Experimental Verification of the Mechanisms for Nonlinear Harmonic Growth and Suppression by Harmonic Injection in a Traveling Wave Tube

A. Singh, J.G. Wöhlbier, J.H. Booske and J.E. Scharer
Electrical and Computer Engineering Department,
1415 Engineering Drive, University of Wisconsin - Madison WI 53706

Understanding the generation and growth of nonlinear harmonic (and intermodulation) distortion in microwave amplifiers such as traveling wave tubes (TWTs), free electron lasers (FELs) and klystrons is of current research interest. It has been widely accepted that similar to FELs, the nonlinear harmonic growth rate scales with the harmonic number in TWTs. Using a custom-modified TWT that has sensors along the helix, we provide the first experimental confirmation of the scaling of nonlinear harmonic growth rate and wavenumber in TWTs. In klystrons, the wavenumber scaling applies to the nonlinear harmonic bunching and associated nonlinear space charge waves. The relative scalings of growth rate and wavelength of a nonlinearly generated harmonic mode versus an injected linear harmonic mode imply that suppression by harmonic injection occurs at a single axial position that can be located as desired by changing the injected amplitude and phase.

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Harmonic (and intermodulation) generation is of significant current interest in traveling wave tubes (TWTs), free electron lasers (FELs), and klystron amplifiers (KLAs) [1–10]. Although these device types differ in their electromagnetic wave guiding properties, they all share common nonlinearities inherent in the physics of ballistic and space charge bunching. In the case of TWTs, for example, a 1-d description would include an equation of motion or force equation, the continuity equation, Gauss' Law, and a wave equation,

$$\frac{\partial v}{\partial t} = -v\frac{\partial v}{\partial z} - \frac{e}{m}\frac{\partial (V_w + V_{sc})}{\partial z}, \tag{1a}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v)}{\partial z}, \tag{1b}$$

$$\frac{\partial^2 V_{sc}}{\partial z^2} = -\frac{\rho}{\epsilon_0}, \tag{1c}$$

$$\frac{\partial^2 V_w}{\partial t^2} - c^2\frac{\partial^2 V_w}{\partial z^2} = cZ_0 A\frac{\partial^2 \rho}{\partial t^2}, \tag{1d}$$

where v is the electron beam velocity, $-e$, m are the electron's charge and mass, ρ is the electron charge density, ϵ_0 is the permittivity of free space, and V_{sc}, V_w are the wave and space charge potentials respectively. The electron beam cross sectional area is represented by A, c represents the phase velocity of a cold circuit wave, and Z_0 represents the interaction impedance.

The near-equivalence of high gain FEL and TWT physics is evidenced by the fact that the 1-d solution for linear growth rates in high gain FELs can be cast in an identical form as Pierce adopted for TWTs using a fluid (Eulerian) treatment for the electron beam. This was first done in [11], as later confirmed in [12]. Reference [13] also derives the Pierce TWT linear dispersion relation for the FEL linear growth rate using a fluid-beam model, although the relationship to TWT physics is not explicitly mentioned. While KLA physics does not include an electromagnetic wave, it shares the same quadratic nonlinear ballistic bunching mechanics described by terms such as $v(\partial v/\partial z)$ in the force equation and by the ρv product in the continuity equation. Such quadratic nonlinearities are responsible for the development of harmonic content in the beam bunching in TWTs [1, 2] as well as FELs and KLAs.

It is, therefore, not surprising that certain observations indicating common dynamics have been made about TWTs, FELs, and KLAs. For example, it has been conventional wisdom that nonlinear harmonic growth rates in TWTs scale approximately with the order of the harmonic. Recently, this was proven analytically for the first time in Ref. [1], wherein it was also shown that there can be exceptions to that conventional wisdom. Also shown in Refs. [1, 2] is the fact that intermodulation distortions arising in TWTs driven by two or more fundamental frequencies evolve from the same quadratic nonlinearities, and therefore, exhibit nonlinear growth rates that scale with the order of the intermodulation product. A similar scaling for harmonic distortion growth rates has recently been described in FEL simulations [3] and analytically derived in Ref. [4]. Experimental measurements of harmonic radiated power versus axial position in FELs have been reported in Refs. [5, 6], but no prior measurements of nonlinear distortion product growth rates have been reported in TWTs.

Harmonic (and intermodulation) distortions are typically unwanted in TWTs or KLAs. One means of suppressing second harmonic distortions in TWTs has been to inject a second wave into the TWT input at the harmonic frequency $2f$, in addition to the power injected at the fundamental frequency f. By varying the amplitude and phase of the signal injected at $2f$, "destructive cancellation" of the $2f$ wave at the output of the TWT can be achieved. It has been demonstrated that a similar technique can be used to suppress intermodulation distortions at the output ports of both TWTs [8, 14] and KLAs [9].

