15-251: Great Theoretical Ideas In Computer Science
Recitation 2

Training Manual

- **Deterministic Finite Automaton (DFA):** A DFA $M$ is a machine that reads a finite input one character at a time in one pass, transition from state to state, and ultimately accepts or rejects. Formally, $M$ is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where $Q$ is the finite set of states, $\Sigma$ is the finite alphabet, $\delta : Q \times \Sigma \rightarrow Q$ is the transition function, $q_0 \in Q$ is the starting state, and $F \subseteq Q$ is the set of accepting states.

- **Regular language:** A language $L$ is regular if $L = L(M)$ for some DFA $M$ ($M$ decides $L$).

- **Turing Machine (TM):** A TM $M$ is a machine that can read and write to an infinite tape containing the input, transition from state to state, and ultimately accepts or rejects. Formally, $M$ is a 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$, where:
  - $Q$ is the finite set of states,
  - $\Sigma$ is the finite input alphabet with $\sqcup \notin \Sigma$,
  - $\Gamma$ is the finite tape alphabet with $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$,
  - $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$ is the transition function,
  - $q_0 \in Q$ is the starting state,
  - $q_{acc} \in Q$ is the accepting state,
  - and $q_{rej} \in Q$ is the rejecting state.

- **Decider TM:** A TM $M$ is a decider if it halts on all inputs.

- **Decidable language:** A language $L$ is decidable (or computable) if $L = L(M)$ for some decider TM $M$.

**Odd Ones Out**

Draw a DFA that decides the language

$$L = \{x : x \text{ has an even number of 1s and an odd number of 0s}\}$$

over the alphabet $\Sigma = \{0, 1\}$.

**Ones Too Many**

Show that the language $L = \{1^n \mid \log_2(n) \in \mathbb{N}\}$ over the alphabet $\Sigma = \{1\}$ is not regular.

**Busy Intersection**

(a) Prove that if $L_1$ and $L_2$ are regular, then $L_1 \cap L_2$ is regular.

(b) Using (a), show that $L = \{w : w \text{ has the same number of 0s and 1s}\}$ over the alphabet $\Sigma = \{0, 1\}$ is not regular.
**Balance in All Things**

Construct a TM that decides the language $L = \{ x : \text{the parentheses in } x \text{ are balanced} \}$ over the alphabet $\Sigma = \{ (, ) \}$.

**Closure Ceremony**

Suppose that $L_1$ and $L_2$ are decidable languages. Show that the three languages $L_1 \cup L_2$, $L_1 \cdot L_2$ and $L_1^*$ are all decidable as well.