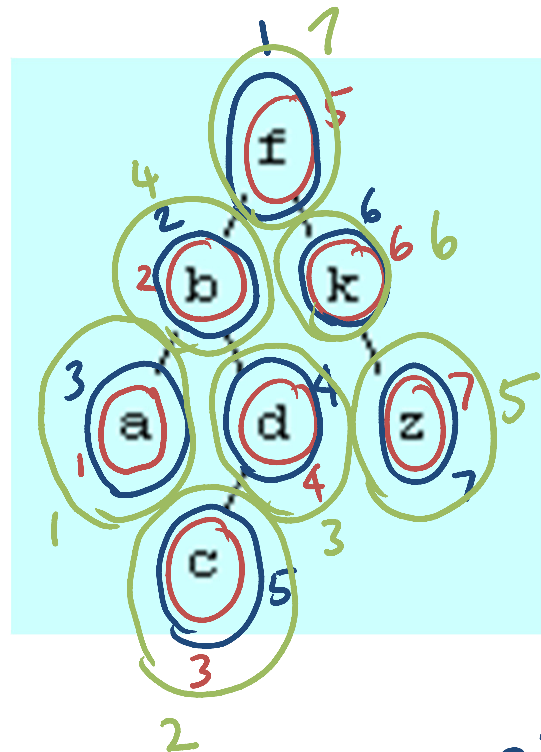
Tree Operations



- Tree Traversals
  - Inorder, PreOrder, PostOrder
  - Level Order
- Insert Node, Delete Node, Find Node
- Order Statistics for BST's
  - Find k^{th} largest element
  - num nodes between two values
- Other operations
  - Count nodes, height of a node, height of a tree, balanced info

Binary Tree Traversals

- Inorder Traversals
  - Visit nodes in the order
    - Left-Root-Right
- PreOrder Traversal
  - Visit nodes in the order
    - Root-Left-Right
- PostOrder Traversal
  - Visit nodes in
    - Left-Right-Root



pre = + 2 3 Prefix
in = 2+3 infix
post = 2 3 + post fix

Inorder Traversal

```
private void inorder(BinaryNode root)
{
      if (root != null) {
         inorder(root.left);
         process root;
         inorder(root.right);
      }
}
```

Preorder Traversal

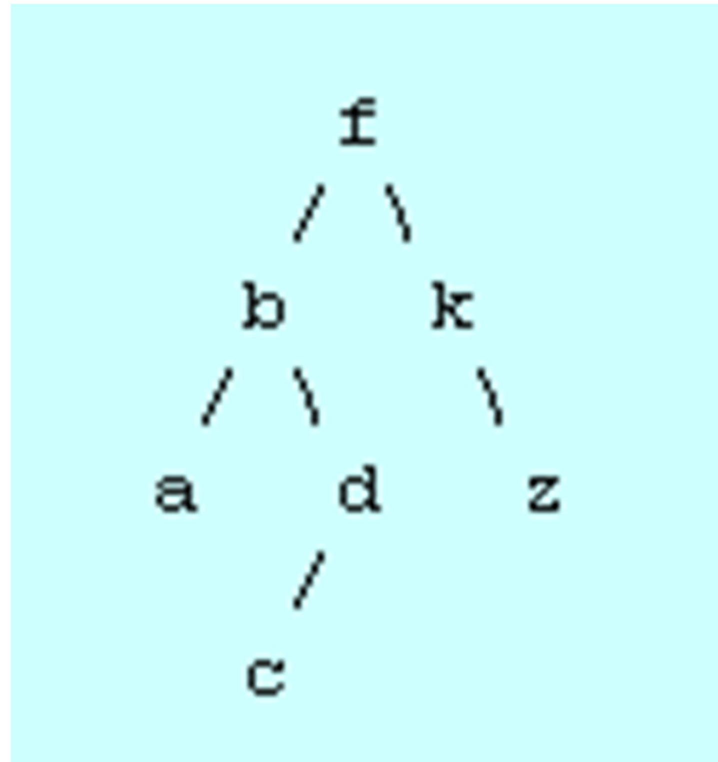
```
private void preorder(BinaryNode root)
{
        if (root != null) {
           process root;
           preorder(root.left);
           preorder(root.right);
        }
}
```

Postorder Traversal

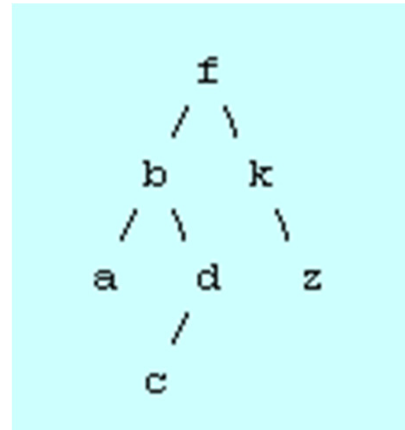
```
private void postorder(BinaryNode root)
{
        if (root != null) {
           postorder(root.left);
           postorder(root.right);
           process root;
        }
}
```

Level order or Breadth-first traversal

- Visit nodes by levels
- Root is at level zero
- At each level visit nodes from left to right
- Called "Breadth-First-Traversal(BFS)"



Level order or Breadth-first traversal



BFS Algorithm

```
enqueue the root
while (the queue is not empty)
{
        dequeue the front element
        print it
        enqueue its left child (if present)
        enqueue its right child (if present)
}
```

Implementation of BST

Insert(N, T)

N → node, T → tree

if $N > T$ insert(N, T.right)

if $N < T$ insert(N, T.left)

if $\underline{T = \emptyset}$ $T = N$

insert(3, $\emptyset$)

Insert(2, $\boxed{3}$)

↓

insert(2, $\emptyset$)

Insert(1, $\boxed{3}$)