Intuitive insights for the physics of harmonic suppression by harmonic injection were given by Mendel [15] and Garrigus and Glick [16] who speculated what the harmonic signal components might look like internal to the TWT. Figure 1, which is similar to Fig. 4 of Ref. [16], illustrates this view.

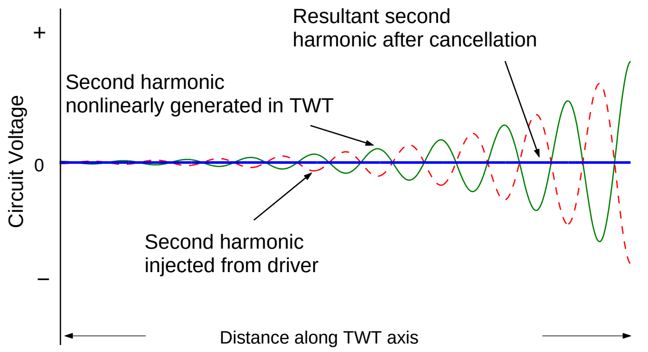

FIG. 1: Earlier hypothesis of mechanism of cancellation by harmonic injection. Similar to Fig. 4 of Ref. [16]. In this view, the injected harmonic cancels the nonlinearly generated harmonic at all positions along the TWT.

Conventional large signal TWT codes ("disk models") have predicted the phenomenon of canceling the second harmonic with harmonic injection [10, 17]; however, the wave at the harmonic frequency in these models cannot be resolved into separate components. Recent theory and numerical simulations [1, 2, 10] have indicated, however, that the harmonic (and intermodulation) distortion suppression by harmonic injection in TWTs results from the fact that the total propagating disturbance of the harmonic (or intermodulation product) is, to a good approximation, a linear superposition of two modes: (1) the nonlinear growing harmonic (intermodulation) mode and (2) a linear growing mode associated with the signal injected at the harmonic (intermodulation) frequency at the TWT's input. This is represented by the analytic solution for the total disturbance at the second harmonic as given by the S-MUSE 1-d nonlinear spectral TWT model [1, 2, 18], considering only the dominant modes,

$$V_{2f}(z,t) = \left\{ A^{\rm dr} e^{(\mu^{\rm dr}+i\kappa^{\rm dr})z} + A^{\rm nl} e^{(\mu^{\rm nl}+i\kappa^{\rm nl})z} \right\} e^{i2\pi f\left(\frac{z}{u_0}-t\right)}, \quad (2)$$

where u_0 is the dc beam velocity, the superscript 'dr' refers to the "driven" or "linear" mode, and the superscript 'nl' refers to the mode at the harmonic frequency $2f$ generated by "nonlinear interactions." This verifies the earlier intuitive notions that the harmonic suppression is a result of destructive interference of the injected harmonic with the nonlinearly generated harmonic. However, the earlier notions envisioned that cancellation occurs at all points along the tube, as is clear from Fig. 1. As indicated in Eq. (2), the linear and nonlinear modes have different growth rates $(\mu^{\rm dr}, \mu^{\rm nl})$ and different wavenumbers $(\beta^{\rm dr} = \kappa^{\rm dr} + 2\pi f/u_0, \beta^{\rm nl} = \kappa^{\rm nl} + 2\pi f/u_0)$. Consequently, cancellation can occur at only one position, which is determined by the input amplitude and phase of the injected signals. This is illustrated in Fig. 2 which is a plot of the S-MUSE analytical solution at the harmonic frequency, Eq. (2).

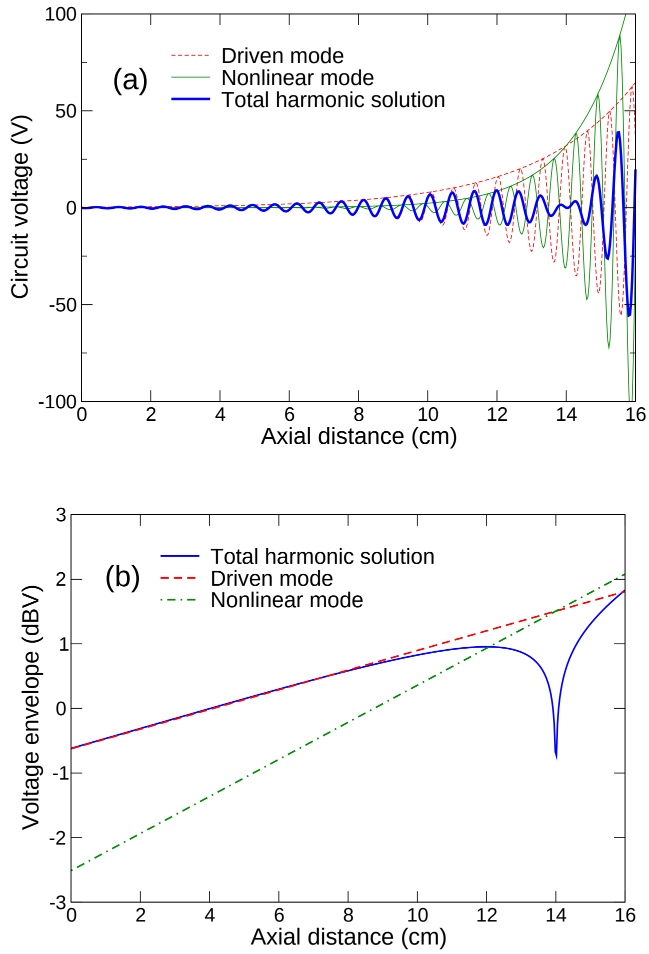

FIG. 2: Illustration of second harmonic suppression by second harmonic injection in a TWT using Eq. (2). Destructive interference of the driven and nonlinear harmonic wave modes results in cancellation of the total solution at a single axial location. The two modes and their sum is shown in (a) on a linear scale, while (b) shows component and sum envelope magnitudes on a log scale. A plot of circuit voltage phase (not shown here) would show an abrupt change of 180° at the cancellation point [10].

Figure 2(a) clearly reveals how the suppression results from a destructive cancellation effect of two modal components with different growth rates and wavenumbers. Figure 2(b) shows the evolution of the envelopes of the second harmonic modes and their sum versus z. It can be seen that the driven mode dominates the solution prior to cancellation, while the nonlinearly generated mode dominates after the cancellation position.

Therefore, it should not be surprising that theory predicts these two modes should destructively interfere at exactly and only one point along the interaction. In fact, this single-point cancellation feature should occur in similar experiments in FELs. Even in KLAs, where the dynamics only involve ballistic beam bunching, an injected beam modulation (which may be represented by two constant amplitude space charge waves) can be made to cancel the nonlinear beam modes ("nonlinear space charge waves") at a single point [9]. What separates the physics of harmonic distortions in TWTs from that of FELs or KLAs, however, is that in the latter devices, both nonlinear and linear mode wavenumbers scale with the frequency of the excitation (e.g., the wavenumber of a second harmonic excitation is approximately twice the wavenumber of the fundamental, $\beta_{2f} = 2\beta_f$, regardless of whether the excitation is a linear or nonlinear mode). In contrast, recent TWT theory predicts that the wavenumber of a nonlinear excitation will differ significantly from the linear excitation [1, 2]. Specifically, the nonlinear mode's wavenumber can be expected to scale approximately with the frequency of the excitation, as with KLAs or FELs, but the linear mode's wavenumber is predicted to differ significantly from such scaling due to the effect of the slow-wave on the beam and the effect of the waveguide's dispersion on the slow-wave.

Using a custom-manufactured TWT, we have been able to experimentally confirm that harmonic suppression by harmonic injection occurs at only one position along the TWT interaction, and that this cancellation point moves as the input signal parameters are appropriately varied. Comparison of the measurements with numerical simulation (see Fig. 3) shows excellent agreement with the theoretically predicted location of suppression. These experiments, therefore, represent the first experimental confirmation of the predicted scalings for nonlinear excitation growth rates and wavenumbers compared with linear excitations.

The experimental device used is a custom-modified research TWT, the XWING (eXperimental WIsconsin Nothrop Grumman) TWT [8], that has multiple sensors along the helix to measure the power in the RF wave as it propagates along the TWT. The sensors are coupled capacitively to the helix at approximately $-40\,\mathrm{dB}$ to avoid significant perturbation of the circuit fields. A drive frequency of 2 GHz was used with 15 dBm input power which corresponds to operation of the XWING close to 1 dB gain compression. Figure 3(a) shows measurements of the evolution of the nonlinearly generated second harmonic without harmonic injection. Next, an injected harmonic at 4 GHz was optimized first for harmonic suppression at the output ($z \approx 14$ cm) as shown in Fig. 3(b), and second at one of the sensors (sensor 4, $z \approx 12.5$ cm) as shown in Fig. 3(c). We found that maximum suppression can be achieved at only a single axial location, and that re-optimizing the injected amplitude and phase moves the maximum suppression point to a different axial location.

In Fig. 3, experimental data are compared to predictions from the LATTE "large signal code" [18, 19]. It has been shown in Refs. [1, 2] that the LATTE code and the S-MUSE theory of Eq. (2) are in very good agreement in describing the scalings of the growth rates and wavenumbers of harmonic and intermodulation distortions.

The experiments demonstrate that maximum suppression occurs at only one axial location and that this location can be shifted by changing the input power and phase of the injected harmonic wave. Thus the experiments confirm the theoretical principle of Eq. (2), that the resultant harmonic wave consists of two modes with different growth rates and wavelengths. In fact, the agreement of experimental and simulation results on the location of suppression is only possible if the theory and experiment are in precise agreement on the relative scalings of the linear versus nonlinear growth rates and wavenumbers. The discrepancies between the experimentally measured harmonic powers and the simulated values have been identified as most likely due to 3-d beam effects (e.g. scalloping) [20]. This would not significantly alter the growth rate or wavenumber scalings, but it would readily explain discrepancies between the absolute power levels on the sensors and computer code predictions.

While Eq. (2) derived from the S-MUSE model is only valid prior to saturation, experimental results and large-signal simulations using the Lagrangian code LATTE are valid for all drive regimes. However, the physics of Eq. (2) is still inherent in the LATTE simulations, at least prior to saturation where S-MUSE and LATTE have been shown to agree. Interestingly, simulations using LATTE in Ref. [10] indicate that the same superposition-of-modes picture applies in saturation as well. The S-MUSE model's value, however, has been the enabling of an analytic solution (e.g., Eqs. (1) and (2)) that clearly reveals the two interfering (linear, nonlinear) modes and explains how their different growth rate and wavenumber scalings conspire to produce the phenomena of harmonic suppression by harmonic injection.

To conclude, this paper presents the first experimental evidence of growth rate and wavenumber scaling for a nonlinearly generated harmonic versus a linear excitation in a TWT. This observation is analogous to similar scalings of nonlinear products in FELs and KLAs, with certain differences unique to the TWT.

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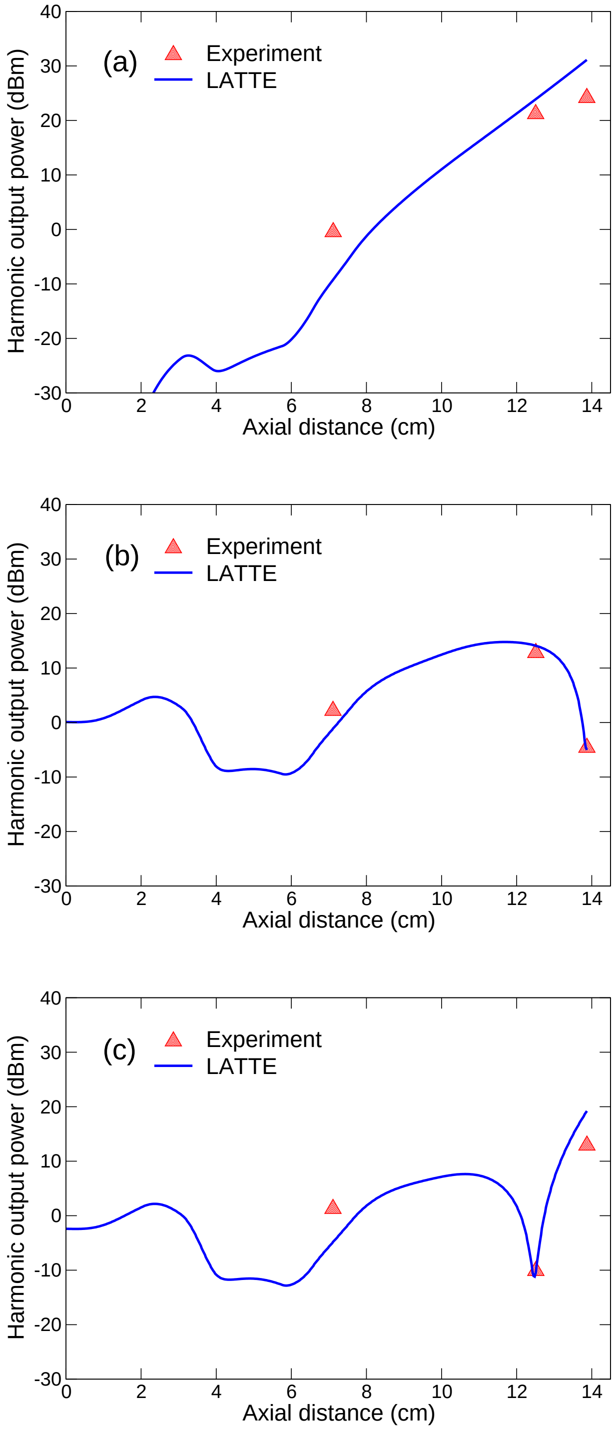

FIG. 3: Experimental and numerical evolution of second harmonic (a) without injection, (b) with harmonic injection obtaining 29 dB suppression at output, and (c) with harmonic injection obtaining 31 dB suppression at sensor 4. (Note that the attenuation experienced by the wave over $z \approx 4-6$ cm is attributed to a circuit sever and is not a result of suppression due to the injected wave.)

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